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Sliding mode observers for fault identification in linear systems not satisfying matching and minimum phase conditions

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The paper studies the fault identification problem for linear control systems under the unmatched disturbances. A novel approach to the construction of a sliding mode observer is proposed for systems that do not satisfy common conditions required for fault estimation, in particular matching condition, minimum phase condition, and detectability condition. The suggested approach is based on the reduced order model of the original system. This allows to reduce complexity of sliding mode observer and relax the limitations imposed on the original system.

Key words: linear systems, faults, identification, disturbances, sliding mode observers

1. Introduction

This work is devoted to the problem of fault diagnosis in engineering systems. The fault diagnosis problem was extensively investigated for the past 30 years (see, e.g., [5, 10, 20, 27]). A variety of tools for fault diagnosis have been developed: diagnostic observers, parity relations, identification. There are many methods of identification, one is based on sliding mode observers (SMO) and uses peculiarities of sliding motion [23] which has many applications in control and observation.

Sliding mode observers are used for fault identification (reconstruction) in different systems: linear [11,12,21,22], nonlinear [6,9,17,25], and descriptor [7],

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for fault tolerant control [1, 8], in practical applications [13, 14, 31]. Sufficient conditions for existence of SMO are that the invariant zeroes of the system must be stable (minimum phase) and the matching condition is satisfied [9]; these conditions could be stringent and limit the applicability of SMO technique.

Two methods have been developed to relax the matching condition. The first method uses high-order sliding mode differentiator [4, 15–17, 26] to generate the derivatives of the outputs which are added to the original system to form a system satisfying the matching condition. The second one uses multiple SMOs in cascade [22], where signals from an observer are used as the output of a fictitious system whose input is the function describing fault; such a process is repeated until the fictitious system satisfies the matching condition. Although both methods are effective, the structure of the fault reconstruction scheme is complicated and large errors could occur. In addition, the system must be minimum phase.

In [2] this condition was relaxed but at the cost of the fault estimate being corrupted by the fault derivative or other dynamics, whereas in [19] the estimation errors are only bounded and asymptotic convergence cannot be achieved. [3] relaxed the minimum phase condition for systems where the fault occurs at the output. In [18, 24] the minimum phase condition is relaxed to only requiring detectability.

Note also that sliding mode observers in [12] and similar papers are constructed based on the original system. As a result, sliding mode observers are of full order.

The novelty of the proposed approach is that SMO is constructed for systems not satisfying matching, minimum phase, and detectability conditions. This arises from the fact that SMO is not constructed for the original system but for its reduced order model. As a result, such a model can be free from some special properties of the original system preventing SMO construction. Besides, the dimension of the observer becomes less than that of the original system.

Consider system described by linear dynamic model

$$
\begin{aligned}
\dot{x}(t) &= Fx(t) + Gu(t) + Dd(t) + L\rho(t), \\
y(t) &= Hx(t),
\end{aligned}
\tag{1}
$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^l$ are vectors of state, control and output, F, G, H, D, and L are known constant matrices, $d(t) \in R$ is a function describing faults: if there are no faults, $d(t) = 0$, if a fault occurs, $d(t)$ becomes an unknown function of time, $\rho(t) \in R^p$ is the unmatched disturbance, it is assumed that $\rho(t)$ is an unknown bounded function of time.

The term $Dd(t)$ may be caused by the change ΔF in the matrix F (or by ΔG in G) due to some failure in the system; in this case we may set $D = 1$ and $d(t) = \Delta F x(t)$ (or $d(t) = \Delta G u(t)$) and identify the function $d(t)$. The term $L\rho(t)$ reflects the external disturbances and modeling errors.

Recall that in [25] and similar papers it is assumed that system (1) satisfies the following conditions: 1) $rank(H[L\ D]) = rank([L\ D])$, 2) all invariant zeros of $(F, [L\ D], H)$ lie in the left half plane; the papers [18, 24] require that the system should be detectable. In the present paper, the problem of fault identification is solved without these conditions. The suggested solution is based on the reduced order model of the original system.

This paper is organized as follows. Section 2 present a solution of the problem including reduced order model design, sliding mode observer design, and fault identification under disturbances. Simulation example is considered is Section 3. Section 4 concludes the paper.

2. Problem solution

2.1. Preliminaries

It is assumed that (F, H) is non-detectable therefore $Ker(V^{(n)}) \neq \emptyset$, where

$$V^{(n)} = \begin{pmatrix} H \\ HF \\ \dots \\ HF^{n-1} \end{pmatrix}$$

and unobservable part of the system is unstable.

Assumption 1 $Im(D) \cap Ker(V^{(n)}) = \emptyset$.

Let r_d be minimal relative degree of the output vector y with respect to the function $d(t)$, y_* be an output corresponding to r_d, and the matrix R_* be such that $R_* y(t) = y_*(t)$. It follows from Assumption 1 that $r_d < \infty$.

Solution of the problem is based on the reduced order model of system (1) generally described by the equations

$$\begin{aligned} \dot{x}_*(t) &= F_* x_*(t) + G_* u(t) + J_* y(t) + D_* d(t) + L_* \rho(t), \\ y_*(t) &= H_* x_*(t), \end{aligned} \tag{2}$$

where $x_*(t) \in R^k$, $k \geqslant r_d$, is the state vector, F_*, G_*, J_*, H_*, D_*, and L_* are matrices to be determined. We assume that $x_*(t) = \Phi x(t)$ for some matrix Φ. It is known [28, 29] that matrices R_* and Φ satisfy the conditions

$$\begin{aligned} &\Phi F = F_* \Phi + J_* H, \qquad R_* H = H_* \Phi, \\ &\Phi G = G_*, \qquad \Phi D = D_*, \qquad \Phi L = L_* . \end{aligned} \tag{3}$$

2.2. Reduced order model design

Consider the method to construct system (2) under $\rho(t) = 0$ which will be used for sliding mode observer design. The matrices F_* and H_* are sought in the canonical form

$$F_* = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix}, \qquad H_* = (\, 0 \;\; 0 \;\; \dots \;\; 0 \;\; 1 \,).$$

Using these matrices, one obtains from (3) equations for rows of the matrices Φ and J_*:

$$\begin{aligned} \Phi_k &= R_* H, \qquad \Phi_i F = \Phi_{i-1} + J_{*i} H, \quad i = k, \dots, 2, \\ \Phi_1 F &= J_{*1} H, \end{aligned} \tag{4}$$

where Φ_i and J_{*i} are i-th rows of the matrices Φ and J_*, $i = 1, \dots, k$. As is shown in [29], equations (4) can be transformed into the single equation

$$R_* H F^k = J_{*k} H F^{k-1} + J_{*k-1} H F^{k-2} + \dots + J_{*1} H.$$

Rewrite it in the form

$$(\, 1 \;\; -J_{*k} \;\; \dots \;\; -J_{*1} \,) W^{(k)} = 0, \tag{5}$$

where

$$W^{(k)} = \begin{pmatrix} R_* H F^k \\ H F^{k-1} \\ \dots \\ H \end{pmatrix}.$$

One has to solve this equation for minimal $k \geqslant r_d$. As a result, the model (2) takes the form

$$\begin{aligned} \dot{x}_*(t) &= F_* x_*(t) + G_* u(t) + J_* y(t) + D_* d(t), \\ y_*(t) &= H_* x_*(t). \end{aligned} \tag{6}$$

Similar to [24], we write down all matrices in (6) in the form

$$\begin{aligned} &F_* = \begin{pmatrix} F_1 & F_2 \\ F_3 & F_4 \end{pmatrix}, \qquad H_* = (\, 0 \;\; 0 \;\; \dots \;\; 0 \;\; 1 \,), \\ &G_* = \begin{pmatrix} G_{*1} \\ G_{*2} \end{pmatrix}, \qquad J_* = \begin{pmatrix} J_{*1} \\ J_{*2} \end{pmatrix}, \qquad D_* = \begin{pmatrix} D_{*1} \\ D_{*2} \end{pmatrix}, \end{aligned} \tag{7}$$

where

$$F_1 = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix} \in R^{k-1\times k-1}, \qquad F_2 = \begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \end{pmatrix} \in R^{k-1\times 1},$$

$$F_3 = (0\;0\;\dots\;0\;1) \in R^{1\times k-1}, \qquad F_4 = 0;$$

the rest of the matrices in (7) have the appropriate dimensions. Introduce a coordinate transformation $z = Tx_*$ with $T = \begin{pmatrix} I_{k-1} & A \\ 0 & 1 \end{pmatrix}$, where $A \in R^{k-1\times 1}$ is selected to make $\overline{F}_1 = F_1 + AF_3$ stable. Since (F_1, F_3) is observable, this matrix exists and is of the form $A := (a_1\; a_2\; \dots\; a_{k-1})^T$.

As a result, we obtain the model in the following form:

$$\begin{aligned} \dot{z}_1 &= \overline{F}_1 z_1 + \overline{F}_2 y_* + \overline{G}_1 u + \overline{J}_{*1} y + \overline{D}_1 d, \\ \dot{z}_2 &= \overline{F}_3 z_1 + \overline{F}_4 y_* + \overline{G}_2 u + \overline{J}_{*2} y + \overline{D}_2 d, \\ y_* &= z_2, \end{aligned} \tag{8}$$

where

$$\overline{F}_1 = \begin{pmatrix} 0 & 0 & \dots & 0 & a_1 \\ 1 & 0 & \dots & 0 & a_2 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & a_{k-1} \end{pmatrix}, \qquad \overline{F}_2 = -\begin{pmatrix} a_1 a_{k-1} \\ a_2 a_{k-1} \\ \dots \\ a_{k-1}^2 \end{pmatrix},$$

$$\begin{aligned} \overline{F}_3 &= (0\;0\;\dots\;0\;1), & \overline{F}_4 &= -a_{k-1}, \\ \overline{G}_1 &= G_{*1} + AG_{*2}, & \overline{G}_2 &= G_{*2}, \\ \overline{J}_1 &= J_{*1} + AJ_{*2}, & \overline{J}_2 &= J_{*2}, \\ \overline{D}_1 &= D_{*1} + AD_{*2}, & \overline{D}_2 &= D_{*2}. \end{aligned}$$

2.3. Sliding mode observer design

Since $\overline{F}_1$ is stable, symmetric positive definite matrices P and Q exist such that $\overline{F}_1^T P + P\overline{F}_1 = -Q$. By analogy with [24], sliding mode observer is sought in the form

$$\begin{aligned} \dot{\hat{z}}_1 &= \overline{F}_1 \hat{z}_1 + \overline{F}_2 y_* + \overline{G}_1 u + \overline{J}_{*1} y + \overline{K}_1 v, \\ \dot{\hat{z}}_2 &= \overline{F}_3 \hat{z}_1 + \overline{F}_4 y_* + \overline{G}_2 u + \overline{J}_{*2} y + k_2 e_2 + k_3 v, \end{aligned} \tag{9}$$

where $e_2 = y_* - \hat{z}_2$, $v = sign(e_2)$, $\overline{K}_1 = P^{-1}\overline{F}_3^T k_1$, $k_1, k_2, k_3 \in R$ are positive numbers:

$$k_3 \geqslant \delta\|\overline{F}_3\| + \beta\|\overline{D}_2\|, \qquad k_1 \geqslant \frac{\delta\beta\|P\overline{D}_1\|}{k_3 - \beta\|\overline{D}_2\|}, \tag{10}$$

β is such that $\beta \geqslant \|d(t)\|$, δ is the value of the norm to which the estimation error e_1 will be bounded.

From (8) and (9) it follows

$$\begin{aligned} \dot{e}_1 &= \overline{F}_1 e_1 + \overline{D}_1 d - \overline{K}_1 v, \\ \dot{e}_2 &= \overline{F}_3 e_1 + \overline{D}_2 d - k_2 e_2 - k_3 v, \end{aligned} \tag{11}$$

where $e_1 = z_1 - \hat{z}_1$.

Lemma 1 *Let the function $e(t)$ satisfies the equation*

$$\dot{e}(t) = \overline{F}e(t) + g(t), \tag{12}$$

where $\overline{F}$ is $p\times p$ stable matrix, $\|g(t)\| \leqslant g_*$ is a bounded function. Then $\|e(t)\| \leqslant \gamma$ for some γ.

Proof. It is known that a solution of (12) is of the form

$$e(t) = \exp\left(\overline{F}t\right)\left(x(0) + \int_0^t \exp\left(\overline{F}(t-\tau)\right) g(\tau)\mathrm{d}\tau\right). \tag{13}$$

Assume for simplicity that $\overline{F}$ has different eigenvalues $\lambda_1, \ldots, \lambda_p$. It is known that in this case

$$\exp(\overline{F}t) = \sum_{k=1}^{p} C_k e^{\lambda_k t},$$

where

$$C_k = \frac{(\overline{F} - \lambda_1 E)\ldots(\overline{F} - \lambda_{k-1}E)(\overline{F} - \lambda_{k+1}E)\ldots(\overline{F} - \lambda_p E)}{(\lambda_k - \lambda_1)\ldots(\lambda_k - \lambda_{k-1})(\lambda_k - \lambda_{k+1})\ldots(\lambda_k - \lambda_p)}$$

$k = 1, \ldots, p$. Let $\max_{k=1,\ldots,p} Re\lambda_k = -a$, $a > 0$. Then

$$\begin{aligned}\|e(t)\| &\leqslant \sum_{k=1}^{p} \|C_k\| e^{Re\lambda_k t} \|e(0)\| + \int_0^t \sum_{k=1}^{p} \|C_k\| e^{Re\lambda_k (t-\tau)} g_* \mathrm{d}\tau \\ &\leqslant \sum_{k=1}^{p} \|C_k\| e^{-at} \|e(0)\| + \sum_{k=1}^{p} \|C_k\| g_* \int_0^t e^{-a(t-\tau)} \mathrm{d}\tau \\ &= \sum_{k=1}^{p} \|C_k\| \left(e^{-at} \|e(0)\| + \frac{g_*}{a}(1 - e^{-at}) \right) \\ &\leqslant \sum_{k=1}^{p} \|C_k\| \left(\|e(0)\| + g_*/a \right) = \gamma.\end{aligned}$$

Theorem 1 *The observer (9) estimates the function $d(t)$ as follows:*

$$\hat{d}(t) = k_3 D_{*2}^{+} v_{eq}(t) \tag{14}$$

if $D_{*1} = 0$,

$$\hat{d}(t) = \overline{K}_1 D_{*1}^{+} v_{eq}(t) \tag{15}$$

otherwise, where $D_{*1}^{+} = (\overline{D}_{*1}^{T} \overline{D}_{*1})^{-1} \overline{D}_{*1}^{T}$ and $D_{*2}^{+} = (\overline{D}_{*2}^{T} \overline{D}_{*2})^{-1} \overline{D}_{*2}^{T}$, $v_{eq}(t)$ is the so-called equivalent output injection signal representing the average behavior of the discontinuous function $v(t)$. According to [12], we use as $v_{eq}(t)$ the continuous approximation

$$v_{eq}(t) = \frac{e_2(t)}{|e_2(t)| + \varepsilon},$$

where ε is a small positive scalar.

Proof. We prove firstly that $\|e_1(t)\| \leqslant \delta$ for some δ. Since $d(t)$ is bounded function and $\|v(t)\| = 1$, then $\|\overline{D}_1 d(t) - \overline{K}_1 v(t)\| \leqslant g_0$ for some g_0. It follows from (11) and Lemma 1 that the error $e_1(t)$ is bounded by $\|e_1(t)\| \leqslant \delta$ for some δ.

Secondly, we prove that by suitable choices of the observer gains $e_2 = 0$ in finite time and sliding motion is achieved. Consider Lyapunov function $V_2 = e_2^2$ and take its derivative using (11):

$$\dot{V}_2 = 2e_2\dot{e}_2 = 2e_2 \left(\overline{F}_3 e_1 + \overline{D}_2 d - k_2 e_2 - k_3 v \right).$$

Since $v = sign(e_2)$, then $2e_2 k_3 v = 2k_3|e_2|$ and

$$\begin{aligned}\dot{V}_2 &\leqslant -2k_2 e_2^2 + 2|e_2| \left(-k_3 + \|\overline{F}_3\| \|e_1\| + \|\overline{D}_2\| \|d\| \right) \\ &\leqslant -2k_2 e_2^2 + 2|e_2| \left(-k_3 + \delta \|\overline{F}_3\| + \beta \|\overline{D}_2\| \right).\end{aligned}$$

If k_3 satisfies

$$k_3 \geqslant \delta\|\overline{F}_3\| + \beta\|\overline{D}_2\|,$$

then $\dot{V}_2 \leqslant 0$ and one can show by analogy with [24] that $\dot{V}_2 \leqslant -c_2\sqrt{V_2}$ for some $c_2 > 0$, and sliding motion ($e_2 = \dot{e}_2 = 0$) happens in finite time.

Thirdly, to prove that by suitable choices of the observer gains $e_1 = 0$ in finite time and sliding motion is achieved, consider Lyapunov function $V_1 = e_1^T P e_1$ and take its derivative using (11):

$$\dot{V}_1 = e_1^T \left(\overline{F}_1^T P + P\overline{F}_1\right) e_1 + 2e_1^T P(\overline{D}_1 d - \overline{K}_1 v).$$

From the second equation of (11) and since sliding motion has occurred ($e_2 = \dot{e}_2 = 0$) it follows that $\overline{F}_3 e_1 = k_3 v - \overline{D}_2 d$. Using $\overline{K}_1 = P^{-1}\overline{F}_3^T k_1$, we obtain

$$\dot{V}_1 = -e_1^T Q e_1 + 2e_1^T P\overline{D}_1 d - 2e_1^T \overline{F}_3^T k_1 v = -e_1^T Q e_1 + 2e_1^T P\overline{D}_1 d - 2(k_3 v - \overline{D}_2 d)^T k_1 v.$$

Since $\|e_1(t)\| \leqslant \delta$, it follows that

$$\dot{V}_1 \leqslant -e_1^T Q e_1 + 2\beta\delta\|P\overline{D}_1\| - 2k_1 k_3 + 2k_1\beta\|\overline{D}_2\|.$$

If k_3 and k_1 are chosen as in (10), then $\dot{V}_1 \leqslant 0$ and it can be shown by analogy with [24] that $\dot{V}_1 \leqslant -c_1\sqrt{V_1}$ for some $c_1 > 0$, and finite convergence of e_1 happens as well. Theorem has been proved. □

When sliding motion is achieved that is $e_1 = \dot{e}_1 = 0$ and $e_2 = \dot{e}_2 = 0$, it follows from (11) that the function $d(t)$ can be estimated by (14) or (15).

The parameters k_1, k_2, and k_3 should be chosen as close as possible to their lower bounds since simulation shows that the high magnification of these parameters prevents to achieve sliding motion.

2.4. Fault identification under disturbances

When $\rho(t) \neq 0$, the reduced order model is constructed to be invariant with respect to the disturbances. The condition $\Phi L = 0$ of invariance with respect to the disturbances can be taken into account in the form $(\,1\;\; -J_{*k}\;\; \ldots\;\; -J_{*1}\,)L^{(k)} = 0$ [28, 29] where

$$L^{(k)} = \begin{pmatrix} R_* HL & R_* HFL & \ldots & R_* HF^{k-1}L \\ 0 & HL & \ldots & HF^{k-2}L \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 \end{pmatrix}.$$

The last equation and (5) result in single equation

$$(\,1\;\; -J_{*k}\;\; \ldots\;\; -J_{*1}\,)(W^{(k)}\;\; L^{(k)}) = 0. \tag{16}$$

Solve this equation for minimal $k \geqslant r_d$ and construct the model (6). The model (8) and observer (9) are constructed by analogy with Subsections 2.2 and 2.3, and the fault can be identified precisely.

In some cases, invariance with respect to the disturbances cannot be achieved, and only the problem of approximate fault identification can be solved here [30].

3. Simulation example

Consider linear control system

$$\begin{aligned}
\dot{x}_1(t) &= -x_1(t) + x_2(t) + u(t),\\
\dot{x}_2(t) &= -x_2(t) + x_4(t) + d(t),\\
\dot{x}_3(t) &= +x_3(t) + x_4(t) + \rho(t),\\
\dot{x}_4(t) &= -x_4(t) + \rho(t),\\
y_1(t) &= x_1(t), \qquad y_2(t) = x_4(t).
\end{aligned} \tag{17}$$

The matrices describing this system are as follows:

$$F = \begin{pmatrix} -1 & 1 & 0 & 0\\ 0 & -1 & 0 & 1\\ 0 & 0 & 1 & 1\\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad G = \begin{pmatrix} 1\\0\\0\\0 \end{pmatrix}, \qquad D = \begin{pmatrix} 0\\1\\0\\0 \end{pmatrix},$$

$$H = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad L = \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix}^T.$$

It can be shown that $Ker(V^{(4)}) = \{(0 \;\; 0 \;\; 1 \;\; 0)^T\}$ and the system is non-detectable. Clearly, $Im(D) \cap Ker(V^{(4)}) = \varnothing$, $r_d = 2$, $y_* = y_1$, and $R_* = (1 \;\; 0)$.

One obtains

$$W^{(2)} = \begin{pmatrix} 1 & -2 & 0 & 1\\ -1 & 1 & 0 & 0\\ 0 & 0 & 0 & -1\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad L^{(2)} = \begin{pmatrix} 0 & 0\\ 0 & 0\\ 0 & -1\\ 0 & 0\\ 0 & 0 \end{pmatrix}.$$

It can be shown that (16) has a solution with $J_{*1} = (-1 \;\; 1)$ and $J_{*2} = (-2 \;\; 0)$; then $\Phi_1 = (1 \;\; 1 \;\; 0 \;\; 0)$, $\Phi_2 = (1 \;\; 0 \;\; 0 \;\; 0)$, $D_* = (1 \;\; 0)^T$, and $G_* = (1 \;\; 1)^T$.

As a result, the model (6) takes the form

$$\begin{aligned}
\dot{x}_{*1}(t) &= -y_1(t) + y_2(t) + u(t) + d(t),\\
\dot{x}_{*2}(t) &= x_{*1}(t) - 2y_1(t) + u(t),\\
y_*(t) &= x_{*2}(t) = y_1(t),
\end{aligned}$$

where $x_{*1} = x_1 + x_2$ and $x_{*2} = x_1$. Choosing $A = -1$ and taking $T = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$, we obtain the model (8) in the form

$$
\begin{aligned}
\dot{z}_1(t) &= -z_1(t) - y_*(t) + y_1(t) + y_2(t) + d(t),\\
\dot{z}_2(t) &= z_1(t) + y_*(t) - 2y_1(t) + u(t),\\
y_*(t) &= z_2(t) = y_1(t),
\end{aligned}
$$

where $z_1 = x_{*1} - x_{*2}$ and $z_2 = x_{*2}$. It follows from this model that $\overline{F}_3 = 1$, $\overline{D}_1 = 1$, $\overline{D}_2 = 0$; since $\overline{F}_1 = -1$, we may set $P := 1$, then $Q = 2$.

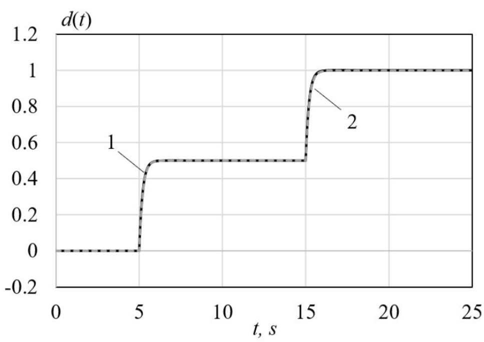
Sliding mode observer is described by equations

$$
\begin{aligned}
\dot{\hat{z}}_1(t) &= -\hat{z}_1(t) - y_*(t) + y_1(t) + y_2(t) + \overline{K}_1 v_{eq}(t),\\
\dot{\hat{z}}_2(t) &= \hat{z}_1(t) + y_*(t) - 2y_1(t) + u(t) + k_2 v_{eq}(t) + k_3 e_2(t),
\end{aligned}
\tag{18}
$$

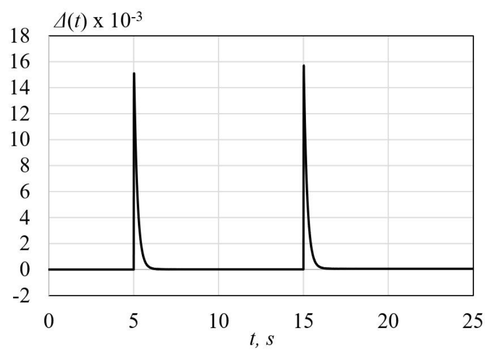
where $e_2 = y_1 - \hat{z}_2$, $v = sign(e_2)$, $\overline{K}_1 = P^{-1}\overline{F}_3^T k_1 = k_1$, $k_1 \geqslant \beta$, $k_2 > 0$, $k_3 \geqslant \delta$, $\delta = \beta + k_1$, β is such that $\beta \geqslant \|d(t)\|$. Since $D_1^+ \neq 0$, the function $d(t)$ can be estimated as

$$
\hat{d}(t) = D_{*1}^+ k_1 v_{eq}(t) = k_1 v_{eq}(t).
$$

For simulation, consider system (17) and the observer (18) with the control $u(t) = sin(t)$, $\rho(t) = 20sin(2t)$, $k_1 = 1.5$, $k_2 = 0.01$, $k_3 = 3$, and $|e_1(0)| = 0$. Simulation results are presented in Figs. 1 and 2 showing behavior of the function $d(t)$, its estimation $\hat{d}(t)$ and the estimation error $\Delta(t) = \hat{d}(t) - d(t)$ for two types of function $d(t)$ – sinusoidal and step-shaped, respectively.

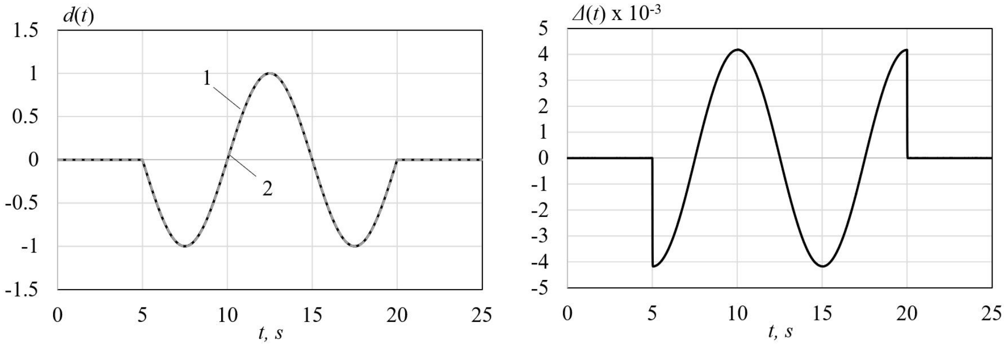


(a)



(b)

Figure 1: Behavior of the step-shaped function $d(t)$ (a,1), its estimation $\hat{d}(t)$ (a,2), and the fault estimation error $\Delta(t) = \hat{d}(t) - d(t)$ (b)



(a) (b)

Figure 2: Behavior of the sinusoidal function $d(t)$ (a,1), its estimation $\hat{d}(t)$ (a,2), and the fault estimation error $\Delta(t) = \hat{d}(t) - d(t)$ (b)

4. Concluding remarks

In this paper, the problem of fault identification for systems under the disturbance that do not satisfy the matching, minimum phase, and detectability conditions is studied. These conditions were reduced to less restrictive one. The suggested method is based on the reduced order model of the original system. A simulation example shows the effectiveness of the proposed method.

The possibility of construction of the observer estimating the fault for systems, in which the unobservable part is not stable, is more theoretical result than practical one. But the suggested method based on the reduced order model is useful *per se* and can be used for fault identification in different systems.

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