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Possibility of surface matching using Hopfield's neural network

In the last decade the usefulness of neural networks (NN) in engineering and commerce has been widely developed.

On the satellite images the different tasks such as cloud, ice, sea-surface thermal feature tracking, spatial resolution improvement, pattern classification etc. have been applied, using neuron networks.

In digital photogrammetry neural networks have been used, at first, in solution of stereo vision problem called stereo matching and to other tasks such as detection, location of object from mobile mapping data and another.

This paper presents an idea of using Hopfield's neural network (HNN) to solve the surface matching problem. First of all the energy function (or cost function) of surface matching would be provided. For this purpose all constraint conditions with the target (or goal) of surface matching will be combined into energy function, which can be applied by Hopfield's neural network in determining the optimal surface matching problem.

INTRODUCTION

Surface matching is a fundamental task that must be solved, whenever we want to compare or merge data sets describing surface of same terrain. For performing surface matching the target functions are, in practice, established [10]; and the process of matching surfaces could be realized on the basic [2].

In the last years there has been an increasing interest in using artificial neural networks for different tasks in remote sensing and digital photogrammetry. The basic of neural network theory could be found in [8, 9].

The Hopfield's neural network (HNN) is one of the sorts of recurrent networks. Neural network, in general, or Hopfield's neural network, in particular, have been found in engineering, commerce and other application, where we have to solve optimization problem [4]. The principle of theory and computation methods of optimization problem can be found in literature as [13].

On the sequential satellite images, HNN has been applied to cloud, ice and sea-surface thermal features tracking [1]; land cover targets identifying at the subpixel scale (super-resolution) [11]. The other applications of NN on satellite images for different tasks such as spatial resolution improvement, pattern classification etc. were given in many publications [14, 15, 16].

In digital photogrammetry the problem of stereovision is the basic task to represent the best three-dimensional description of the locations of the objects. The strategy of stereo matching using artificial neural network has been a main of the investigation directions. In last decade the HNN used to stereo matching have been realized on the four layers so-called feature detecting, pattern matching, stereo fusion and compound decision layer [6]. Other solutions of using HNN for orienting stereo pairs, detection and location of objects from mobile mapping image, automated name placement for point feature in cartography were presented in [3, 5, 12].

In this paper the proposed approach for surface matching using HNN constructed on data sets derived by different sensor images such as Aerial, LIDAR, SAR, IFSAR and other one have been developed. The first surface created from point data set obtained by accurate measurements, considered as a pattern, is generated in TIN model (or DEM model). The second by measurements, for example, of LIDAR, SAR...treated as a candidate, is divided into quadrilateral models. The correspondence problem between normal vectors (as the features) of the two single TIN model and quadrilateral models is formulated as the minimization of energy function. This energy function defines the surface matching problem to be solved by HNN, which includes the goal of matches and imposed constraints. The neurons being interconnected and matches being done simultaneously, HNN finds global solution of the optimization problem.

1. *Hopfield's neural network*

To use and understand the Hopfield's neural network some general information of model are presented.

1.1. Generalized neuron model

Neural networks are the modern calculation systems that process information on the base as mimicking the human brain. Fundamental element of neural network model proposed by McCulloch in 1943 is summing input signals with corresponding weights and giving obtained sum under the operation of activation function (Fig.1). In effect output signal *i* of neuron is written as follows:

$$y_i = f\left(\sum_{j=1}^{m} w_{ij}u_j + w_{i0}\right) \equiv f(x_i) = \begin{cases} 1 & \text{for } x_i>0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where: u_j (j = 1, 2,...,m) – input signals, w_{ij} – synaptic weights (interconnection strengths between neurons i, j), y_i – output signals, w_{i0} – threshold value, $f(.)$ – the activation function.

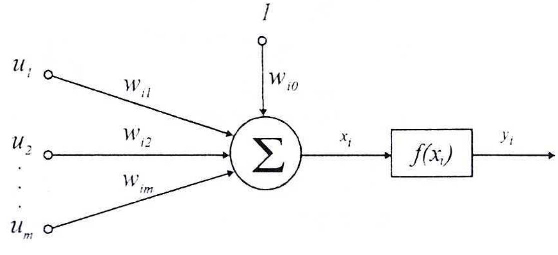

Fig. 1. Neuron model

On the beginning of using neural network the activation functions were in unit type (1). Currently, we apply the continuous nonlinear activation function named sigmoid unipolar function (2) or bipolar (hyperbolic tangent) (3)

$$f_u(x_i) = \frac{1}{1 + \exp(\beta . x_i)} \tag{2}$$

$$f_b(x_i) = \text{tg}h(\beta . x_i) \tag{3}$$

where: β – steepness of the function

The sigmoid function has special feature of its differential. For the case of the unipolar function we have

$$\frac{df_u(x_i)}{dx_i} = \beta f_u(x_i)[1 - f_u(x_i)] \tag{4}$$

and in the case of bipolar function

$$\frac{df_b(x_i)}{dx_i} = \beta[1 - f_b^2(x_i)] \tag{5}$$

The graphs of unipolar and bipolar activation functions are presented in Fig. 2.

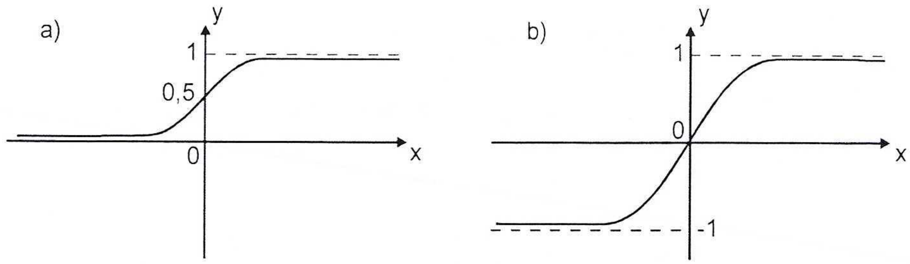

Fig. 2. The graphs of unipolar (a) and bipolar (b) activation function

On dependence of connecting neurons and of their mutual operation there are different types of network. Every type of neural network is tied with its corresponding method of weight learning [8, 9].

1.2. Hopfield's neural network

Hopfield's neural network is a sort of recurrent network [8]. It is commonly used to solve the behavior of such an array of electronic components that can be derived from Kirchoff's current law [4] in the differential equation:

$$\frac{du_i}{dt} = -\frac{u_i}{\tau} + \sum_{j=1}^{m} w_{ij} y_j + I_i \tag{6}$$

where: u_i – total weighted input signal at neuron i, t – the time evolution of each neuron i, τ – constant for neuron i, $y_j = f(u_j)$ – output signal – neuron that is a function of the u_i, I_i – external bias on neuron i, w_{ij} – the synaptic weights (interconnection strengths) from neuron j to neuron i, m – number of neurons in the network.

The set of differential equations (6) described the time evolution of the network. From a set of initial neuron output the state y_j of the network varies with time until convergence to a stable state where neuron output stops varying with time. The weights and biases determine the neuron outputs at this stable state. The stable state of the network correspond consequently to the local minimum of the Lapunov's energy function E [4, 12]:

$$E = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij} y_i y_j + \sum_{i=1}^{n} y_i I_i \tag{7}$$

In the convergence process the value E always decreases with regardless of the determined weights w_{ij} and the input biases I_i (the initial state of network). Basing on the equation (6) (7) the system of equations describing the dynamics (rate of change of neuron input) of the Hopfield's network can be rewritten:

$$\frac{du_i}{dt} = -\frac{\partial E}{dy_i} \tag{8a}$$

After doing differential of equation (7) we obtain the net value of i-th neuron as:

$$\frac{du_i}{dt} = -\sum_{j=1}^{m} w_{ij} y_j + I_i \tag{8b}$$

If the weights w_{ij} and biases I_i are arranged such that they represent an energy function with minimum of energy occurring at the stable state of the network the Hopfield's network can be used for energy minimization problem. The energy function describing the minimization problem has to be presented correctly and reached to minimum at the solution. In practical application, for determining optimal weights of network the calculation method based on the Lapunov's energy function (7) will be used. For concrete task of problem the target function will be accurately established. It is considered as the weight function. Through its comparing with the common form of the energy function we can obtain the equation system by which the values of weight will be determined.

If we like to determine Hopfield's network value HV_{ij} of the i-th row and j-th column neuron (Fig. 4) we can transform equation (8b) to ready formula following [12]:

$$HV_{ij} = -\sum_{k=1}^{m} \sum_{l}^{n} w_{ijkl} y_{kl} - \frac{1}{2} w_{ijkl} + I_{ij} \tag{8c}$$

The details of HNN have been presented in [4].

2. *Surface matching problem*

Surface matching problem is represented as follows: there are two sets of point obtained with different sensors but describing the same physical surface. In general, the points of two sets are in different reference systems and different distribution and are not identical. Our task depends on the comparing, merging surface data taken from two sets of points. The solution of this task is known as surface matching. Suppose further that between two sets there is an existence of mathematical, simply 3D similarity transformation. To use the Hopfield's neural network for surface matching problem, first we have to build the target function as a component of energy function, which is mathematically connected between the two sets of points. Inside of target function (as a component of total energy function) it is necessary to combine all the other evaluations (constraints) into total energy function. It is a system to evaluate the overall fitness between features of two sets. Minimum of total energy corresponds to a set of good matches.

In applications [3, 5, 11], an energy function describing mathematically the problem of matching surface is coded into Hopfield's neural network which converges to minimum of energy function.

2.1. Feature similarity

Suppose surface patches of set S1 in TIN models as a pattern were generated (Fig. 3a). The equation of planar surface patch (triangle) is expressed as

$$AX + BY + CZ + D = 0 \tag{9}$$

where A, B, C, D – coefficients calculated on the base of co-ordinates of three nodes of triangle.

The A, B, C are the co-ordinates of normal vector $\boldsymbol{N}$ of TIN, i.e., $\boldsymbol{N} = [A\ B\ C]^T$. Similarly, in the set S2 as a candidate we divide it into group per four points (Fig. 3b). The quadrilateral has independent four triangles with their corresponding normal vectors. The co-ordinates of four normal vectors were also calculated on the base of Eq. (9).

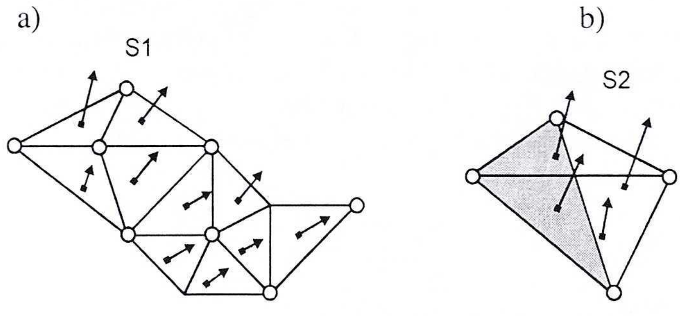

Fig. 3. Normal vectors of TIN models in set S1 (a) and of quadrilateral models in set S2 (b)

In this paper we consider the chosen points, surface patches and normal vectors as the features for correspondence and transformation. The problem of matching entire surfaces is due to the correspondences of quadrilateral model in set S2 with TIN model in set S1. If any quadrilateral model in set S2 could be belonged to certain TIN model of set S1, the their normal vectors would be equal each to other. The target function is based on the minimizing co-ordinates differences of two normal vectors:

$$\upsilon = (sRN_{tj} + R_o) - N_i \tag{10}$$

where $\upsilon = [\upsilon_{Atij}\ \upsilon_{Btij}\ \upsilon_{Ctij}]^T$ – the residuals in co-ordinate of two normal vectors; s – scale factor, R – the orientation matrix of two reference systems, $N_{tj} = [A_{tj}\ B_{tj}\ C_{tj}]^T$ – the normal vector of t-th triangle in j-th quadrilateral ($t = 1, 2, 3, 4; j = 1, 2, 3, ...l...n$), $R_o = [X_o\ Y_o\ Z_o]^T$ – the translation vector of two reference systems of set S1 and S2, $N_i = [A_i\ B_i\ C_i]^T$ – the normal vector of i-th TIN in set S1 ($i = 1, 2, 3...k...m$).

Suppose small rotation angles $d\omega$, $d\varphi$, $d\chi$ and small scale factor ds the equation (10) will be written in the scale form as follows:

$$\begin{bmatrix} \upsilon_{Atij} \\ \upsilon_{Btij} \\ \upsilon_{Ctij} \end{bmatrix} = \begin{bmatrix} A_{tj} & B_{tj} & C_{tj} & 0 & -1 & 0 & 0 \\ B_{tj} & A_{tj} & 0 & -C_{tj} & 0 & -1 & 0 \\ C_{tj} & 0 & -A_{tj} & B_{tj} & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} ds \\ d\chi \\ d\varphi \\ d\omega \\ dX_o \\ dY_o \\ dZ_o \end{bmatrix} + \left(\begin{bmatrix} A_{tj} \\ B_{tj} \\ C_{tj} \end{bmatrix} - \begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix} \right) \tag{11}$$

The values υ_{Atij} υ_{Btij} υ_{Ctij} are the residuals of co-ordinates of normal vectors, obtained after transformation. They represent the differences between features. The average discrepancy between fitted features (mean square difference – *MSD*) can be computed as:

$$MSD_{tj} = \sum_{t=1}^{4} (\upsilon_{A_{tij}}^2 + \upsilon_{B_{tij}}^2 + \upsilon_{C_{tij}}^2) \tag{12}$$

For every quadrilateral in the set S2 we have 12 equations in the form (11) to determine 7 unknowns. Basing on the principle of least squares solution to obtain the best fit between features the transformation can be solved by minimizing the sum of the squares residual.

2.2. Hopfield neural network solution for surface matching

Two-dimensional array *U*, can be constructed to present all of the possible matches between the features *T* in set S1 and features *q* in set S2. The rows of array represent *i* features (*i* =1, 2, 3,...*k*,...*m*) of normal vectors of TIN models. The columns of array represent *j* features (*j* =1, 2, 3,...*l*,...*n*) of normal vectors of quadrilateral models in the S2 (Fig.4)

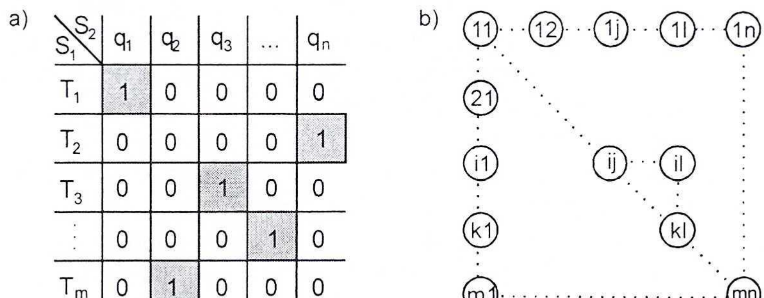

Fig. 4. A two-dimensional array of features (a) and of neural network (b)

The number 1 in the array (Fig. 4a) means selected points in the correspondence between *T* and *q*. On the contrary, number 0 means without a correspondence.

By application of Hopfield's neural network for matching corresponding features of two sets, the neurons would be also arranged in two-dimensional array similar to (Fig. 4b). In (Fig.4b) there are m and n neurons in row and column, respectively. The values of neurons can be 0 or 1. They represent negative or positive correspondence, respectively.

The equation (7) for a 2D neuron array in Fig. 4 can be now expressed in the new form:

$$E = -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{n} w_{ijkl}y_{ij}y_{kl} + \sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}y_{ij} \qquad (13)$$

The y in equation (13) describes the state of the neuron array and w is the connection strength (weight) between neurons, and I is a constant input (bias).

2.2.1. Building energy function of surface matching

In order to define the energy function, we describe the problem of surface matching as the sum of constraint conditions and the optimal goal (target) as follows [1, 3, 11]:

$$Energy = Constraints + Goal \qquad (14)$$

When the energy function is arranged in this particular way, the constraints become a part of the minimization process, which means that the constraints do not need to be treated separately, just weighted by their importance to the problem. The Hopfield's network process finds the minimum energy that represents a compromise between the goal and the constraints. In our problem the constraints and goal are following:

A. Constraint conditions:

1) one-to-one match constraint: every group of four points (quadrilateral) belongs only to one surface patch (triangle) of TIN.
2) orientation consistence constraint: It means that after performing transformation of surface S2 into S1 every feature has same orientation.

B. Optimal goal is based on the feature similarity: It means two features are exactly similar each to other; the mean square differences are minima.

A1. One-to-one match constraint:

Looking at the Fig. 4 we can consider that for every row and column there is only one neuron with positive configuration (number 1). It means that the connection strength (weight) between neurons in each row will be equal to 1 for $i = k$ and $j \neq l$, but equal to 0 in others. Similarly, we examined the weight in each column. The sum of energy in rows and columns can be represented as follows:

$$E_c = E_1 + E_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{n} y_{ij}w^{r}_{ijkl}y_{kl} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{n} y_{ij}w^{c}_{ijkl}y_{kl} \qquad (15)$$

where: $w^{r}_{ijkl} = \begin{cases} 1 \text{ for } i = k, j \neq l \\ 0 \text{ otherwise} \end{cases}$, $w^{c}_{ijkl} = \begin{cases} 1 \text{ for } j = l, i \neq k \\ 0 \text{ otherwise} \end{cases}$

A2. Orientation consistence

If pair of feature in set S1 and S2 is fitted together, it has an orientation similar to the other a pairs of corresponding features. The orientation can be calculated by using equation (11). The binary relationships of two matching pair can be classified as follows: when two features j and l of set S2 is matched to corresponding features i, k in the set S1, then transformation orientation between j, l and i, k should be consistent. It means the orientation between feature k and j, i and l is the same as ik and jl (Fig.4). For the scale between feature k and j, i and l or scale between vectors ik and jl the orientation consistence is same. The relationship of neuron can be modeled by energy function as:

$$E_0 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{n} y_{ij} w^{0}_{ijkl} y_{kl} \tag{16}$$

where: $w^{0}_{ijkl} = \begin{cases} 1 \text{ for } i = j,\ k = l \\ 0 \text{ otherwise} \end{cases}$

B) Optimal goal: Feature similarity

Two features are similar; it means they have approximate values with a predefined threshold. The values of two features may be shape, diameter, volume, area, and length. In our problem the features are the normal vectors of planar surface (triangle, see equation 11). To confirm the good fitness of pairs of feature the mean square differences MDS_{ij} (Eq. 12) has to be calculated. It can not be larger than a predefined threshold T. If this condition should be fulfilled, the energy could be decreased to minimum. The energy function for this purpose is modeled as:

$$E_s = \sum_{i=1}^{m}\sum_{j=1}^{n} f(MSD_{ij} - T)\, y_{ij} \equiv \sum_{i=1}^{m}\sum_{j=1}^{n} I^{s}_{ij}\, y_{ij} \tag{17}$$

where: $I^{s}_{ij} = f(MSD_{ij} - T) \equiv \text{(Eq. 2)} = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x \leq 0 \end{cases}$

Basing on the Eq. (14), (15), (16), (17) the total energy of Hopfield's network is:

$$E = P_c E_c + P_0 E_0 + P_s E_s = P_1 E_1 + P_2 E_2 + P_0 E_0 + P_s E_s \tag{18}$$

where: P_1, P_2, P_0, P_s – constants weighting the various energy parameters (price coefficients).

2.2.2. Feature correspondence

Comparing Eq. 18 with (13) the obtained equation system is as follows:

$$\begin{Bmatrix} w_{ijkl} = -\ 2(P_0 \cdot w_{ijkl}^{0} + P_1 \cdot w_{ijkl}^{r} + P_2 \cdot w_{ijkl}^{c}) \\ I_{ij} = P_s \cdot I_{ij}^{s} \end{Bmatrix} \tag{19}$$

By substituting Eq. (19) to Eq. (8c) we can calculate the Hopfield value *HV* of the energy to determine the output in the operation of each neuron (i, j) if

$$\begin{aligned} &HV_{ij} > 0 \text{ set the output of neuron } ij \text{ to } 1; \\ &HV_{ij} < 0 \text{ set the output of neuron } ij \text{ to } 0; \\ &HV_{ij} = 0 \text{ Neuron } ij \text{ remain unchanged.} \end{aligned} \tag{20}$$

The final state of the neuron array (Fig. 4b) represents the correspondence fitness between feature groups when system reaches the lowest energy.

The automatic process of surface matching using Hopfield neural network can be represented on the working system schema as follows.

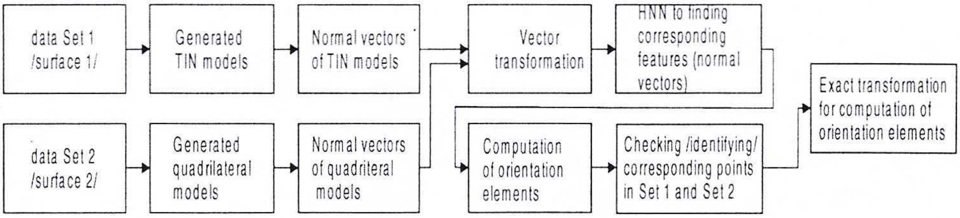

Fig. 5. Workflow of feature correspondence in surface matching.

As on the Fig. 5 we can consider that Hopfield neural network is a good tool of the decision for quickly determining a correspondence between two features from two sets.

CONCLUSION

Surface matching using Hopfield's neural network has a significant meaning in the cases when we like to merge and compare two surfaces of same object or terrain, represented by two data sets of points derived from different sensor images. In general, the two sets of the same physical surface are in different density, different reference systems and the points are not identical in two sets. For surface matching using Hopfield's neural network the surface created on the first data set has to be generated into the TIN models and the surface from second data set also has to be generated into quadrilateral models. The technique of surface matching using HNN has the advantage that the complex matching problem is solved in a simple way by minimization of the energy. The energy function includes two constraint conditions so-called one-to-one match; orientation consistency and the goal of transformation related with the minimization of target function. The features applied to solve surface matching using HNN are the normal vectors of TIN model generated on the first data set (first feature group) and of quadrilateral models generated on the second data set (second feature group). Basing on the condition (20) we can confirm the best correspondence fitness between feature groups when system HNN reaches the lowest energy.

Surface matching using HNN do not only has been applied in merging two data sets of same physical terrain for finding a deformation but also to be used to automatic navigation.

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• IAPRS – International Archives for Photogrammetry and Remote Sensing.
• PE&RS – Photogrammetric Engineering Engineering and Remote Sensing.
• IEEE - Institute of Electrical and Electronics Engineers.

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Możliwość zastosowania sieci neuronowej Hopfield'a do spasowania powierzchni terenu

S t r e s z c z e n i e

Przy porównaniu dwóch powierzchni terenu, które są reprezentowane dwoma zbiorami danych punktów otrzymanych z różnych sensorów tego samego obszaru wykorzystano sieć neuronową Hopfiel'da HNN (Hopfield's Neural Network) dla uzyskania rozwiązania optymalnego, tzn. kiedy funkcja energii tej sieci zdąża do minimum. Funkcję tę wyznaczono z sumy warunków zapewniających jednoznaczność cech należących do odpowiednich elementów powierzchni oraz z funkcji celu. Stabilny stan sieci HNN jest określony wówczas, gdy funkcja energii dąży do minimum. W takim przypadku zadanie dopasowania dwóch powierzchni jest najlepsze.

Люонг Чын Ке

Возможности применения нейронной сети Хопфильда для приспособления поверхности местности

Р е з ю м е

Сравнивая две поверхности местности, представленные при помощи двух множеств данных пунктов, полученных при использовании разных сенсоров для той-же самой области, была использована нейронная сеть HNN Хопфильда (Hopfield's Neural Network) для получения

оптимального решения, в котором функция энергии этой сети стремится к минимум. Эта функция определена из суммы условий обеспечивающих однозначность черт элементов поверхности, а также из функции цели. Стабильное состояние сети HNN является определяемым тогда, когда функция энергии стремится к минимум. В таком случае решение задачи приспособления двух поверхностей получаетця лучше всего.