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Innovative design of spur gear tooth with infill structure

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Abstract. The progress of additive manufacturing technology brings about many new questions and challenges. Additive manufacturing technology allows for designing machine elements with smaller mass, but at the same time with the same stiffness and stress loading capacity. By using additive manufacturing it is possible to produce gears in the form of shell shape with infill inside. This study is carried out as an attempt to answer the question which type of infill, and with how much density, is optimal for a spur gear tooth to ensure the best stiffness and stress loading capacity. An analysis is performed using numerical finite element method. Two new infill structures are proposed: triangular infill with five different densities and topology infill designed according to the already known results for 2D cantilever topology optimization, known as Michell structures. The von Mises stress, displacements and bending stiffness are analyzed for full body gear tooth and for shell body gear tooth with above mentioned types of infill structure.

Key words: infill, spur gear tooth, stress analysis, stiffness, numerical analysis.

1. Introduction

Gears are the most commonly used machine elements and they are crucial parts of most mechanical power transmissions. Early examples of gears in China date back to the 4th century BC [1]. A gear driven device for calculating the motion of stars and planets was discovered in Europe and it was concluded that it dates from the Greek period. It is generally accepted that gears were invented by the Greek mechanics of Alexandria in the third century BC. They had many different applications in the “machinery” of that time. They were also widely used in the Roman world with two main applications: in heavy-duty machines such as mills and irrigation wheels, where they transmitted considerable power, and in small-scale water-clocks, calendrical instruments and automata. Some of them were extraordinarily sophisticated, including the differential and hypoid gear. They became the ancestors of all modern gearing [2]. From Greek period up to the industrial revolution, there was no major development concerning gears. Modern gears with involute curve profile were developed in the late 19th century and at the beginning of the 20th century. Before, the development of gears was mostly limited by low manufacturing possibilities – it was almost impossible to manufacture gears with involute curve profile. Involute gears are commonly used today. After the invention of involute gears there has been no major development concerning gears up to this day. Today, the focus of research on gears is on optimization of gear profile [3–5]. Paper [3] presents an analysis of the toothing modification of spur gears. The aim of the modification of the profile on the tooth tip is to compensate for the tooth deflection under loading and to improve the conditions of gear operation. Paper [4] presents research on the maximum radial stress which is obtained by the optimization of gear design. Radial stress decreases by about 25% in contrast with the maximum radial stress of the gear with the same thickness profile. Angular correction of the gear tooth profile is investigated in paper [5] in order to establish the impact of optimizing width in the helical cylindrical gear on bearing and durability. Ongoing studies also concern vibration and noise of gears, as in paper [6], where research was carried out to investigate the relation between the design of gears and the noise which they produce. Similar analyses are carried out in papers [7, 8]. Topology optimization of gears is also within research interests of several authors. Topology optimization of machine parts has become an increasingly interesting field in recent years, since additive manufacturing enables manufacturing of optimized parts [9]. A completely new field of mechanical design was born, popularly called the design for additive manufacturing. Additive manufacturing has a significant impact on many of areas of science [10]. The goal of topology optimization is to find minimum mass of gear in correlation to allowed stresses [11]. Paper [12] presents a new approach aiming to reduce gear vibration and weight by modifying its body structure. As it can be seen in papers [11] and [12], and other similar papers, most researchers try to optimize gear body, while none of them try to optimize gear tooth (Fig. 1 from [12]). Extended research in the field of topology optimization can be found in [13].

In the first paper to deal with the topology optimization of spur gear tooth, authors try to optimize spur gear tooth by introducing new design of spur gear tooth as shell body element without infill structure. Paper [14] proves that gear tooth can be produced as a shell body gear tooth with the reduction of mass (10.27% per gear tooth), and at the same time with the stresses

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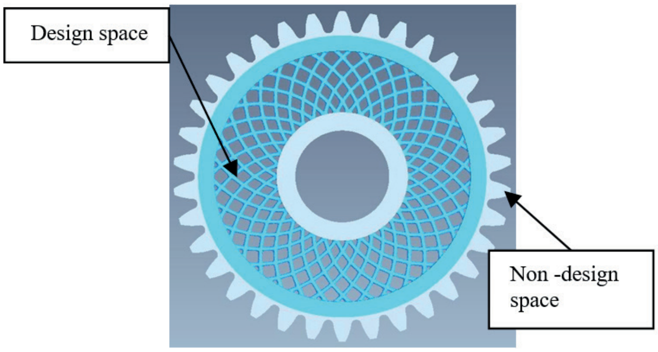

Fig. 1. Design space and non-design space [12]

in allowed limits. This paper is an extension of work presented in paper [14]. The aim of the research described in this paper is to optimize spur gear tooth by introducing new design of spur gear tooth as shell body element, but this time with infill structure. The main thesis is that it is possible to additionally reduce mass of spur gear tooth in correlation to minimum stresses and displacements allowed by reducing the shell thickness and placing infill structure inside. The idea is that it is possible to remove some of the material from the inside of gear tooth and to replace that material with some type of infill structure. Two new infill structures are proposed: triangular infill with five different infill densities and topology infill designed according to results for 2D cantilever topology optimization already known from literature, known as Michell structures [15].

2. Methodology of research

The optimization of spur gear tooth in this paper is carried out using structural numerical finite element method (FEM) for full body spur gear tooth and for spur gear tooth with infill structure inside. The von Mises stress, displacements and bending stiffness are analyzed. Numerical structural analysis is often used for stress analysis and for visualization of stress distribution in solid body structures [16–18]. Numerous papers deal with numerical structural analysis of gears and gears teeth. Bending stress analysis of bevel gears is investigated in paper [19]. Research about stress distribution of gear tooth due to axial misalignment condition is carried out in paper [20]. Research of error analysis on finite element modeling of involute spur gears is done in paper [21]. Finite element analysis of a spur gear tooth using Ansys and stress reduction by stress relief hole is done in paper [22]. Static analysis bending stress on gear tooth profile by variation of gear parameters with the help of FEA is carried out in paper [23].

2.1. Definition of problem. Observing the results of von Mises and principal stresses in papers [20, 22, 23], it can be noticed that there are some areas on spur gear tooth where stresses have lower values. Same results can be seen in many other studies on stress distribution in gear tooth. The idea of this research is that it is possible to remove some material from these areas, and replace it with the infill structure, while maintaining the values of stresses and displacements within the permitted limits. From the same papers [20, 22, 23] it can be seen that there are two main areas where stress increases: in contact area and in tooth root area. Contact stress, also known as Hertz pressure [24] always go deeper inside gear tooth in relation to tooth root stress (also known as bending stress). A more detailed explanation can be found in [14]. Up to today, all gear teeth have been full body elements, yet in this research gear tooth is designed as shell body with shell thickness all around the tooth and with infill structure inside the tooth. Two new infill structures are proposed, triangular infill in amount of 20%, 40%. 60% and 80% (Fig. 2) and topology infill in amount of $\approx 20\%$ and $\approx 60\%$ (Fig. 3). This way, the mass of the tooth (and the whole gear in total) will be reduced. These types of shell body gear teeth can be manufactured only using additive manufacturing. This study attempts to answer one more question, which is

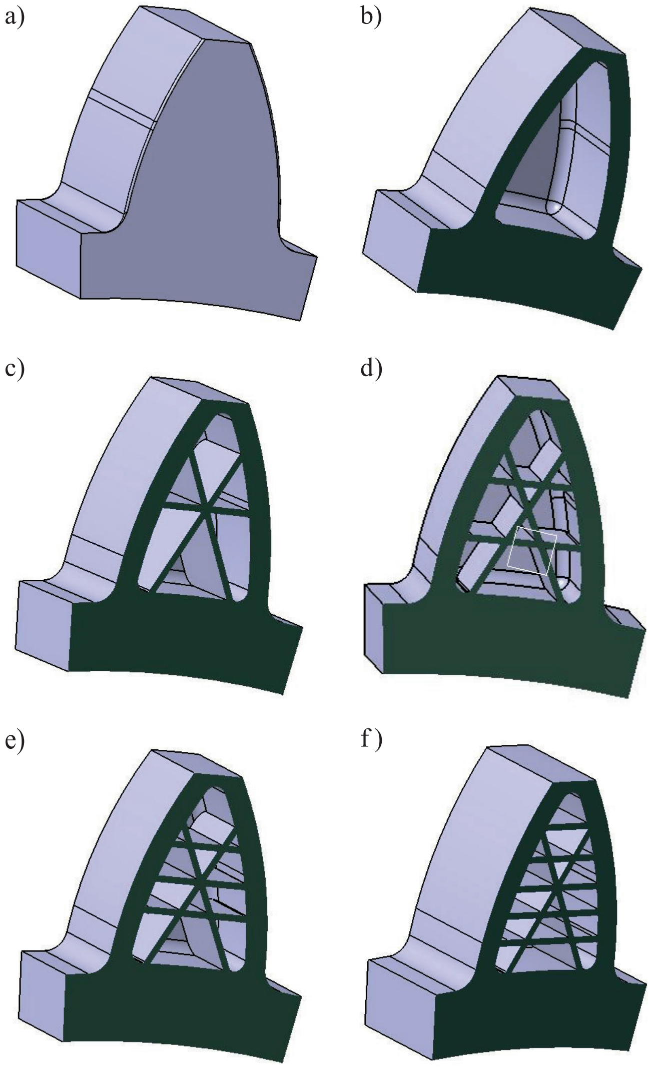

Fig. 2. Full body spur gear tooth (100% infill) (a), shell body spur gear tooth without infill structure (b), with triangular infill structure inside with 20% infill (c), 40% infill (d), 60% infill (e) and 80% infill (d) (shown with a section view)

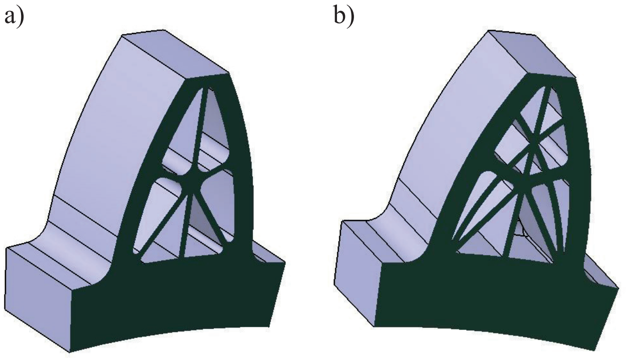

Fig. 3. Shell body gear tooth with topology infill structure inside with ≈ 20% infill (a) and ≈ 60% infill (b)

often asked these days by gear manufacturers who want to produce gears using additive manufacturing. That question is which type of infill structure of spur gear tooth should be chosen and with how much density, taking in consideration that values of stresses and displacements need to be within permitted limits. To carry out this research, CAD models and FEM models of full body and shell body spur gear teeth need to be developed.

2.2. Development of CAD models of gear teeth. To be able to compare results and to validate FEM model, geometrical characteristics and material properties of gear (and gear tooth at the same time) are same as geometrical characteristics and material properties of gear analyzed in paper [23]. Spur gears teeth are modeled as full body spur gear tooth (Fig. 2a) and shell body spur gear tooth with and without infill structure inside (Figs. 2b–2f, 3a and 3b). Shell body spur gear teeth with infill structure are shown with a section, because the inside of the tooth has to be shown. Root radius on all teeth is 1 mm. Radius inside shell body teeth are 0.5 mm for radius around the shell and 0.2 mm for critical places on infill structure (only spur gear tooth in Fig. 6b have radius in amount of 0.4 mm on critical places in infill structure). These radiuses are applied in infill structures inside the shell body spur gear tooth to reduce the influences of stress concentration [25–26]. Radius on the outer edges on all teeth are 0.1 mm.

Table 1
Geometrical characteristics of chosen gear

Geometrical characteristics	Units	Value
Number of teeth	–	24
Module	mm	4
Pitch circle diameter	mm	96
Base circle diameter	mm	90
Pressure angle	°	20
Addendum circle diameter	mm	104
Circular pitch	mm	12.56
Face width	mm	10

Material of gear is standard steel with modulus of elasticity 210 GPa and yield strength 350 MPa.

Triangular type of infill is proposed taking in consideration experience from steel metal structures and knowledge from statics. It is already known that metal structures with triangular shape have good stiffness. Also this type of infill is proposed because it is easy to generate it or to model it. Second infill type which is proposed in this research is topology infill which is generated according to results for 2D cantilever topology optimization known from literature, also known as Michell structures [15] (Fig. 3).

All walls of all infill structures have 0.3 mm thickness, with the exception of infill structure shown in Fig. 3b. In this case the main "horizontal" wall has 0.8 mm thickness. All infill structures are manually modeled because all commercial software have a problem with disposition of infill inside the model in correlation to the stiffness of that model. For example, for shell body spur gear tooth used in this research (Fig. 2d) and for 40% automatically generated infill using Ultimaker CURA software, infill structure will have disposition shown in Fig. 4. It can be seen that this infill structure does not have the best disposition when the best stiffness of produced spur gear tooth is required. Similar results can be obtained for other values of infill density and for other commercial software.

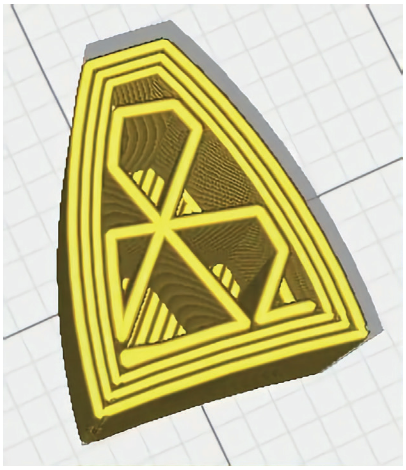

Fig. 4. Shell body gear tooth with 40% infill generated using Ultimaker CURA commercial software

2.3. Development of FEM models of gear teeth. For numerical structural stress analysis of spur gear teeth, finite element method is used. FEM model of shell body tooth is shown in Fig. 5.

To develop the FEM model, the first step is to choose finite element type. Linear tetrahedron volume element with 4 nodes is chosen with size of 0.3 mm and absolute sag 0.03 mm. More about tetrahedron element type can be found in [27]. Second step is to define boundary conditions. As earlier mentioned, for this study only gear tooth is taken in to consideration, rest of the gear is considered as rigid body. Because of this assumption lower surface of gear tooth is constrained with fixed constrains

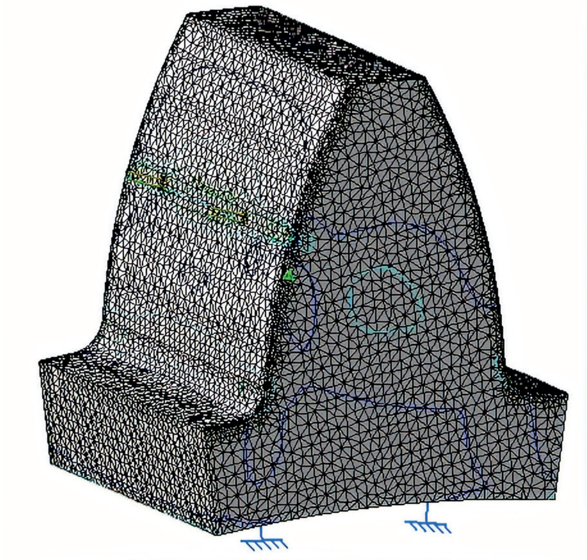

Fig. 5. FEM model of spur gear tooth with infill structure

(Fig. 5). The final step is to apply loads. Loads are applied according to Lewis equations [26] in form of force density load ($F = 2500$ N) on the small surface with the width of 0.4 mm (Fig. 6). Spur gear tooth is maximally loaded at the moment when only one tooth pair is in contact. This occurs when the contact line of two gears is on the pitch circle diameter of gears [24]. Force F is applied at the angle of 20° to horizontal axis. Loads are applied in the same way for all spur gear teeth.

3. Results

3.1. FEM model validation. After the formulation of CAD and FEM model, FEM model needs to be validated. Validation of FEM model is carried out by comparing results for full body tooth with the results from [23]. The von Mises stress in gear root area is compared for gear root radius 0 mm and force load 2500 N. Figure 7 shows results from [23] and Fig. 8

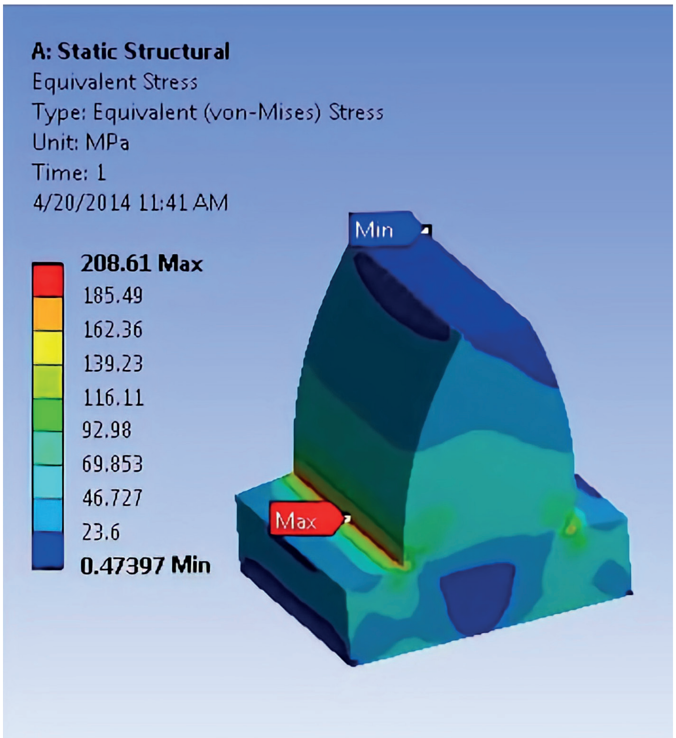

Fig. 7. The von Mises stress results in gear root area from [23]

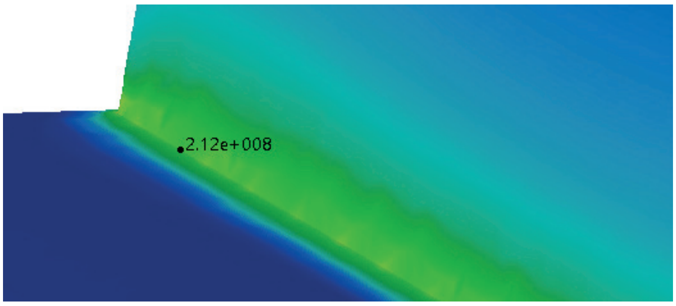

Fig. 8. The von Mises stress results in [Pa] in gear root area from this research

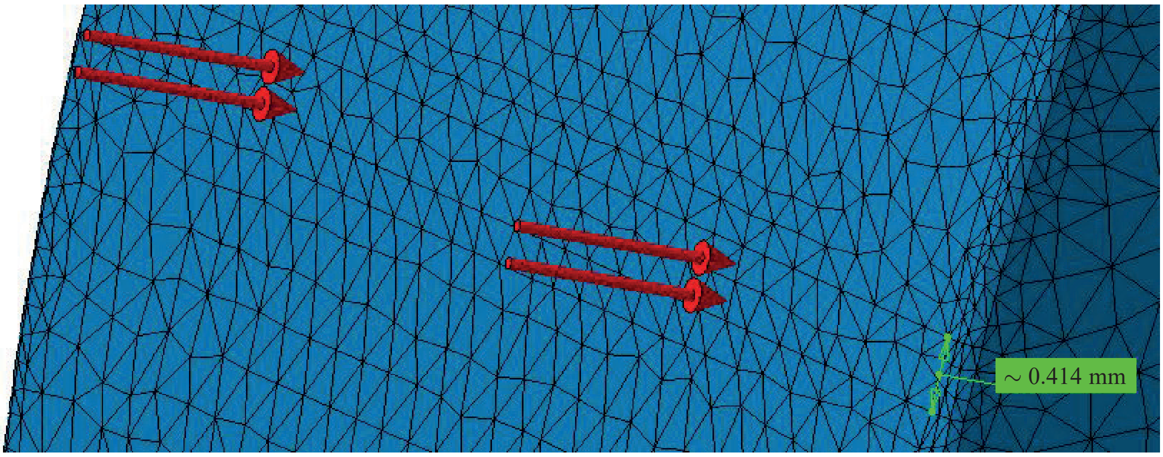

Fig. 6. Contact surface of applied loads

shows results from this study. The von Mises stress in [23] is 208.61 MPa and in this research it is 212 MPa. This way FEM model used for this research is validated. It is important to notice that results from [23] can be taken for validation because they are confirmed by analytical results in [23]

3.2. Numerical structural FEM analysis of spur gear teeth. To find out whether it is possible to remove some of the material from the inside of spur gear tooth and replace it with infill structure, numerical structural FEM analysis is carried out for all spur gear teeth showed in Fig. 2 and Fig. 3. The von Mises stress, displacements and bending stiffness are analyzed. To be able to measure and compare results for von Mises stress from different spur gear teeth, it is necessary to define critical points or areas where the measurements will be done. Analysis showed that for shell body spur gear tooth with infill structure it is not enough to look only to von Mises stresses at the contact of two teeth and at the root of the tooth, like in the case of full body spur gear tooth (Fig. 9). Other places on the spur gear tooth need to be analyzed (Fig. 10 and Fig. 11).

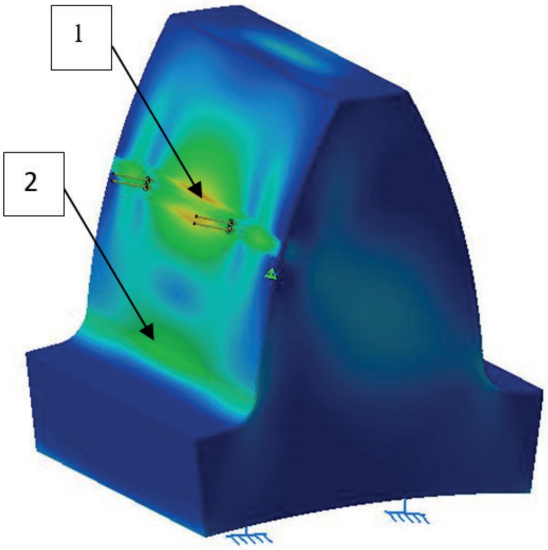

Fig. 10. The von Mises stress of shell body spur gear tooth. Places of analysis (increased stress)

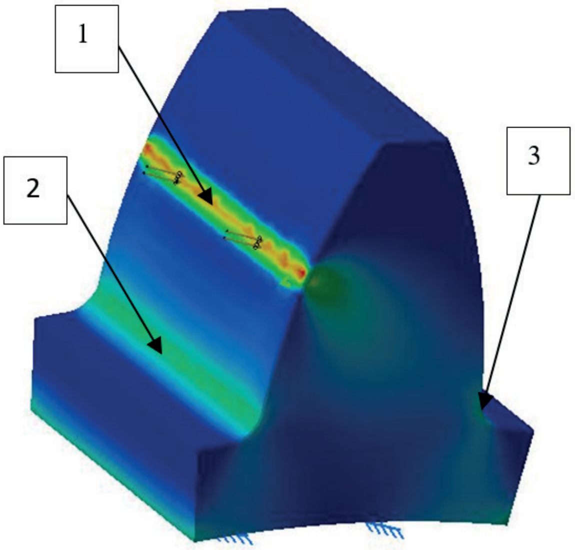

Fig. 9. The von Mises stress of full body spur gear tooth. Places of analysis (increased stress)

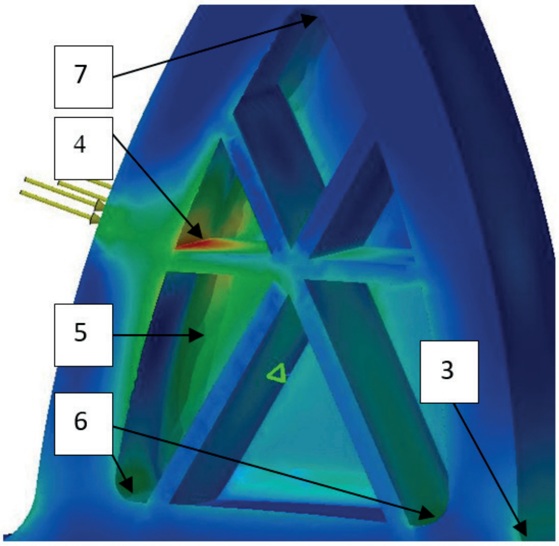

Fig. 11. The von Mises stress of shell body spur gear tooth with 20% triangle infill structure inside. Places of analysis (increased stress)

In Fig. 10 it can be noticed that from the outside of the spur gear tooth (places 1 and 2) stress is not uniformly distributed along the tooth like in the case of full body spur gear tooth. In the middle part of spur gear tooth, the area of stress distribution is increased. Inside of the spur gear tooth with infill structure of 20% is shown in Fig. 11. Stress distribution is very complex in this case, primarily because of the stress concentration, which occurs on all edges of infill structure.

The von Mises stress is increased at the same places for 40, 60, and 80% triangle infill structure. For topology infill structure, stress distribution outside of the spur gear tooth is the same as for triangle infill, but stress distribution inside of the shell body spur gear tooth with topology infill is different (≈ 20% infill in Fig. 12 and ≈ 60% infill in Fig. 13). From Fig. 12 it can be noticed that place 4 is moved from the edge of infill structure to the body of the "horizontal" wall. This is the result of the bigger radius (0.4 mm) which is added to the edge of connection of infill structure and shell of the spur gear tooth. Better von Mises stress distribution can be noticed in Fig. 13. This is the result of the thicker "horizontal" wall (0.8 mm).

Result values of von Mises stress, maximal displacement and bending stiffness for all analyzed spur gear teeth are shown in Table 2. From these results, it can be concluded that values of von Mises stress decrease with the increase of the% of infill structure. These values cannot be reduced down to the values which they have when 100% infill is applied (full body gear

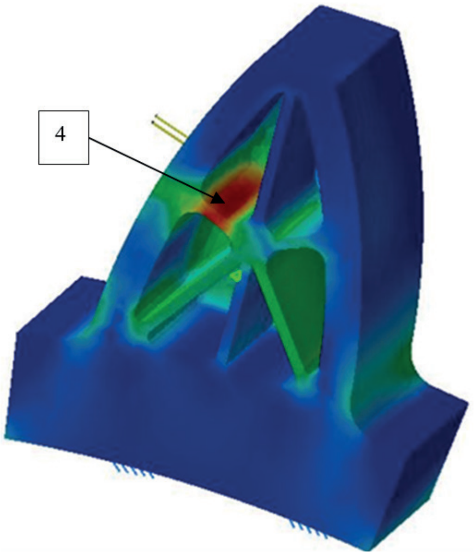

Fig. 12. The von Mises stress of shell body spur gear tooth with ≈ 20% topology infill structure inside

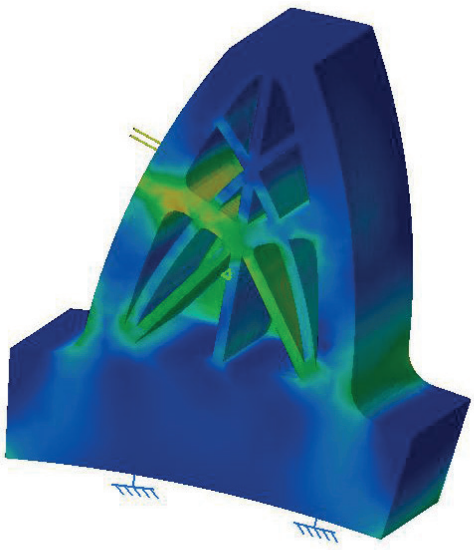

Fig. 13. The von Mises stress of shell body spur gear tooth with ≈ 60% topology infill structure inside

Table 2
Result values of von Mises stresses in MPa for different % of infill structure and for $F = 2500$ N

	Infill [%]	The von Mises stress for all places of analysis [MPa]							Maximal displacement [mm]	Bending stiffness [N/mm]	Reduced mass [%]
		1	2	3	4	5	6	7			
Triangular infill	0%	960	686	390	994	1240	1090	628	0.0341	73313.7	37.7
	20%	450	304	336	927	475	408	145	0.0171	146198.8	20,6
	40%	490	311	338	832	641	352	35	0.0170	147058.8	19.7
	60%	415	303	329	730	476	328	26	0.0149	167785.2	18.1
	80%	408	295	320	709	440	286	22	0.0140	178571.4	17.2
Topology infill	≈ 20%	388	227	260	548	235	273	60	0.0149	167785.2	19.1
	≈ 60%	421	210	248	260	165	163	43	0.0128	195312.5	15.6
Full body gear tooth	100% infill	392	176	188	–	–	–	–	0.0076	328083.9	0

tooth). Most problematic places of analysis are 4 and 5. Place 4 has big values of von Mises stress because it is the nearest place to the place of loads and because of stress concentration which appears at the edge between the infill structure and shell of the spur gear tooth. From the results, the following conclusions can be drawn:

- Topology infill is much better to use in comparison to the triangular infill. Topology infill with ≈ 20% infill has better values for von Mises stress, maximal displacement and bending stiffness than the triangular infill with 80% infill structure. This is a significant conclusion for future research.
- Topology infill with ≈ 60% infill structure has values very close to the values of full body gear tooth (around 30 MPa bigger for von Mises stress and around 5 μm bigger for maximal displacement). Values of von Mises stress are bigger than the yield strength (350 MPa) of the material, but that is also the case for full body spur gear tooth. This is the proof that spur gear tooth can be designed and manufactured as shell body spur gear tooth with infill structure. With additional research (which will be mentioned later), these values can be the same as for full body spur gear tooth.
- For topology infill with ≈ 60% infill structure, maximum von Mises stress is at the place of analysis number 1; it is not inside the spur gear tooth. This is significant because in this case, it is easy to carry out experimental analysis of the stress in future research.

To obtain better stress distribution inside spur gear tooth produced using additive manufacturing with infill structure inside, future research needs to be done. One of possible directions in future research is to make analysis of shell thickness of spur gear tooth. In this research, shell thickness is taken as 1 mm. It could probably be increased in order to obtain better stress distribution inside the spur gear tooth. Also, in this research all inside walls of infill structure are orientated in the direction of the shaft axis of spur gear tooth; maybe some of the walls need to be placed perpendicularly to the shaft axis.

4. Conclusion

Gears can be considered as one of the most commonly used machine elements, with long history of research about them. Gear tooth as a part of gear is usually considered as optimized as possible; many authors would claim that there is nothing to be done with gear tooth shape to improve it. This is true for the outside shape of the gear tooth, but the inside of gear tooth can be optimized, especially for gear teeth with bigger dimensions. Additive manufacturing has opened a completely new area of manufacturing for machine elements. All machine elements which were considered as optimal as possible in comparison to the optimal stiffness or minimum mass can now be reinvented using the additive manufacturing technology. Additive manufacturing enables production of elements with complex shapes and with holes or empty spaces inside the elements, popularly called shell body elements. That empty space can be filled with some kind of infill structure. This study has proved that it is possible to reduce the weight of spur gear tooth and at the same time get good values of von Mises stresses, maximal displacement and bending stiffness. Additional research needs to be done if those values needs to be the same as for full body spur gear tooth. Commercial software for preparing models for additive manufacturing should include options for best infill disposition in correlation to best stiffness of parts. For that, future research and development needs to be done, because that means that software for preparing models for additive manufacturing should provide numerical analysis. A better solution is to include tools for preparing models for additive manufacturing directly in computer aided design (CAD) software, such as CATIA and Solid Works, and in finite elements method (FEM) software, such as ANSYS.

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