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A Study of Directional Patterns of Ultrasonic Parametric Array

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The paper presents results of numerical calculations and experimental data on the directional pattern of two 38-element parametric arrays composed of ultrasound sources. Two types of antenna arrays are considered, namely with parallel and coaxial connections of ultrasonic transducers (elements). The results of selecting and functional testing of unit elements are described in this paper. It is found that in the coaxial element connection of the antenna array, the level of side lobes is higher than that in the parallel element connection.

Keywords: antenna array; ultrasonic transducer; primary frequency; differential frequency; parametric array.

1. Introduction

First findings in the field of parametric arrays were published in the 1960's and 1970's. They were based on the idea of the non-linear interaction of acoustic waves. The work of Westervelt (1963) studied the interaction area between two high-frequency signals and the generation of the differential frequency signal caused by the nonlinearity of the wave propagation medium. The theoretical justification of the parametric effect was given in Berktay (1965). At that date in Russia, our researchers were also involved in the similar investigations (Zabolotskaya *et al.*, 1966; Zverev, Kalatchev, 1970; Rudenko, 1974; and others).

Due to the fact that it was considered to be very difficult to generate a parametric effect in air, the above-mentioned works developed the theory of parametric arrays for underwater research. In 1975, researchers Bennett and Blackstock (1975) described the possibility of generating a parametric effect in air.

The work of Japanese scientists (Yoneyama, Fujimoto, 1983) was devoted to the creation of the aerial parametric array assembled from piezoceramic acoustic transducers. The special mention must be made of the contribution of Japanese researchers to the investigation of various configurations of aerial parametric arrays, that is supported by the works of Aoki *et al.* (1991) and Kamakura *et al.* (1994).

Different configurations of aerial parametric arrays and their investigations were presented in Gan *et al.* (2011), Alunno and Yarce Botero (2016), and Gan *et al.* (2012) along with measuring results on directional patterns of created parametric arrays and the sound pressure level observed at differential and primary frequencies, which depends on the distance to the transducer. Another example is the work of Gudra and Opieliński (2002), which gives measuring results on directional patterns for two different parametric arrays operating at the primary frequencies of 40 and 160 kHz, respectively. One parametric antenna array comprises two channels. Each channel consists of ultrasound transducers (elements) and receives the signal of the appropriate primary frequency.

In all these publications, the experimental findings are mostly related to measurements of the directional pattern of parametric arrays. Several publications present the results of theoretical calculations and experimental data, however, there is no detailed interpretation of these findings. As it is known, the construction of the antenna array including the parametric array, requires the selection of its optimum parameters, namely the element arrangement, the aperture amplitude distribution, and the element spacing. These parameters influence both the width of the main lobe and the level of the side lobe of the directional pattern. In practical design of the antenna array, it is advisable to consider the individual frequency response of the unit elements, whose parameters are different due to their production quality. Therefore, it is relevant to combine the optimum configuration of the antenna array with the specified parameters of its directional pattern.

The aim of this paper is to study the directional patterns of two types of 38-element parametric arrays with parallel and coaxial connections of the unit elements. General information is given to the directional pattern of antenna arrays and results of its theoretical calculations.

2. Methods and equipment

A parametric array is an array whose operation is based on the non-linear interaction of the high frequency signals with the wave propagation medium. The formation of the differential frequency in the parametric array occurs in two cases, at the interaction between two high frequency acoustic waves and at the amplitude-modulated high frequency primary wave. In this work we study the first variant of the array configuration. In this experiment, we design two antenna arrays 10 × 10 cm in size, with different element connection patterns. The next subsection describes the selection of elements, their connections and research procedure.

2.1. Selection of ultrasound transducers

Piezoceramic ultrasound transducers are usually used for the formation of parametric arrays and should correspond to such criteria as functioning at continuous signal, creation of high acoustic pressure, and operation at specified primary frequencies.

A broad range of piezoceramic ultrasound transducers is widely marketed. They differ in the maximum level of the developing sound pressure, sensitivity of received signal, and resonance frequency. So, the selection of a transducer becomes complicated. Besides, the instability of parameters is observed when selecting transducers of the same designation. The spread of parameters is mostly connected with the process conditions, i.e. cleanliness of materials used, stability of temperature conditions etc.

Different types of piezoceramic ultrasound transducers are analyzed, and their parameters are given in Table 1. For the selection of the required transducer, we considered ten transducers of each designation, and conducted test measurements of the sound pressure level and resonance frequency stability. It is found that the type 400SR160 ultrasound transducer from Pro-Wave Electronics Corporation (Taiwan) possesses the most stable parameters, and is, therefore, suitable for our experiment.

Figure 1 presents the diagram of the resonance frequency of 200 ultrasound transducers of the type 400SR160. According to this figure, the selected transducers possess such stable parameters as the resonance frequency and the sound pressure level within this frequency.

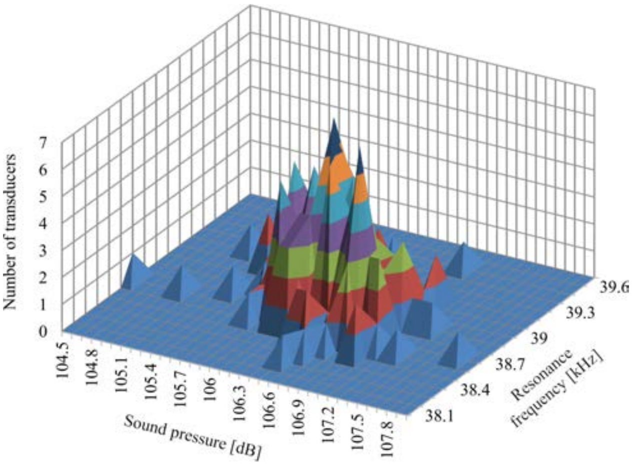

Fig. 1. Testing results for resonance frequency of ultrasound transducers.

76 ultrasound transducers having parameters: resonance frequency 39 kHz ± 200 Hz, sound pressure 106 dB ± 0.3 dB (at 10 V_{rms}), are used in the experi-

Table 1. Parameters of ultrasound transducers.

Parameters	Transducer types			
	MA40S4S	TR2516R1/T1	MCUSD16A40S12RO	400SR160
Resonance frequency [kHz]	40±1	25	40	39±1
Max sound pressure level [dB]	120	115	110	120

ment. This number of transducers allows us to create two antenna arrays with a hexagonal arrangement of elements such that the size of the array does not exceed 105 × 108 mm.

2.2. Experimental models of parametric arrays

Two types of 38-element parametric arrays are proposed. The first type consists of two circuits with 19 elements each and relates to the parallel connection pattern. The second type includes the inner circuit with 18 elements and the outer circuit with 20 elements and relates to the coaxial connection pattern. The model of the element arrangement in the parametric array with 38 elements is shown in Fig. 2.

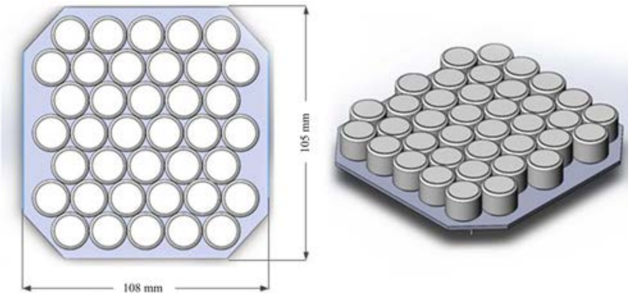

Fig. 2. Three-dimensional model of parametric array with 38 elements.

According to Fig. 3, the parallel connection pattern includes Circuit 1 and Circuit 2, whereas the coaxial connection includes the inner Circuit 1 and the outer Circuit 2.

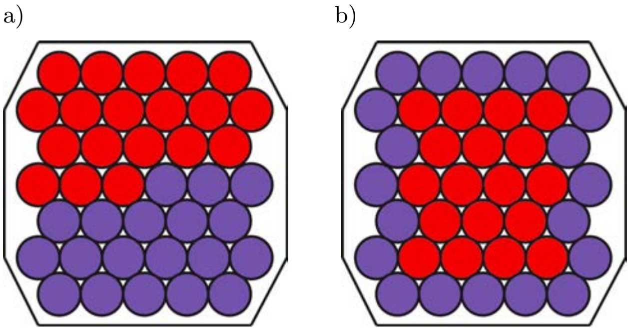

Fig. 3. Element connection patterns: a) parallel connection; b) coaxial connection. Red and violet balls indicate Circuit 1 and Circuit 2, respectively.

2.3. Research procedure

The first-order sound level meter Ekofizika-110HF (Ekophizika-110A, n.d.a.) with the ultrasound measuring module was used as a receiving measuring device. Also, a precision capacitance microphone was used to measure the sound pressure level within the ultrasonic frequency range. A signal generator was used for generation of signals received by the parametric array, which were then recorded by the oscilloscope. The flow chart of the experimental setup is presented in Fig. 4.

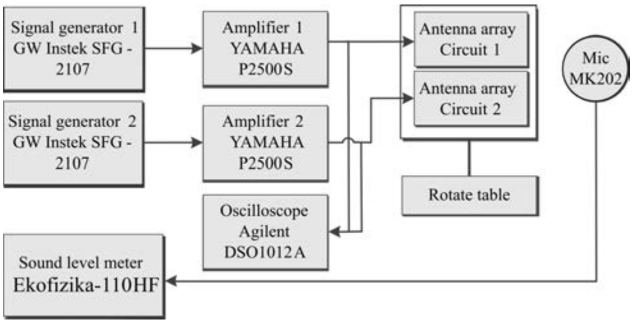

Fig. 4. Flow chart of the experimental setup.

The aim of this work is to study the directional patterns of two types of parametric arrays with different patterns of the element connection. Investigations were carried out in the laboratory almost without ambient acoustic noise.

3. Comparison of experimental data with theoretical calculations

3.1. Theoretical basics of antenna array construction

Theoretical calculations of the antenna array directional pattern in the far-field region are reduced to the computation of the acoustic field which represents the sum of fields of each element. When the elements are identical and have an equally oriented field, their directional pattern can be found from

$$D(\theta,\varphi) = D_1(\theta,\varphi)\cdot D_\Sigma(\theta,\varphi), \tag{1}$$

where $D_1(\theta,\varphi)$ is the directional pattern of the unit element, $D_\Sigma(\theta,\varphi)$ is the antenna array pattern.

For the type 400SR160 ultrasound transducer the directional pattern is:

$$D_1(\theta,\varphi) = \frac{2J_1(k\cdot a\cdot\sin(\theta))}{k\cdot a\cdot\sin(\theta)}\cdot\frac{2J_1(k\cdot a\cdot\sin(\varphi))}{k\cdot a\cdot\sin(\varphi)}, \tag{2}$$

where J_1 is the Bessel functions, a is the transducer diameter.

In this way, the antenna array pattern for the flat array of $m\times n$can be written as

$$D_\Sigma(\theta,\varphi) = \sum_{i=0}^{m-1}\sum_{g=0}^{n-1}|P_{i,g}|\,e^{jk\cdot(i\cdot d_x\sin(\theta)\cos(\varphi)+g\cdot d_y\sin(\theta)\sin(\varphi))}, \tag{3}$$

where $P_{i,g}$ is the amplitude of the unit element, d_x and d_y are distances between the neighbour elements on OX- and OY-axes, i and g are coordinates of the unit element.

Figure 5 shows the geometry for calculating the angles for the antenna array. The antenna arrays considered in the article have a triangular array lattice.

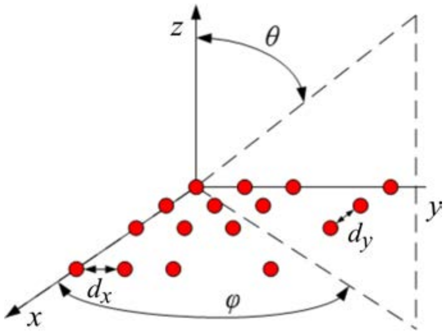

Fig. 5. Directional patterns of antenna array with parallel element connection, at primary and differential frequencies.

Within the far-field region of the parametric array, the differential frequency wave appears due to the interaction between the divergent waves of the primary frequency. The directional pattern on the differential frequency is the product of the directional patterns of the parametric array on the primary frequency (Gan *et al.*, 2006):

$$D(\theta)_s = D_1(\theta)D_2(\theta), \tag{4}$$

where $D_1(\theta)$, $D_2(\theta)$ are the directional patterns of the parametric array on the primary frequency.

3.2. Experimental results

Figures 6 and 7 depict measurement results for the directional patterns of antenna arrays with respectively parallel and coaxial element connections.

Consider Figs 8 and 9 which show the results of theoretical calculations and experimental data for Cir-

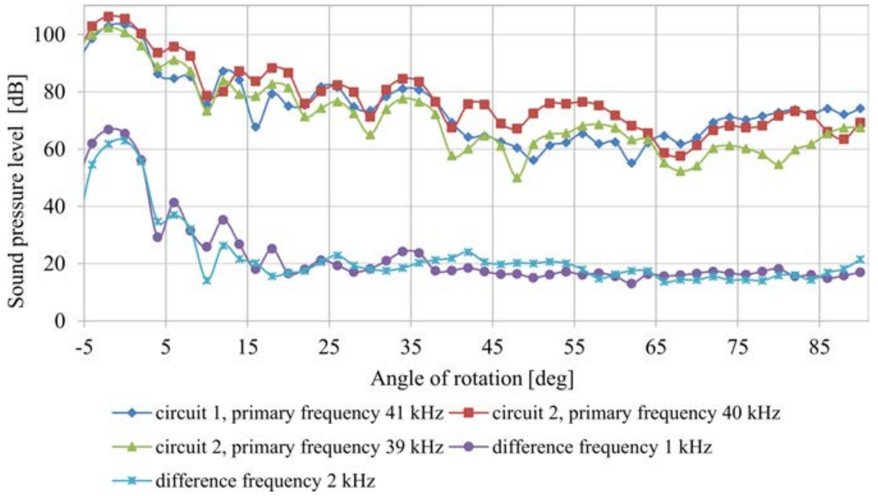

Fig. 6. Directional patterns of antenna array with parallel element connection, at primary and differential frequencies.

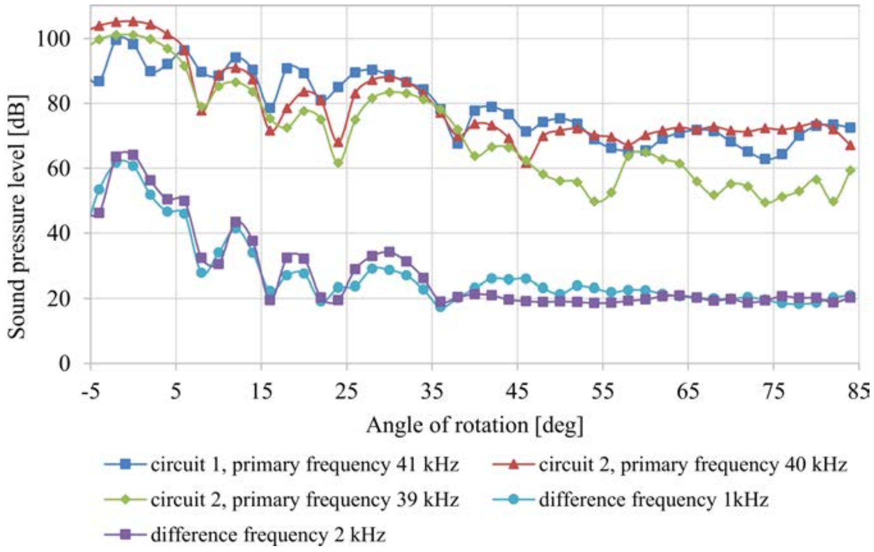

Fig. 7. Directional patterns of antenna array with coaxial element connection, at primary and differential frequencies.

cuit 1 with the parallel element connection and inner Circuit 1 with the coaxial element connection.

Figure 10 presents the results of theoretical calculations and experimental data on the directional pattern

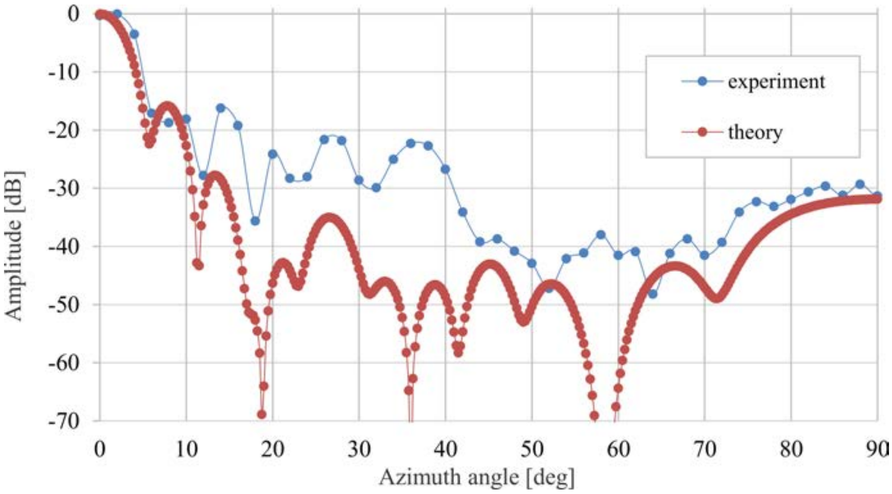

Fig. 8. Comparison of theoretical calculations and experimental data for Circuit 1 with the parallel element connection at 41 kHz frequency.

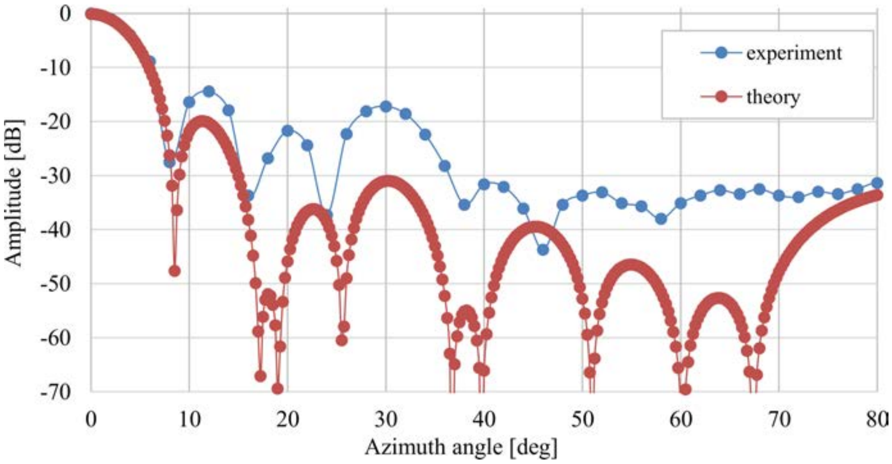

Fig. 9. Comparison of theoretical calculations and experimental data for Circuit 1 with the coaxial element connection at 40 kHz frequency.

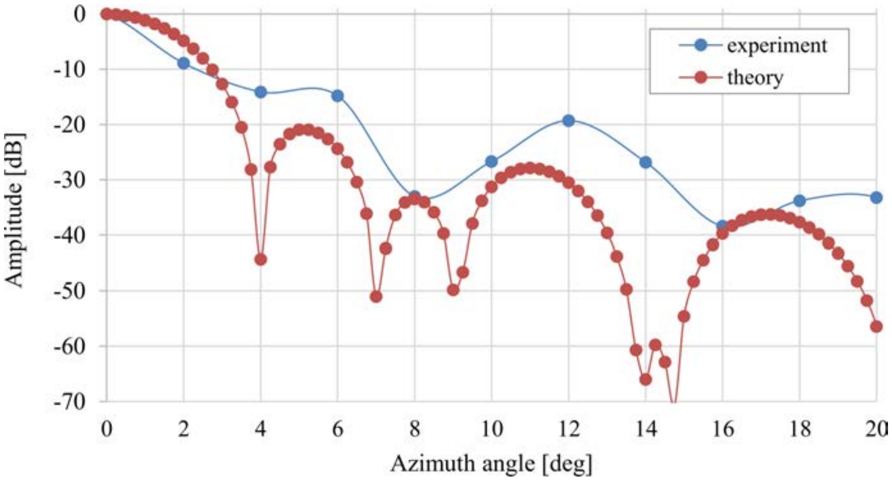

Fig. 10. Comparison of theoretical calculations and experimental data for coaxial element connection at 2 kHz differential frequency.

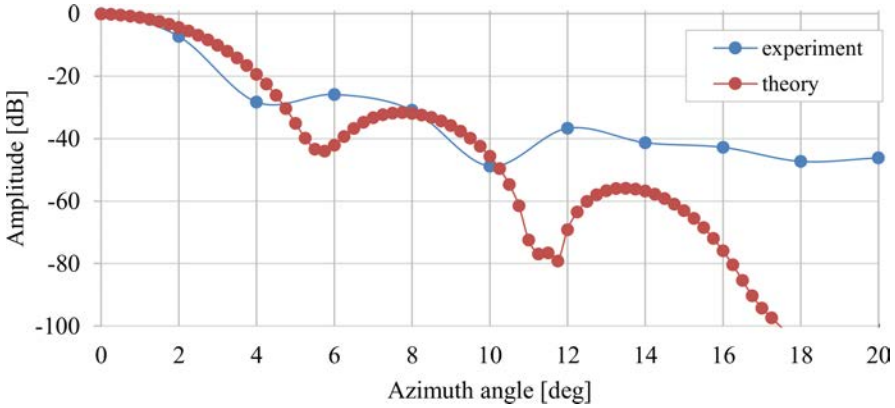

Fig. 11. Comparison of theoretical calculations and experimental data for parallel element connection at 2 kHz differential frequency.

on the differential frequency of 2 kHz for the inner Circuit 1 with the coaxial element connection within 0–20 degrees of the selected azimuth angle.

Figure 11 presents the results of theoretical calculations and experimental data on the directional pattern on the differential frequency of 2 kHz for the Circuit 1 with the parallel element connection within 0–20 degrees of the selected azimuth angle.

As it can be seen from Figs 9 and 10, the coaxial element connection is less suitable for the use in directional loudspeakers or the data transmission because of its higher level of side lobes which is also higher in the channels of this connection pattern.

4. Discussion

In this work, the focus of attention is two 38-element parametric arrays with different patterns of the element connection. It is shown that the level of side lobes in the coaxial element connection is higher than that in the parallel. This difference occurs due to the fact that in the coaxial connection, the both outer and inner circuits possess the differential pattern with a higher level of side lobes than that in the parallel connection. A careful comparison between our calculations and experimental results shows good agreement within 0–20 degrees of the selected azimuth angle. Within the range of 20–90 degrees of the azimuth angle, we observe the same waveform, experimental values being higher than theoretical ones.

A good agreement between theoretical calculations and experimental data is established nearby the main and the first lobes of the directional pattern in both circuits. Table 2 summarizes the correlation factors for theoretical and experimental results on the element connection patterns of the parametric array.

The difference between data presented within 15–90 degrees of the azimuth angle is conditioned by the experimental conditions which include the parameters spread of the unit elements and also probable acoustic wave re-reflections from the laboratory walls.

Table 2. Correlation factors for theoretical and experimental results.

Frequency (circuit)	Azimuth angle [deg]			
	0–20	20–50	50–90	0–90
Parallel element connection				
41 kHz (Circuit 1)	0.80	0.19	0.61	0.70
40 kHz (Circuit 2)	0.83	0.27	−0.33	0.64
39 kHz (Circuit 2)	0.87	−0.02	−0.37	0.67
1 kHz (differential frequency)	0.92	−0.17	0.23	0.80
2 kHz (differential frequency)	0.84	−0.10	−0.10	0.75
Coaxial element connection				
41 kHz (inner Circuit 1)	0.52	0.26	−0.07	0.60
40 kHz (outer Circuit 2)	0.95	0.20	−0.03	0.67
39 kHz (outer Circuit 2)	0.97	0.29	0.20	0.75
1 kHz (differential frequency)	0.89	0.33	0.38	0.77
2 kHz (differential frequency)	0.87	0.06	−0.64	0.75

5. Conclusions

In this paper, we considered the theoretical calculations of the directional pattern of the parametric array both on primary and differential frequencies. The obtained experimental results showed a good agreement with theoretical ones. In our next investigations, we plan to improve the power of parametric arrays due to the increase in the number of elements and their optimum arrangement. The proposed parametric arrays are intended for the creation of loudspeakers with concentrated sound beam.

The parameters of the proposed antenna arrays include a 3 dB width of the directional pattern for both element connection patterns; 1 and 2 kHz differential frequency equalling 4 ± 0.2 degrees; −20 dB level of the first side lobe for coaxial connection; −25 dB level of the first side lobe for parallel connection. The first side lobe locates within 5–15 degrees of the selected azimuth angle, whereas the second side lobe and all the consecutive ones locate below −30 dB in both element connections of the parametric array.

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