GEODEZJA I KARTOGRAFIA
t. LI, z. 4, s. 191–211 (2002)

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Generalisation of a coast line – a multi-fractal approach

Simplification of a shape of a coastline is one of the best-described issues of quantitative generalisation. Schematisation of a coastline shape is a process, which may be relatively easily described by means of an algorithmic formula. However, the majority of algorithms consider only geometric aspects and river and road networks are generalised by means of the same parameters. Many described methods of direct transfer of subjective ways of manual generalisation to computer systems have turned out to be ineffective. Application of fractal analysis is an attempt aiming at objective implementation of a process of automated cartographic generalisation by means of selection of parameters of algorithms of simplification of lines, preceded by analysis of local geometric features of modelled objects. The, so-called mono-fractal dimension of objects, commonly used in cartometric analysis, D_f, specifies the averaged level of filling of available space only. The multi-fractal dimension of analysed objects, as, for example of a coastline, determined by means of a method proposed by the author, specifies the multi-fractal spectrum of dimensions, $D(q)$. The range of obtained values of the parameter $D_f (1.05 \div 1.42)$ allows for assumption that the coastline has multi-fractal properties.

In this paper the author proposes development of new descriptive and research tools, which may be used for investigation of local geometric features of objects presented on a map, as well as for simplification of shapes of objects in the process of cartographic generalisation.

Cartographic generalisation

In 1866 the German cartographer, E. Sydow, defined three basic issues of cartography, which he called ''reefs'' in the process of map generation:

1) presentation of the spheroid Earth's surface on a plane,
2) presentation of the terrain relief,
3) cartographic generalisation.

Although these issues were described in the middle of the 19^{th} century, they are still considered as the basic issues of the contemporary cartography. A map contains two types of deformations of its content. One type of deformations results from presentation of the non-developable Earth's surface on a plane. Such deformations can be accurately defined when the form of projection functions is known. The second type of deformations results from generalisation of the map content.

A map, as a mean of transfer of chronological information (information concerning location of objects and phenomena in the geographic space as well as relations resulting from such locations) is limited by the capacity of information channels (Ratajski, 1989). Information capacity of a map results from the scale, destination, applied methods of presentation and technical limitations. Therefore, in the process of cartographic transfer, reduction of source information by means of its generalisation is required.

The term generalisation originates from the Latin term *generalis*. The essence of cartographic generalisation is ''selection of the most important and important elements and their rational generalisation'' (Saliszczew, 1998), aiming at presentation of certain real features on a map, with particular respect to basic, typical and characteristic features of the reality.

Two various orientations may be found in professional literature, which consider generalisation as art, as well as the opposite orientation, which considers this process as a set of algorithmic rules, which may be automated. Following M.-J. Kraak and F. Ormeling (1998) generalisation is connected with unavoidable loss of certain amounts of source information. On the other hand, A. Makowski (2001) states, that generalisation should be identified with a cartographic method of modelling of the reality. Map production should sufficiently consider data acquired in the process of observations of the reality, resulting from explicit definition of features and destination of a map. Following this concept, generalisation cannot be identified with loss of information due to geometric simplifications. Generalisation is a method of model generalisation of data, performed in order to meet certain goals. The basic feature of the process of generalisation is maintenance of the basic structure and nature of geographic data. Similar opinions are presented by W. Ostrowski (2001), who aliases the process of cartographic generalisation with generation of a cartographic model, used for research of a selected fragment of the geographic space. Considering the complexity of this space, adaptation of the created model to perceptual and intellectual abilities of a map user, by means of rational simplification and generalisation of source data, is required.

Following D. E Richardson (1999) generalisation is a mechanism, which allows for abstracting and compressing of real data, with respect to graphic presentation and meanings. Abstracting is understood as a process of distinguishing important features of the geographic space and their relations. Therefore, in an artistic sense this process reminds creation of caricature, where some characteristic features of a modelled object are exaggerated and other features, which are considered less important, are neglected.

Molenaar (1996) specifies generalisation as a ''process of abstracting of geographic information presentation during the change of a map scale''. This process consists of two stages:

- generalisation of meanings, which relates to abstracting of information and settlement of rules,

- graphical generalisation, i.e. utilisation of geometric algorithms of simplification of shapes and graphic symbolisation.

However, generalisation cannot be considered as a mechanical procedure of sequential application of deterministic rules; it should be considered as a process "based on understanding" (R. Weibel, 1995). Familiarisation with a structure of generalised objects requires cartometric analysis of their shapes, mutual relations and spatial differentiation. Many authors attempted to solve the problem of cartographic generalisation, creating theoretical models of the generalisation process (Brassel and Weibel, 1988; Shea and McMaster, 1992). Work of L. Ratajski (1989), who defines the term of the so-called generalisation thresholds is considered as one of the best models of generalisation. Shea and McMaster (1989) proposed the model of generalisation based on three basic issues:

1. *Why do we generalise?* (definition of goals),
2. *When do we generalise?* (definition of situation),
3. *How do we generalise?* (selection of a method and specification of its parameters)

This model has been considered as an effort toward standardisation of the process of cartographic generalisation.

Development of computer technology, which occurred in the second half of the 20th century, contributed to numerous attempts to automate the process of cartographic generalisation. Development of computer-based generalisation origin from the middle of the sixties (Tobler, 1966). Due to the complexity of the process of cartographic generalisation, the majority of researches relate to automated reduction of the number of points that determine a line, by means of utilisation of simplifying operators. The simplifying operator (A. Iwaniak, 1998) is an elementary transformation of the map content, which may be expressed by a mathematical formula or by explicit description of the algorithm. The process of automated cartographic generalisation may be determined as sequences of such transformations performed with the use of appropriate values of parameters of a given method.

Five categories of algorithms of reduction of the number of points which create a linear object are mentioned in professional literature (T. Chrobak, 1999): independent point algorithm (for example the n-th point method), procedures of local processing (for example the algorithm of perpendicular distance), unconditional local processing (for example Reumann-Witkam algorithm), conditional expanded local processing (for example Lang algorithm) and global procedures (Douglas-Peucker: D-P algorithm).

Z. Wang and J. C. Muller (1998) proposed a method of generalisation of linear object based on analysis of shapes of objects. The basis of this process is recognition and maintenance of the basic structure of a curve. Analysed shape is considered as a set of elementary graphical objects (of various shapes, sizes and curvature) which are generalised by means of operators of deletion, simplification, smoothing and zooming. The majority of simplification algorithms is based on the concept of the, so-called, characteristic points (G. Dutton, 1999). This issue has been widely discussed in professional literature; for example in the opinion of F. Piątkowski (1961), generalisation of linear objects should be based on the, so-called, generalising points, characteristic for each topographic line – the start and end of the line and extreme points of deviation of the curvature. R. McMaster (1991) lists a series of disadvantages of utilisation of operators of simplification of lines:

- no functional relation between source and resulting data,
- lack of a shape optimisation method of open polygons with consideration of similarity relations of shapes.

Such problems may be partially solved by means of a method developed by T. Chrobak (1999) – the so-called, elementary triangle method. It allows for selection of characteristic points of a topographic line, and also for maintenance of graphical similarity of a simplified object to source data, by means of creation new points, depending on the level of complexity of a curve.

Topfer radical law

This paper presents an attempt to objectivize criteria of determination of the Douglas--Peucker (1973) global method of line simplification. The D-P algorithm is based on two terms: the basic line and the tolerance zone. The basic line (the basic section) connects the starting and ending points of a simplified line and the tolerance zone is a user-defined parameter of this method. The wider the tolerance zone is the stronger simplification of a shape of a generalised line. The Douglas-Peucker algorithm is determined as follows:

- orthogonal distance to the basic line is determined for each point of the source line,
- if all distance are shorter than the assumed tolerance zone, the source line is simplified to the basic line,
- otherwise, the point which is located within the longest distance from the basic line, becomes the point of division of the source line into two new lines,
- for each of them the whole procedure is repeated.

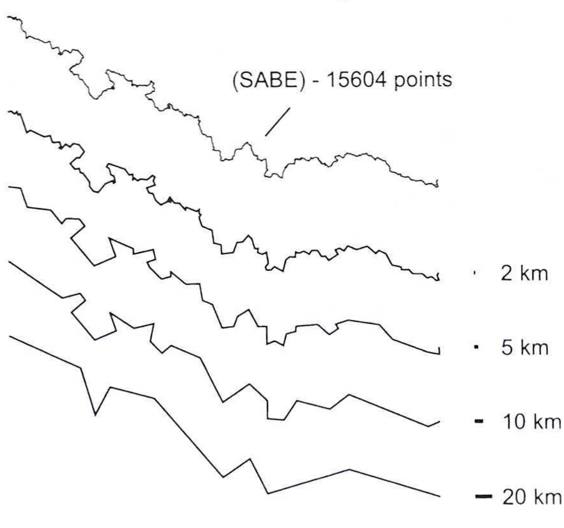

Fig.1. Generalisation of the southern frontier of Poland with the use of Douglas-Peucker algorithm

Obtained results depend on the width of the tolerance zone, the number of source points, their distribution and density (Fig. 1). Correct generalisation of linear objects performed with the use of the global Douglas-Peucker algorithm, requires many initial cartometric analyses. Determination of the optimum parameter of the method for the given scale interval – the width of the tolerance zone – is not a trivial problem due to non-linear relation of this parameter with the length (and shape) of the simplified line (G. Dutton, 1999). The attempt of objectivisation of this method consists of utilisation of the, so-called, radical law. T. Topfer (1966) presented a method of cartographic generalisation based on the ''radical law''. This law is described by the formula:

$$n_2 = n_1 \cdot \sqrt{\left(\frac{M_1}{M_2}\right)} \qquad (1)$$

where: n_2 – the number of elements on a resulting map, n_1 – the number of elements on a source map, M_2 – denominator of the resulting map scale, M_1 – denominator of the source map scale.

Topfer has proposed modification of this formula in his later works:

$$n_2 = n_1 \cdot \sqrt{\left(\frac{M_1}{M_2}\right)^x} \qquad (2)$$

where: $x = 1$ for point objects, $x = 2$ for linear objects, $x = 3$ for surface objects.

This author stresses that his method may be only used for objectivisation of some aspects of quantitative generalisation of topographic maps. Topfer has generalised the outline of a plan of Vienna, decreasing the number of sides of an outline, according to the specified formula, resulting from modification of the map scale. He stated that various coefficients should be applied in the formula, depending on types of generalised objects.

However, acceptance of such an assumption does not solve the problem of objectivisation of selection of values of parameters of generalisation algorithms; it requires generation of a set of tables of obligatory values of coefficients, for example 1.2 – for administrative borders, 1.4 – for river networks etc. This method also requires the introduction of additional assumptions concerning the homogeneity of the structure of a river network or a coastline. Therefore it should be stated that methods of global ''recognition of shapes'' do not always lead to determination of appropriate parameters of generalisation due to high spatial diversification of source data. So, in the case of objects of high diversification of the structure, it is necessary to expand cartometric global analysis by a method of local characteristics of a shape and function of such objects (G. Dutton, 1999).

Fractal analysis

Fractal geometry, developed by B.B.Mandelbrot (1982) is one of contemporary methods of analysis of the structure of spatial objects. In 1610 Galileo stated that geometry is the language of nature and its alphabet consists of triangles, circles and other geometric figures. Many shapes, which naturally exist, such as clouds, coastlines or chains of mountains, are too complex to be described by means of the language of classical Euclidean geometry. Observations natural phenomena, such as river networks or coastlines prove, that large-scale structures consist of smaller components, which are similar in their form and construction. Such structures, which cannot be described by means of algebraic formulae, were defined as geometrically ''shapeless'' ones for a long time.

Simple Euclidean shapes loose their structure after magnification. A highly magnified circle reminds a straight line without any curvature. However some shapes exist, which present complexity of structure independently of the level of magnification. Benoit Mandelbrot called them *fractals*. Let us consider a shape of a developed coastline, presented on a map (Fig. 2). If a map at a larger scale is available for an arbitrary fragment of the coast, similar distribution of bays and headlands may be observed. Smaller bays and headlands occur in every bay; in these bays smaller bays occur etc. New details of the structure similar to the image of the entire coast are observed at every level of observations (I. Stewart, 1996).

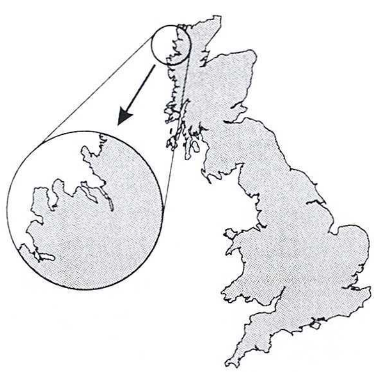

Fig. 2. A fragment of a coastline of the Great Britain

Mathematical figure of similar, although strictly deterministic properties, is Koch's curve. The Swedish mathematician, Helge von Koch, specified recurrent description of generation of this curve as well as its unusual features. Let us assume the unit section (Fig. 3) as the starting object. Let us divide it into three equal parts and insert an equilateral triangle without a base, instead of the central part. The resulting figure occurs in minified copies in successive recurrent steps. The same algorithm is applied for each resulting section. The final shape of a curve (after infinite number of iteration steps) does not consist of any straight sections. This curve has infinite length!

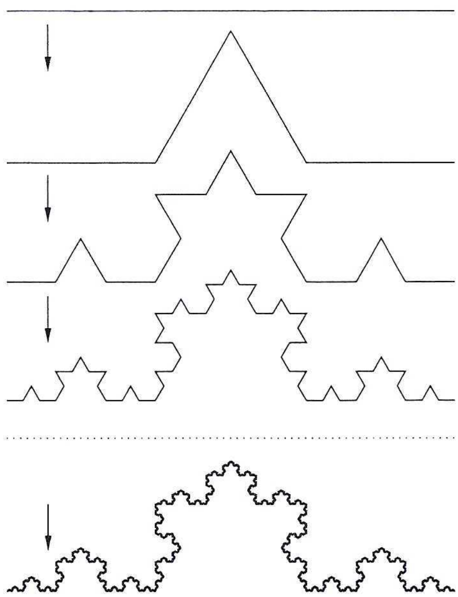

Fig. 3. Algorithm of generation of the Koch's curve

It is obvious, that the natural coastline cannot be modelled by means of this curve since the nature does not carve coastlines in the form of equilateral triangles. However, the deterministic Koch's curve, as well as the real coastline, have similar structure after magnification. The magnified fragment of a coast still looks as a coast cut with bays and headlands. This property is called self-similarity. In the case of Koch's curve it has explicit nature, in the case of a coastline it has statistical nature. This property is the basic property of fractals. The term *fractal* originates from the Latin adjective *fractus* and the verb *frangere*, which mean breaking and creation of irregular parts. Following Mandelbrot this word is also related to the English term *fragmented*, so it means both, an irregular shape as well as a shape, which consists of parts creating a complicated and self-similar structure.

Another, interesting property of fractals is their dimension, which is understood as a coefficient of complexity of the structure, as well as the level of filling of the available space. From the topological point of view, a point is a zero-dimensional object, a line is a one-dimensional object and a figure, as a square, is a two-dimensional object. Homeomorphic transformations (i.e. continuous, mutually explicit transformation, which continuously has its reverse transformation), which act with an object, does not change its topological properties (Fig. 4). That is why since topological transformations do not preserve shapes of objects. This means that, foe example, a section and the Koch's curve are topologically equivalent and characterised by the topological dimension $D_t = 1$.

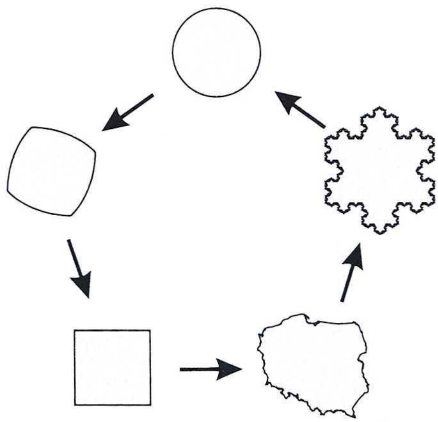

Fig. 4. Topological equivalency of flat figures

Fractal dimensioning

Therefore, in order to be able to differentiate objects of complex geometry, another definition of a dimension is required. Mathematically, the strict definition of a fractal dimension of an object (D_f) is relatively complicated. However, in many cases, the dimension specified by means of a box method, described below, may be sufficient approximation of this definition. Let us consider a geometric object located on a regular network of the grid dimension equal to s. This is implementation of a classical example of Mandelbrot of 1967 – fractal dimensioning of the Great Britain's coastline (Fig. 5). The number of grids, which contain (or only touch) figures is marked as N. This number obviously depends on the size of squares, which create the network – $N(s)$. Let us repeat this procedure several times for various sizes of the grid s and present the result on a logarithmic diagram. The directional coefficient of the straight of regression, determined by means of the least square method, is marked as D_p – as a box, fractal dimension.

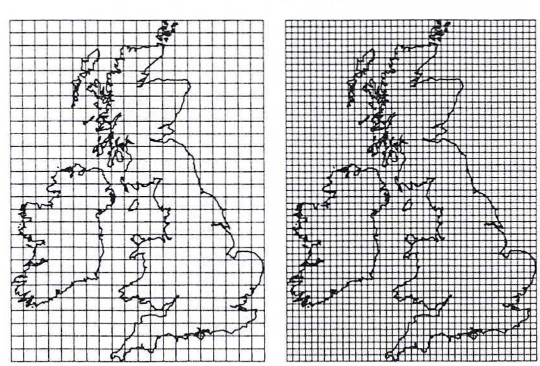

Fig. 5. Calculation of fractal dimension of the Great Britain's coastline by means of the box method

For the Great Britain's coastline $D_p \approx 1.28$. Therefore it turns out that the Great Britain's coastline has more complex structure than the infinitely crooked Koch's curve ($D_p = \log 4/\log 3 \approx 1.26$). When the fractal dimension is determined by means of the box method, it is important that the root mean square error of standard adjustment or of the determination coefficient R^2 is simultaneously calculated. The R^2 considerably lower than 1 (e.g. $R^2 < 0.9$) means that the hypothesis concerning fractal nature of an investigated object should be rejected.

Obviously, tools of fractal geometry are not a universal method of description of the nature. B. Mandelbrot (1982) and J. Feder (1988) state that the geometry of the nature is the geometry of fractals. However, mathematically "clean" recurrent deterministic fractals do not occur in the nature. Similarly, ideal circles, ellipses or straight lines do not occur in the nature. However, the ellipse equation is a good approximation of a planetary orbit, as the random Koch's curve (in which the direction of insertion of an equilateral triangle is randomly specified) allow for coastline modelling (R. Olszewski, 2001).

Random fractals are characterised by statistical self-similarity (each fragment of an object is characterised by the same probability distribution function). This means, that every fragment is visually similar to the entire figure, with the accuracy of the scale of observation (P. Burrough, 1993).

Fractal dimension D_f of linear and surface objects contains unique information concerning metric parameters of such objects. Fractal dimension of an object points how metric parameters of an object will change in the process of cartographic generalisation (A. M. Berlant, 1998). Linear objects of low fractal dimension ($D_f \rightarrow 1$) (administrative boundaries, road networks) maintain their length during the change of scale. Linear objects of high fractal dimension ($D_f \rightarrow 2$) (meandering rivers, coastlines) rapidly "lose" their length in the process of generalisation (B. Klinkenberg, 1992).

Determination of the value of fractal dimension of mapped objects allows for modification of values of parameters of generalisation algorithms (B. Buttenfield, 1989). Q. Wang, Y. Hu and J. Wu (1995) attempted to practically apply fractal analysis in the process of cartographic generalisation. The authors developed a method of determination of parameters of an algorithm of simplification of shapes of a line depending on its fractal dimension D_f. The objective of this work was to investigate functional or statistical relations between "the level of generalisation", i.e. the value of generalisation interval (quotient of denominators of the resulting and the source map – M^1/M^2) and the parameter of the line simplification algorithm. However, many important problems of cartographic generalisation, as for example, the issue of heterogeneity of source data, were not solved by this work.

Mono-fractal generalisation

Let us consider shapes of the Polish administrative boundaries and the Great Britain's and Island's coastlines (Fig. 6). Source data originates from the European SABE Programme (Seamless Administrative Boundaries of Europe). For this data, the geometric accuracy of point location is not higher than 30 m. For example, the Great Britain's coastline is composed of 14,770 points (1.41 for each kilometre of the line length). For such source data and attempt was made aiming at objective determination of parameters of the process of cartographic generalisation. The level of generalisation of a coastline is always lower than of other map elements; this line marks the border between two basic areas of the Earth's surface: lands and waters. Generalisation of a coastline, river networks and hipsometry is often considered as the determinant of the given school of cartography (W. Pawlak, 1971). This process is based on geographic analysis of the nature and structure of such objects.

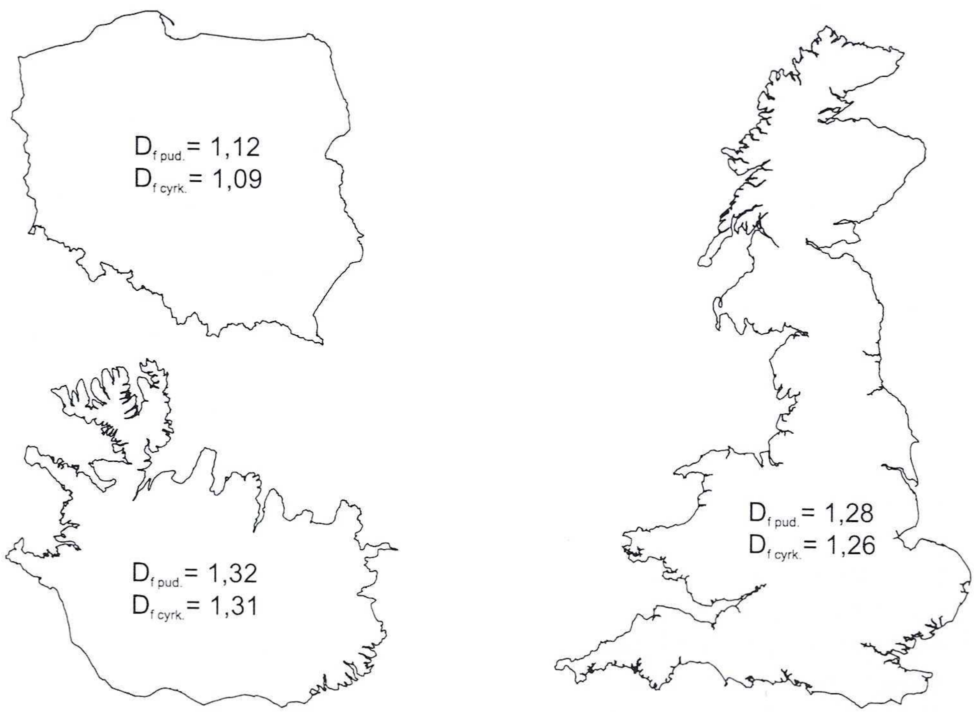

Fig. 6. Source data

The general basis of the process of generalisation is hierarchic diversification of elements of a set, as well as diversification of internal structure of these elements. Details disappear following decreasing of a map scale. Sequence of such disappearing corresponds to internal diversification of the structure of simplified objects.

Every process of generalisation of shapes of lines includes the less important details (at the given level). Importance of details is understood as importance of a given detail for "geometric similarity" acquired from the shape geometry at the higher level of generalisation (W. Pawlak, 1993). This results in unequal or selective neglecting details, without disturbing "the geometry of the higher level shape".

T a b l e 1. Fractal dimension of source data

	D_f Box method	D_f Compasses method
Island	1.31 (R^2 = 0.99)	1.33 (R^2 = 0.98)
Poland	1.12 (R^2 = 0.99)	1.09 (R^2 =0.99)
Great Britain	1.28 (R^2 = 0.99)	1.26 (R^2 = 0.99)

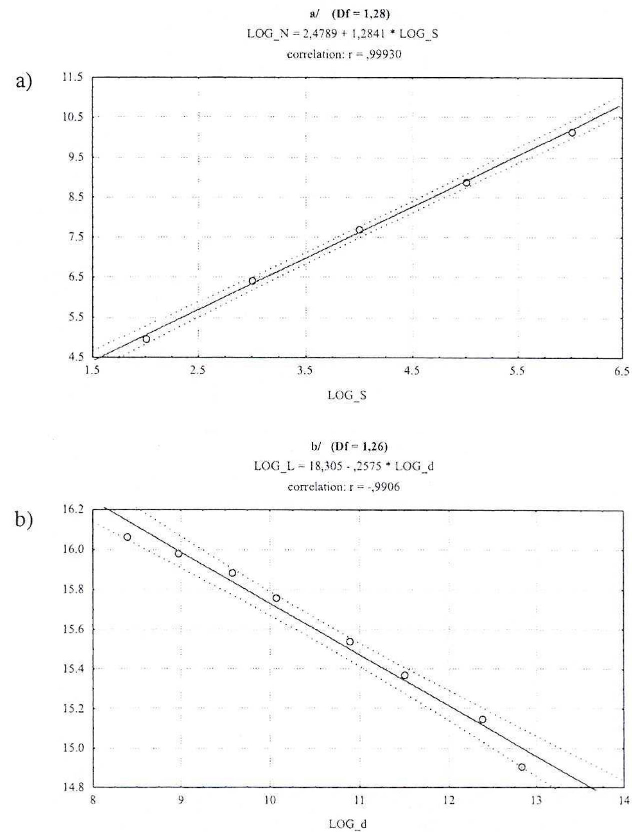

Fig. 7. Calculation of fractal dimension of the Great Britains coastline by means of box: a — and compasses, b — methods

The global Douglas-Peucker algorithm was assumed as the shape generalisation method. Fractal dimension of source data was calculated by means of the box method as the, so-called, compasses method, with the direct utilisation of properties of the Douglas-Peucker algorithm (table 1). When the width of the zone of tolerance d in the D-P algorithm is changed, the resulting image of various shape and length L is obtained. Presenting results of measurements on a logarithmic diagram and calculating the directional coefficient of the regression line, we obtain the compasses fractal dimension of an investigated shape:

$$L(d)\ c \cdot d^{1-D} \tag{3}$$

where: c – constant

After finding logarithms of both sides of the equation we obtain:

$$\mathrm{Log}(L(\{d\}) = \log(c) + (1-D)\log(d) \tag{4}$$

Knowing the scale interval, i.e. the quotient of denominators of the resulting and the source maps – (M_2/M_1) and the length of the curve on the source map – L_1, it is possible to calculate its length on the resulting map L_2, using the formula based on the modified radical Topfer law. The exponent x in the original formula (2) may be considered as equivalent with the topological dimension of an object, increased by 1: $x = D_t + 1$. However, in the opinion of J. C. Muller the radical Topfer law should be modified, considering the fractal dimension of generalised objects.

$$L_2 = L_1 \cdot \left(\frac{M_2}{M_1}\right)^{1-D_f} \tag{5}$$

Substituting L_2 to the formula (4) we obtain:

$$d_2 = e^{\frac{\log L_2 - \log C}{1-D}} \tag{6}$$

So the modified radical Topfer law allows for determination of the definite value of the parameter of the line simplification method, for example the width of the tolerance zone in the D-P algorithm, with the use of the specified scale interval. This method may be obviously applied for other line shape simplification algorithms (e.g. Lang algorithm). H. Zongyi, Ch. Tao, P. Xiaoping, G. Lizhen (2001) developed the method of generalisation of the coastline and the river network based on analysis of fractal dimension of these objects. The proposed method allows for simplification of shapes of objects, and also – in the case of river networks – for selection of the specified number of elements.

However, the described method is based on an assumption that such objects as river networks or coastlines are homogenous, i.e. they have homogenous spatial structure, which is characterised by the constant value of fractal dimension. However, in the case of such extensive objects as coastlines, this assumption is false.

Basing on analysis of cartometric deformations of lengths and angles of the Australian coastline, resulting from generalisation, W. Pawlak (1971) came to the conclusion that the type and size of such deformations depend not only on the scale and destination of a map and on the characteristic of the cartographic school, but also on spatial location. This author notices that deformation s of various fragments of the Australian coast are diversified. For the entire Australian coastline reduction of length when transferring from 1 : 5 000 000 to 1 : 80 000 000 equals to 64%, reaching 50% for its north-west part and only 8% for its south-west part. This diversification is connected with higher development of this part of the coastline (Fig. 8).

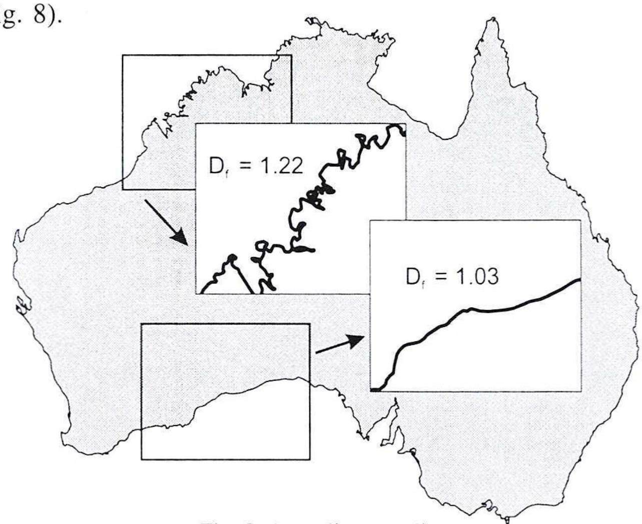

Fig. 8. Australian coastline

The coastline is a spatially heterogeneous object. This means it is impossible to perform correct automatic generalisation of the entire shape of the coastline, according to unified criteria. For the south-west part, of lower level of terrain relief diversification (D_f = 1.03) considerably slower degradation and disappearing of details of secondary meaning may be observed than in the case of the north-west part, of much higher relief diversification (D_f= 1.22). This means that the mono-fractal dimension, calculated for the entire object of high geographic extension, consist information concerning the "average" complexity of the spatial structure only.

Let us consider now the shape of the Great Britain's coastline (Fig. 9). For the entire coast D_f equals to 1.28. The fractal dimension of the eastern part of this coast, determined by means of the box method, equals to 1.17. For the western part, of much higher level of development, it is equal to 1.36. Utilisation of the D-P algorithm with the parameter of the width of the zone of tolerance specified by means of formula (6) and the value D_f= 1.17 and D_f= 1.36 allows to obtain two various shapes of the coastline. This leads to the conclusion that the fractal dimension of an object is not the sufficient factor of complexity of its geomorphologic spatial structure, and, therefore, it is not an objective factor of appropriate selection of parameters of cartographic generalisation.

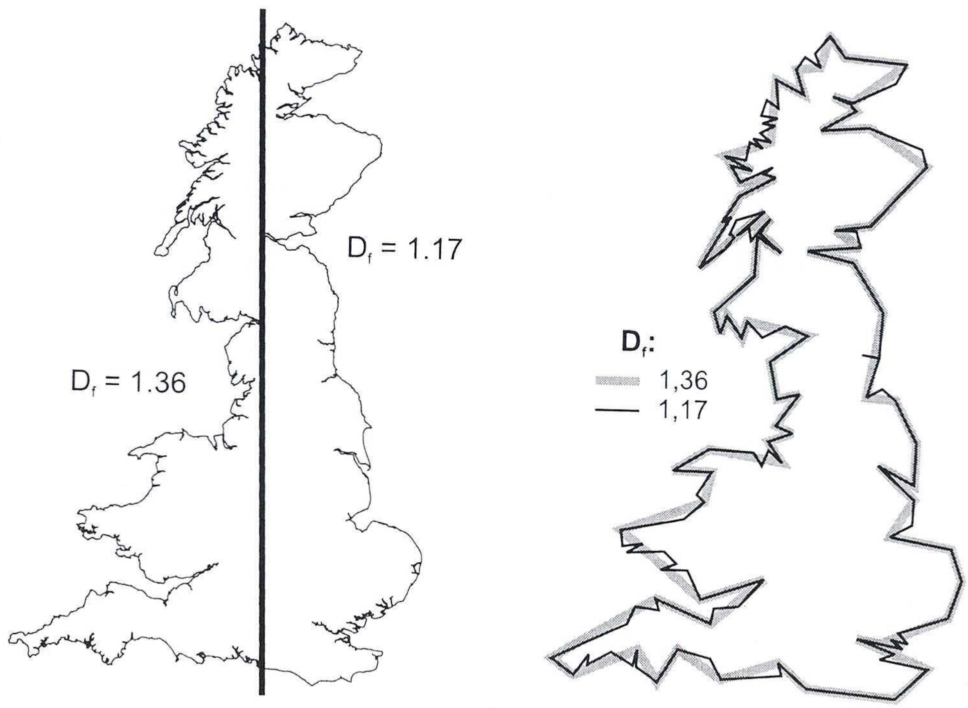

Fig. 9. Generalisation of the coastline of the Great Britain by means of the D-P method

Multi-fractal generalisation

Mono-fractal dimension, as the basic measure of fractal analysis, contains information on the average level of complexity of the structure of geometric objects and its self-similarity. In the case of figures, which are strictly (deterministically) self-similar, D_f – by the contrast with the topological dimension D_t – is the factor of the level of filling of the available space. Deterministic fractals are homogenous, the fractal dimension of an arbitrary fragment of a real object, determined by means of a specified method, does not depend on an assumed scale of observations (the size) or on selection of this fragment.

Real objects and natural phenomena, such as geomorphologic forms or natural coastlines, are self-similar only in the statistical sense. Stochastic fractals are often heterogeneous – and spatially diversified. Their fractal dimension is the function of spatial location. The D_f value, calculated for a mountain chain or a river network, depends both on the size of the test site as well as on its spatial location. The total fractal dimension of figures, which are statistically self-similar, informs about "average" properties of an object at the assumed scale of observations. Therefore, the obtained value does not correctly characterise the entire structure (Voss, 1990).

Multi-fractal dimensioning is the solution of this problem (Feder, 1988). It consists of determination of a set of numerical meters, which contains information on distribution of the fractal dimension for the given structure. Utilisation of multi-fractals means the transfer from geometry of sets to investigation of geometric properties of measurements (Falconer, 1997). It is also the transfer from characteristics of objects, defined by the number to functional characteristics (Mandelbrot, 1982).

In the "classical" fractal approach objects are considered as scaled geometric sets. Multi-fractal technique consists of identification of spatial distribution of investigated phenomena with the mathematical field of variable density of a measured parameter (Pecknold, Lovejoy, Schertzer, Hooge, 1997). In the practice of calculation, determination of the continuous spectrum of the numerical range of D_f values consists of determination of the finite number of parameters.

R. Voss (1990) proposed to base the numerical procedure of multi-fractal dimensioning on the technology of a movable basic field of variable size. The investigated object may be expressed in the form of a set of points located in the *n*-dimensional Euclidean space. For such a set, the probability density function may be defined – $P(m, L)$. This function defines the probability of occurrence of m points in the basic field of the side (or radius) equal to L. The analysis is performed for various scales of observations, specified by the size of the basic field L. The function $P(m, L)$ is normalised:

$$\sum_{m=1}^{N} P(m, L) = 1 \tag{7}$$

for all values of L.

This function may be considered as the standard statistical distribution, and q order moments may be defined for this function:

- The moment of the first order is called the "mass dimension" of the object:

$$M(L) = \sum_{m=1}^{N} m \cdot P(m, L) \tag{8}$$

- In general, the moment of the q order ($q \neq 0$) is expressed by the formula:

$$M^q(L) = \sum_{m=1}^{N} m^q \cdot P(m, L) \tag{9}$$

- The moment of zero order defines the entropy of configuration, when the space is divided into fields of size of L:

$$S(L) = \sum_{m=1}^{N} \ln m \cdot P(m, L) \tag{10}$$

For fractal objects the following relation occurs: $M^q(L) \; \alpha \; L^{Dq}$ (for $q \neq 0$) and $e^{S(L)} \; \alpha \; L^D$ (for $q = 0$) (R. Voss (1990). The fractal dimension connected with the q order moment may be defined by means of the linear regression method on a logarithmic diagram:

$$D_q = \frac{1}{q}\langle\frac{\partial \log M^q(L)}{\partial \log L}\rangle \qquad \text{for } q \neq 0 \tag{11}$$

$$D_0 = \frac{1}{\log e}\langle\frac{\partial S(L)}{\partial \log L}\rangle \qquad \text{for } q = 0 \quad \text{(Voss, 1990).} \tag{12}$$

The D_f dimension of deterministic fractals, defined basing on moments of an arbitrary order is constant and equal to the dimension of self-similarity. The fractal dimension of self-similar objects, in the statistical sense, determined by means of the discussed method, defines multi-fractal spectrum of dimensions – Dq. Multi-fractal analysis allows for decomposing of the initial object into locally ''dense'' and ''loose'' areas (Milne, 1996), which are characterised by various properties of self-similarity.

Theoretical methods of multi-fractal dimensioning, described above, were adapted for the purpose of analysis of coastlines. The majority of analytical procedures described in professional literature (Voss, 1988) may be applied for source data recorder in raster forms. In order to determine the multi-fractal spectrum Dq of investigated coastlines, registered in the basic GIS system as vector layers, own numerical applications were developed, which define q order moments of the $P(m, L)$ function.

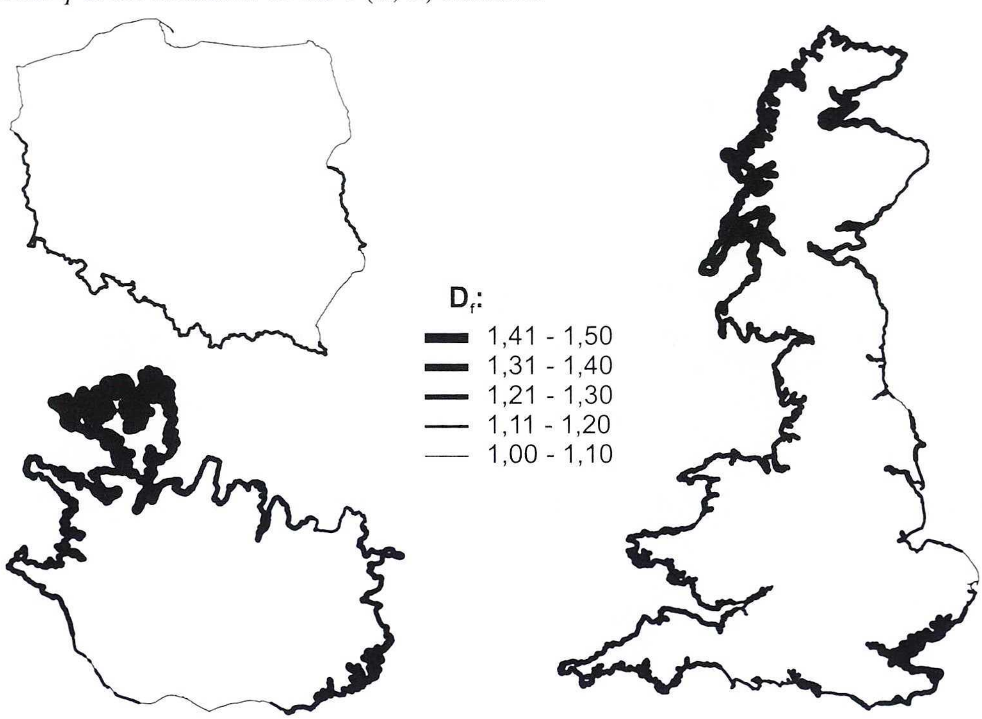

Fig. 10. Multi-fractal dimensioning of source data

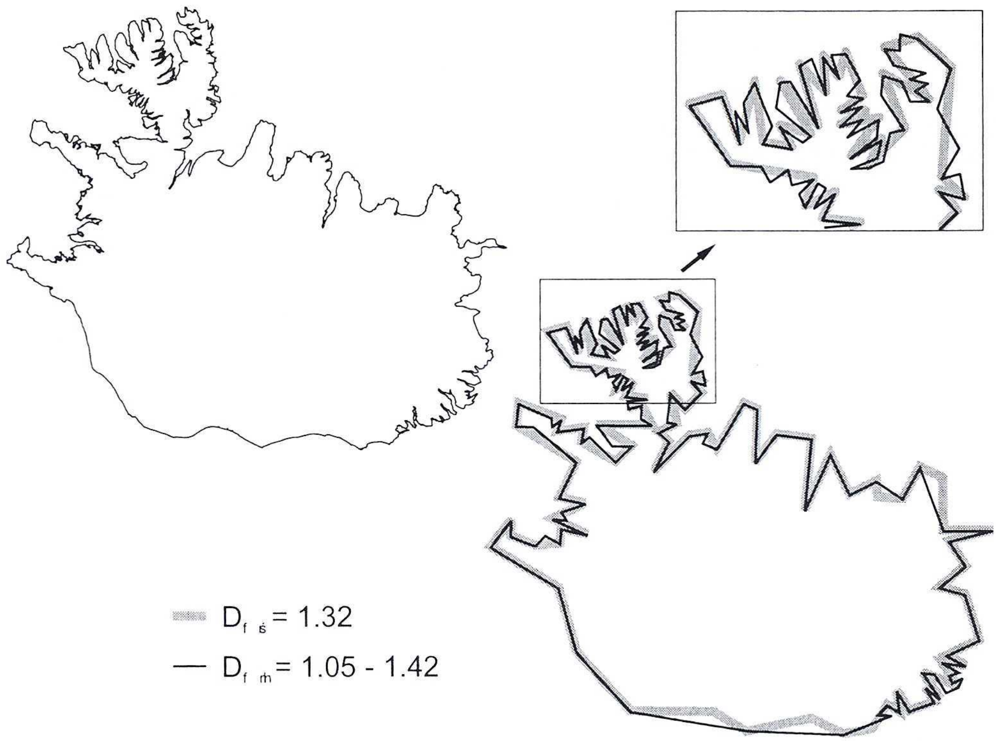

Fig. 11. Multi-fractal generalisation of the Island coastline

Coastlines (continuous linear objects) were substituted by a discrete set of evenly located points (every 50 m). For such defined source set the application generates movable basic fields in the shape of circles. Utilisation of a circular field results from its optimum shape. In the opinion of W. A. Czerwiakow (1978) the circular field, as the most compact figure, is the best basis for measurement of values of phenomena, which occur around an arbitrary point (a circle centre).

For every point of measurement the application defines 5 circular fields of such radii, that every time the area is increased two times. For each figure the number of model points, contained within this figure, is specified.

For every point of measurement the probability density function may be defined – $P(m, L)$. This function defines the probability of occurrence of m points in the basic field of the radius L (the scale of observation $L = 100$ km was assumed for the discussed work). The application calculates moments of the function $P(m, L)$ for $q = -3, -2, -1, 0, 1, 2, 3$. On this basis the fractal dimension of the coastline is calculated for the assumed scale of observation.

The above procedure was applied for determination of the multi-fractal dimension of source data (Fig. 10). Significant part of Polish boundary is characterised by the dimension D_f close to 1. The exceptions are: the river boundary along the Bug river and the mountainous watershed boundary. Analysis of geometry of the Great Britain's and Island's coastlines are interesting. The northwestern coast of Island and the fragment of the western coast of Scotland are characterised by the fractal dimension close to 1.4. They are fragments of a coastline, which is well developed and which is under intensive processes of sea erosion. Therefore, multi-fractal dimensioning, which defines the local complexity of structure of objects, may be a tool of geographic analysis. It is the basis for objective and spatially diversified selection of parameters of algorithms of generalisation of shapes of modelled objects (Fig. 11). This does not mean the possibility to substitute subjective methods of manual generalisation with the automated multi-fractal generalisation, but it allows for objective analysis of shapes of objects. This is the solution which may be included in the strategy of ''enhanced intelligence'', proposed by Weibel (1991), an intermediate method between utilisation of algorithms with assumptive parameters and expert systems, which are currently developed.

CONCLUSIONS

Simplification of shapes of lines is one of issues of quantitative generalisation, which have been well described in professional literature. Schematisation of a line shape is a process, which may be relatively easily expressed by means of an algorithmic formula. However, the majority of algorithms considers only geometrical aspects and generalises road or river networks, using identical parameters. Many attempts of direct transfer of subjective methods of manual generalisation to computerised systems turned to be ineffective. However, satisfactory results were obtained for particular operations, related to quantitative aspects of generalisation, as, for example, simplification of shapes of lines (A. Iwaniak, 1998; T. Chrobak, 1999). Utilisation of fractal analysis is an attempt of objectivisation of the process of automated cartographic generalisation by selection of parameters of line simplifying algorithms, preceded by analysis of local geometric properties of modelled objects.

The dimension of deterministic fractals D_f (e.g. the Koch's curve) defined based on moments of an arbitrary order has the constant value, which is equal to mono-fractal dimension of self-similarity for this object. This means that – independently on spatial location – every fragment of the Koch's curve has the same geometric properties, equal complexity of shapes and fills the available space at the same level. The fractal dimension of self-similar objects, defined by means of the discussed method in the statistical sense only (for example coastlines) defines multi-fractal spectrum of dimensions – $D(q)$. Extension of obtained values of the parameter D_q ($1.05 \div 1.42$) allows to assume that a coastline has multi-fractal properties. The mono-fractal dimension – D_f specifies the averaged level of filling of the available space. Geometric properties of real coastlines depend on their spatial location, size of the analysed site and on the level of cartographic generalisation of source data.

The terms of fractal geometry, presented above, allow for creation of new descriptive and research tools, which have been applied in widely understood Earth sciences since the seventies. Many physical processes were explained by means of fractal analysis, however fractal geometry has been applied in cartography and geomorphology mainly for the needs of description of existing terrain forms and not of the process of their generation (Falconer, 1997). Fractals are not the set of completed geomorphologic or hydrographic models, but they are a set of new ideas, concerning the method of shaping and visualisation of abiotic components of the natural environment (Klinkenberg, 1992).

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Received Juna 13, 2002
Accepted October 28, 2002

Robert Olszewski

Generalizacja linii brzegowej – próba podejścia multifraktalnego

S t r e s z c z e n i e

Upraszczanie kształtu linii należy do najlepiej opisanych zagadnień generalizacji ilościowej. Schematyzacja kształtu linii jest procesem, który stosunkowo łatwo można ująć w postaci formuły algorytmicznej. Większość algorytmów uwzględnia jedynie aspekt czysto geometryczny, generalizując sieć rzeczną i drogową wykorzystując identyczne parametry. Wiele opisanych w literaturze sposobów bezpośredniego przeniesienia subiektywizacji procesu automatycznej generalizacji kartograficznej poprzez dobór parametrów algorytmów upraszczania linii, poprzedzony analizą lokalnych własności geometrycznych modelowanych obiektów.

Powszechnie stosowany w analizie kartometrycznej tzw. wymiar monofraktalny obiektów – D_f, określa jedynie uśredniony stopień wypełnienia dostępnej przestrzeni. Wymiar multifraktalny analizowanych obiektów, np. linii brzegowych wyznaczany zaproponowaną przez autora metodą, określa multifraktalne spektrum wymiarów – $D(q)$. Rozpiętość uzyskanych wartości parametru D_f (1,05 ÷ 1,42) pozwala sądzić, iż linia brzegowa ma własności multifraktalne.

Autor zaproponował opracowanie nowych narzędzi opisowych i badawczych, służących zarówno badaniu lokalnychwłasnościgeometrycznych prezentowanych na mapie obiektów, jak i upraszczaniu ich kształtu w procesie generalizacji kartograficznej.

Роберт Олшевски

Генерализация береговой линии — попытка мультифрактального подхода

Р е з ю м е

Упрощение формы лини это лучше всего описанный вопрос количественной генерализации. Схематизация формы линии это процесс, который сравнительно легко может быть представлен в виде алгоритмической формулы. В большинстве алгоритмов учитывается только чисто геометрический аспект, проводя генерализацию речной и дорожной сети, используя идентичные параметры. Много из описанных в литературе способов непосредственного переноса субъективных методов ручной генерализации в компьютерные системы оказалось в многих случаях неэффективным. Применение фрактального анализа это попытка объективизации процесса автоматизированной картографической генерализации путём подбора параметров алгоритмов упрощения линии, предшествованного анализом местных геометрических качеств моделированных объектов.

Повсеместно применяемый в картометрическом анализу т.н. мультифрактальный размет объектов — D_f определяет только усредненную степень заполнения доступного пространства. Мультифрактальная размерность анализированных объектов, нп. береговой линии, определённая представленным автором методом, определяет мулятифрактальный спектр размерности — $D(q)$. Разброс полученных величин D_f (1,05 1,42) даёт возможность судить, что береговая линия имеет мультифрактальные свойства.

Предложена разработка новых описательных и исследовательских инструментов, предназначенных как для местных исследований геометрических свойств представляемых на карте объектов так и упрощению их формы в процессе картографической генерализации.