METHODS OF DC/DC CONVERTER TRANSFER FUNCTION MEASUREMENTS, BASED ON DATA ACQUIRED IN THE TIME DOMAIN

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Abstract

The measurement of frequency characteristics, like magnitude and phase, related to a specific transfer function of DC–DC converters, can be a difficult task – especially when the measured signal approaches the boundary of a small-signal model validity (*i.e.* 1/3 of the switching frequency f_S).

It is hard to find a paper where authors mention the measurement techniques they use to draw frequency characteristics. Meanwhile the presence of noise in the output signal does not enable to directly measure the gain and the phase shift between the input and output signals. In such situations additional analysis is required in order to achieve a reliable result. This paper contains a description of a few methods that can be used to analyse measured signals in order to determine the gain and the phase shift of a specific transfer function. They enable to verify mathematical models in a wide range of frequencies (up to 1/3 f_S). The methods use signals measured in the time domain and can be implemented in mathematical software such as Matlab or Scilab.

Keywords: DC–DC converters, frequency characteristics, transfer function measurement.

1. Introduction

Verification of small-signal models, that are used to describe DC/DC converters, is usually done by comparison of the theoretical and measured frequency characteristics of a specific transfer function. Sometimes, in order to verify a model, a large-signal simulation is performed in software such as OrCAD or SABER [1]. The theoretical values of gain and phase shift are calculated based on a specific small-signal model [2, 3, 5] while the measured characteristics can be acquired either with specialized frequency analysers or by analysis of input and output signals in the time domain. In this paper the input signal v_{IN} and the output signal v_{OUT} refer to signals corresponding to a specific transfer function (*e.g.* using the notation presented in Fig. 1 the input and output signals for the line-to-output transfer function are the small-signal components of the input voltage source v_G and the output voltage v_O, respectively). As for the type of the input signal it can be either a pure sinusoid or a multi-frequency signal. When using a pure sinusoid, each point of a frequency

characteristic is measured separately – which takes time, but also gives the most accurate results [6]. On the other hand, a multi-frequency signal (such as a Dirac pulse) enables fast evaluation of frequency characteristics [6, 7], but also seems to be less accurate at higher frequencies.

Nevertheless, drawing a frequency characteristic based on signals acquired in the time domain can be a challenging task if a specialized frequency response analyser is not available. The reason for that is noise, related to the switching frequency f_S. The noise is actually a residue that is left after the inductor current ripple is filtered by the output capacitor, therefore it contains harmonics higher than the switching frequency, though the harmonics related to the switching frequency have the largest amplitude and thus the highest impact on the measured signals. The noise exists in an output voltage, and it becomes noticeable for the signal frequencies which are far below the boundary of a small-signal model validity (*i.e.* 1/3 of the switching frequency) (Fig. 2a). If the amplitude of the noise is relatively small it is still possible to determine the gain and the phase shift between the input and output signals. For higher frequencies of the input signal the output signal amplitude is attenuated to a level which is comparable to or even much smaller than the noise amplitude (Fig. 2b). Low SNR (*Signal to Noise Ratio*) makes the phase measurement impossible. The input signal v_{IN} presented in Figs. 2a and 2b has been scaled, therefore its amplitude should not be treated as the actual amplitude of the input voltage.
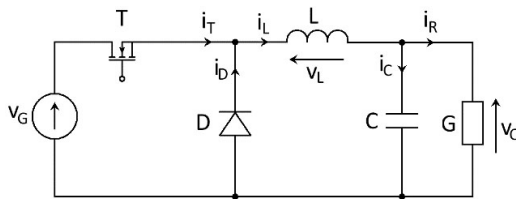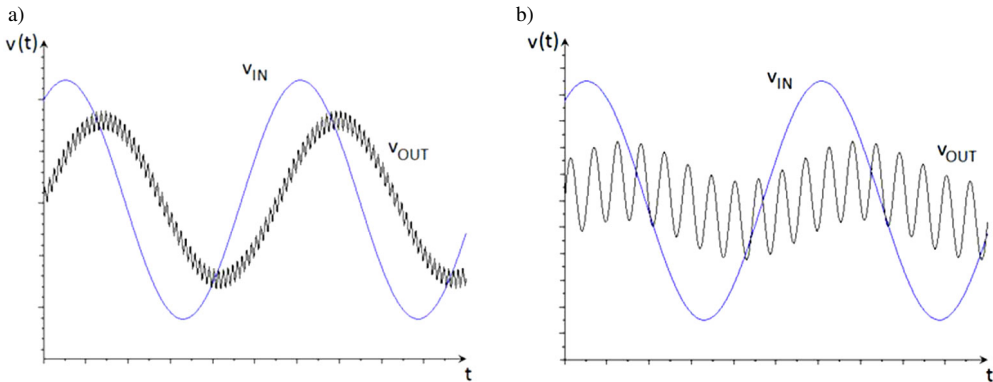

Fig. 1. An example of a BUCK converter.

Fig. 2. Input and output signals in a BUCK converter a) frequency of the input signal was equal to $1/50\,f_S$; b) frequency of the input signal was equal to $1/5\,f_S$.

The problem of the presence of noise in the output signal can be solved by additional calculations performed on the measured data. More detailed information regarding the calculations are presented in the following sections.

2. Methods of transfer function measurement

2.1. Averaging waveforms over one switching cycle

Small-signal models of DC–DC converters are calculated by averaging currents and voltages over one switching cycle [2–4]. The same idea can be used to remove the high-frequency noise from a signal measured in the time domain. In order to do so, the samples of the measured signal, contained in one switching cycle (T_S), should be replaced with a single point that represents an averaged value of the waveforms over the cycle (Fig. 3a). The point should be placed in the middle of the corresponding cycle in order to avoid an additional phase shift. If this requirement is not fulfilled, an additional phase shift will appear in a frequency characteristic. The discrepancies between theoretical and measured values will be noticeable for relatively low frequencies (above $1/10\ f_S$).
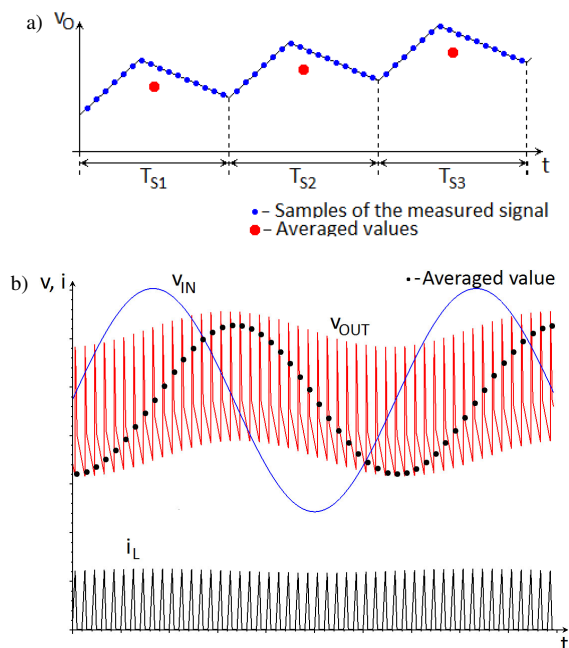
Fig. 3. The idea of averaging the measured signal over one switching cycle:
a) the principle; b) an example of signals in a BOOST converter.

All of the points that represent the averaged values form a curve which subsequently can be used to determine the gain and phase shift between the input and output signals (Fig. 3b). The described algorithm is easy to implement, but unfortunately it has some drawbacks. To be specific, each switching cycle is represented by a single point only. The number of switching cycles existing in one period of the output signal decreases with an increase of the signal frequency. When the frequency approaches higher frequencies, the number of averaged points drops to a level which is insufficient to determine the accurate value of the phase shift (Fig. 4). Large discrepancies can be observed for the input frequencies higher than 1/5 of the switching frequency.

Figure 4 shows that the averaging method described in this section should not be used for high frequencies (above $1/5\ f_S$). Moreover, the method can be used only for converters with a fixed
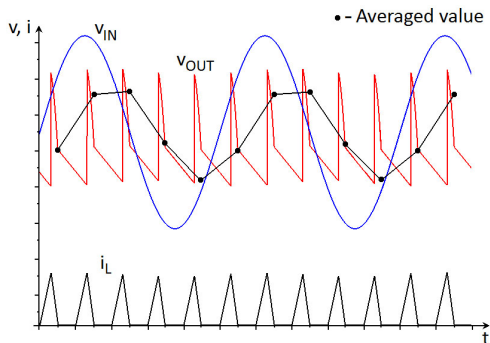
Fig. 4. Averaging over one switching cycle when the input signal frequency
in a BOOST converter is equal to 1/5 of the switching frequency.

switching cycle; therefore, its use is limited to converters with a constant switching frequency. The next section describes a different method that can be used to obtain frequency characteristics based on signals measured in the time domain.

2.2. Zero-phase filtration (ZPF)

The zero-phase filtration has been used in [8] to calculate frequency characteristics presented there. The filtration requires a few operations which can be done with mathematical software such as Scilab or Matlab (the last one contains a built-in function named *filtfilt* that can perform the filtration). The algorithm of the filtration is presented in Fig. 5.
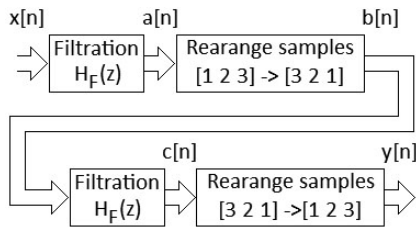

Fig. 5. The algorithm of zero-phase shift filtration.

First, a simple filtration is done with a low-pass filter that has a transfer function H_F. The result of the filtration is moved to the next step where the order of data is changed – the last sample becomes the first one etc. After that another filtration is performed with the same filter H_F. The output data of the second filtration are then rearranged once again in order to obtain the final result. An example of unfiltered and filtered data is presented in Fig. 6.

The digital zero-phase-shift filtration does not affect the phase of a measured signal. The reason is that the filtration is performed two times in "both directions" of the acquired data, therefore the phase shifts which appear after each filtration cancel each other. As for the magnitude, a filter with right parameters must be selected in order to suppress the unwanted frequencies and leave the magnitudes of relevant frequencies unchanged.

Using the ZPF can help suppress the unwanted noise, and thus enables to calculate the gain and the phase shift between the input and output signals. Unfortunately, sometimes the result of
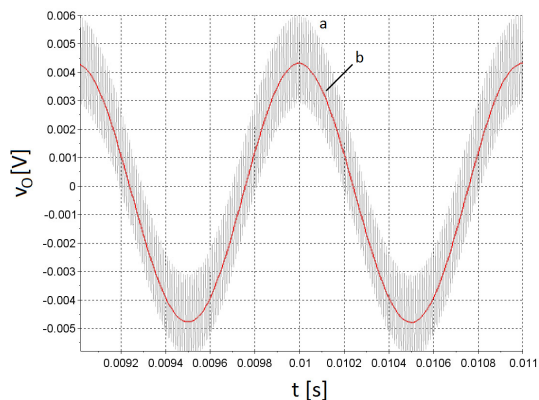
Fig. 6. Comparison of (a) unfiltered and (b) filtered data.

filtration can still suffer from the high-frequency noise which can affect calculation of the gain and phase (Fig. 7). Such noise is observed especially in converters where the output capacitances with large ESR are used. The noise does not enable the accurate measurement of the amplitude and the phase shift. Therefore, the results acquired with this method should always be analysed in order to evaluate whether the noise was present in the data after filtration.
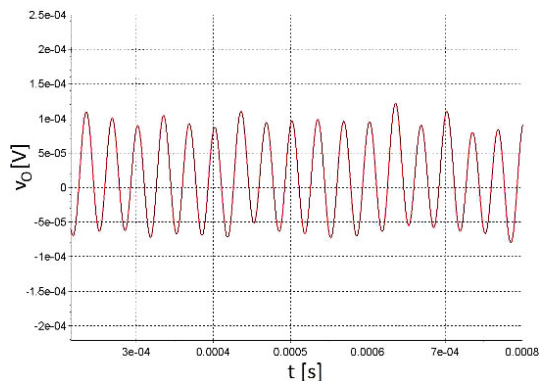

Fig. 7. The presence of unwanted noise in the filtered signal, after zero-phase filtration with the 3rd order Butterworth filter, when frequency approaches $1/3 \, f_s$.

2.3. FFT analysis

Another method that can be used to solve the problem of the noise presence in the output signal, requires the FFT analysis. The idea of the method is extracting the useful signal from the noisy output with the FFT analysis. Subsequently, the extracted signal can be compared with the input signal in order to obtain information on the magnitude and the phase shift. The FFT analysis can be done with the mathematical software mentioned in the previous section. As a result of the FFT analysis a vector is created. The vector contains complex values which correspond to each harmonic of the analysed signal in a frequency range from 0 to 1/2 of the sampling frequency (Fig. 8).
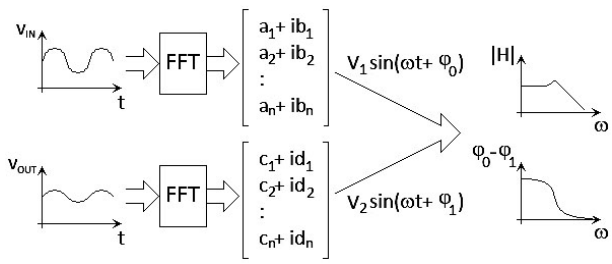
Fig. 8. The idea of calculation of frequency characteristics based on the
time-domain data and FFT.

A harmonic that corresponds to the frequency of the input signal has to be found – a complex value assigned to the harmonic is then used to calculate the magnitude and the phase shift of the analysed signal. If for some reasons the phase shift of the input signal is unknown, the FFT analysis of both input and output signals can be performed.

The FFT enables to calculate accurate frequency characteristics in a short period, even if the noise amplitude is much greater than the amplitude of the useful signal.

2.4. Analysis of step response

The methods presented in the previous sections require separate measurements of the input and output signals in the time domain for various frequencies. The measurements can be time consuming, although they provide accurate results. The method presented in this section can be used for fast evaluation of converter frequency characteristics, although their range is limited because of hardware restrictions. The method uses the response of a circuit to a step change of the input signal. Basically, the derivative of a step change is a Dirac pulse which has its harmonics with a gain of 1 in the whole spectrum of frequencies. Thus, calculating the derivative out of the step response yields a pulse response. Next, the FFT can be used to extract the information regarding the amplitude and the phase shift of the measured system. The algorithm is presented in Fig. 9.
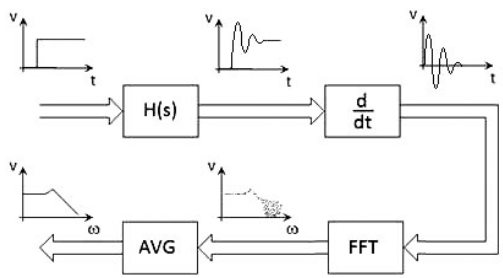

Fig. 9. The algorithm used for extraction of frequency characteristics
out of the response of a circuit to a step change of the input signal.

The result of FFT needs to be averaged in order to get rid of the quantization error and reach the final frequency characteristic. Although this method can be used for fast evaluation of the frequency characteristics, it has some drawbacks related to hardware. For higher frequencies,

where the output signal amplitude is attenuated to a very low level (below the resolution of ADC), the method gives wrong results, which is presented in the next section.

A similar approach is presented in [7, 9], where measurement of the control-to-output transfer function is performed using a *pseudorandom binary sequence* (PRBS) which basically is an approximation of white noise. This method uses the cross-correlation calculation between the PRBS and the output signal in order to obtain the system pulse response. The problem of low accuracy in higher frequencies was solved in [9] by using a pre-emphasis filter that boosts the amplitude of higher frequencies in the input signal, which improves SNR in the output signal. Using the pre-emphasis filter comes with some restrictions and thus it cannot be used in all situations [9].

3. Simulation examples

This section contains examples of frequency characteristics acquired by a large-signal simulation performed in OrCAD software. An ideal BOOST converter operating in the CCM was used for the simulation. The converter was chosen because of a well-known and valid model [2, 3, 10] which is essential for comparison with the simulation results. A circuit that was used for the simulation is presented in Fig. 10. Such a method of converting the control signal to PWM is well known and it had been used in the past [8, 11].
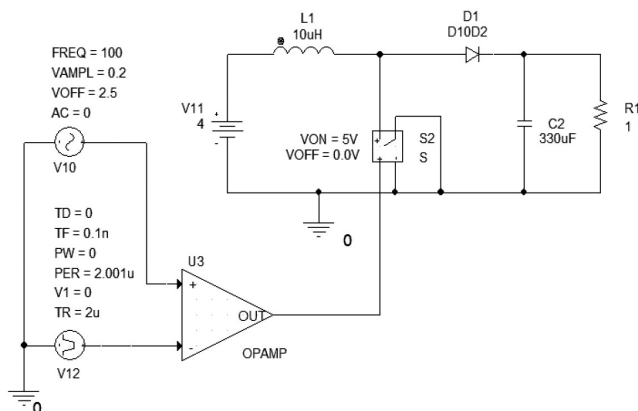

Fig. 10. A circuit used for the simulation.

The values of parameters used for the simulation are as follows: $L = 10$ μH; $C = 330$ μF; $R_o = 1\,\Omega$; $V_G = 4$ V, $f_S = 500$ kHz, the steady-state value of the duty cycle was equal to $D_A = 0.5$; while the small-signal value of the duty cycle had an amplitude of 0.04. After acquiring the input and output signals in the time domain for different frequencies, frequency characteristics have been calculated using the methods presented in the previous sections. The frequency characteristics presented in Figs. 11a, b, c, d have been achieved by averaging, filtration, FFT and analysis of a step response, respectively. The figures contain the gain and phase characteristics. The phase characteristics are more sensitive though, and thus are better to show any discrepancies between the methods. Solid lines in Figs. 11a, b, c, d represent theoretical values calculated based on the model presented in [2, 3].

According to Fig. 11, the ZPF and FFT methods give similar results, although FFT seems to be more accurate. On the other hand, averaging is quite accurate for lower frequencies, but
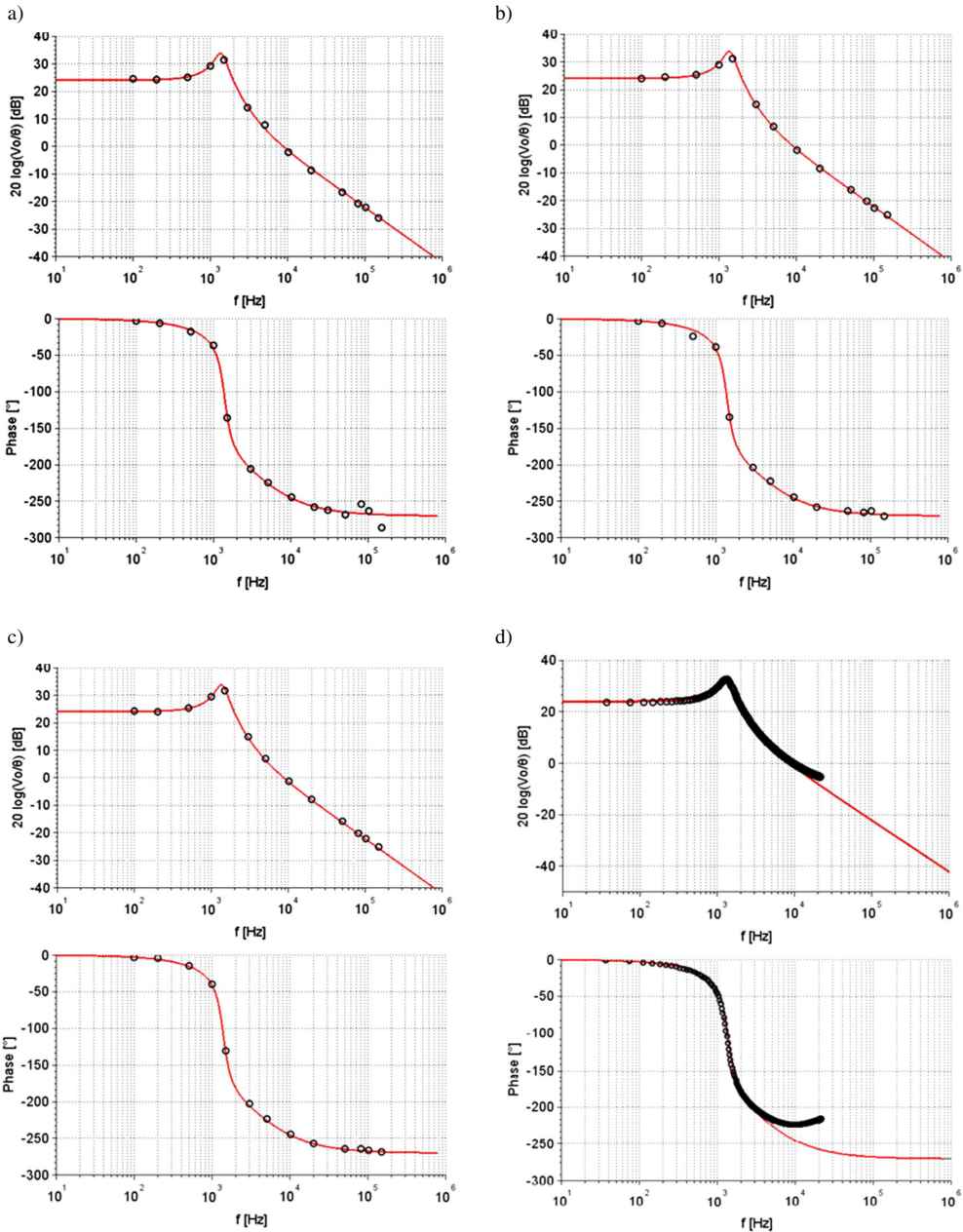
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a)

b)

c)

d)

Fig. 11. Frequency characteristics of a control-to-output transfer function: theoretical (solid lines), and achieved with analysis of transient simulations using methods presented in the previous sections (•) – a) averaging over one switching cycle; b) zero-phase filtration; c) FFT; d) analysis of a step response.

its accuracy deteriorates for frequencies above 1/5 f_s, as predicted. Also, the analysis of a step response is less accurate for higher frequencies, but it has an advantage of fast measurement in comparison with other methods.

4. Measurement and calculation of frequency characteristics of real BUCK converter

In this section the frequency characteristics of the control-to-output transfer function have been measured and compared with those obtained with theoretical calculations. In order to measure the frequency characteristics, a BUCK converter operating in the discontinuous conduction mode has been designed and built. The measuring circuit is presented in Fig. 12.
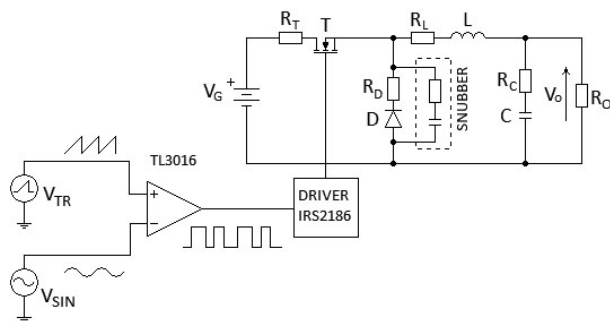

Fig. 12. The measuring circuit of the BUCK converter operating in the discontinuous conduction mode.

The values of parameters of the components were as follows: $L = 90.8$ μH; $C = 108.8$ μF; $R_o = 198$ Ω; $R_C = 18.6$ mΩ; $R_L = 121$ mΩ; $R_D = 281$ mΩ; $R_T = 39$ mΩ, $V_G = 8$ V; $f_S = 100$ kHz; the steady-state value of the duty cycle was equal to $D_A = 0.2$. The signals at the input of TL30166 comparator had the following amplitudes: $V_{TR} = 5$ V; $V_{SIN} = 0.2$ V. The sinusoidal signal contained an offset $V_{OFFSET} = 4$ V. A snubber consisting of $R_S = 100$ Ω; $C_S = 4.7$ nF was used during the measurements, in order to eliminate oscillations that appear when the inductor current falls to zero [12]. More information regarding snubbers can be found in [13]. The FFT analysis was chosen for calculation of the gain and the phase shift between the input and output signals.

A small-signal model, presented in [14, 15], has been used for the calculations of theoretical frequency characteristics.

Figure 13 shows the frequency characteristics of the measured circuit. Solid lines refer to the theoretical characteristics, whereas dots represent values that have been acquired with the FFT analysis of the measured data.
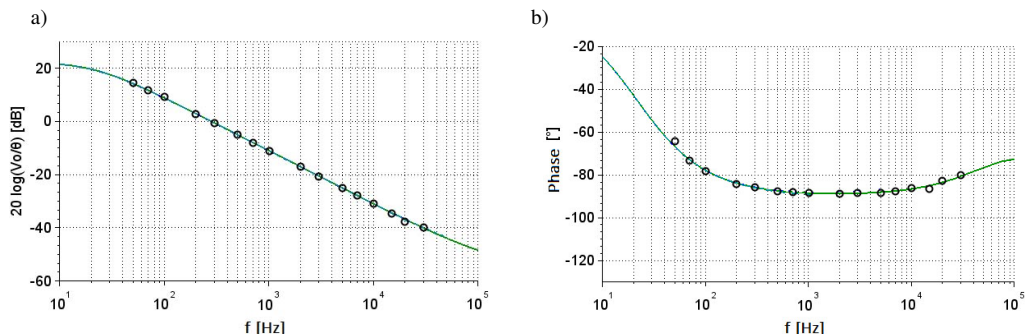

Fig. 13. Frequency characteristics of a control-to-output transfer function.

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5. Conclusions

Measurement of DC/DC converter frequency characteristics is a challenging task, especially when a specialized instrument for this purpose is not available. There are some ways to calculate the frequency characteristics, based on data acquired in the time domain. A few methods suited for this task have been compared in this paper. Based on the simulations presented in this paper the FFT method seems to be the best choice because of good accuracy, in comparison with other presented methods, and its functionality (it is easy to implement). However, for fast evaluation of the frequency characteristics in a lower range of frequencies (*i.e.* below 1/10 f_S) the analysis of the step response is sufficient to acquire the characteristics with good accuracy. Fast evaluation can also be done with a more advanced method presented in [9], but its use can be restricted for some converters. Moreover, a special care must be taken when designing the pre-emphasis filter.

In the last section the FFT method was used to calculate the frequency characteristics of a real converter. The calculations were made based on the measurement results of the input and output signals in a BUCK converter operating in the discontinuous conduction mode. The results of the operation were compared with the characteristics calculated based on a model presented in [14, 15]. A good consistency can be observed between the model and the measured characteristics, which proves that the method can be used for obtaining the frequency characteristics based on signals measured in the time domain.

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