archives
of thermodynamics
Vol. **43**(2022), No. 3, 29–45
DOI: 10.24425/ather.2022.143170

Correlations for the thermal conductivity of selected steel grades as a function of temperature in the range of 0–800°C

RAFAŁ WYCZÓŁKOWSKI[a*]
DOMINIKA STRYCHALSKA[a]
VAZGEN BAGDASARYAN[b]

[a] Czestochowa University of Technology, Department of Production Management, Armii Krajowej 19, 42-200 Czestochowa, Poland

[b] Warsaw University of Life Sciences, Institute of Civil Engineering – SGGW, Nowoursynowska 166, 02-787 Warsaw, Poland

Abstract Reliable knowledge of thermo-physical properties of materials is essential for the interpretation of solidification behaviour, forming, heat treatment and joining of metallic systems. It is also a precondition for precise simulation calculations of technological processes. Numerical calculations usually require the knowledge of temperature dependencies of three basic thermo-physical properties: thermal conductivity, heat capacity and density. The objective of this work is to find a correlation that fits the thermal conductivity of selected steel grades as a function of temperature (within the range of 0–800°C) and carbon content (within the range of 0.1–0.6%). The starting point for the analysis are the experimental data on thermal conductivity taken from literature. Using the method of least squares it was possible to fit an equation which allows calculating the thermal conductivity of steel depending on the temperature and carbon content. Two kinds of equations have been analyzed: a linear one (a linear model) and a second degree polynomial (a non-linear model). The thermal conductivity obtained by linear and nonlinear models varies on average from the measured values by 3% and 2.6% respectively.

Keywords: Thermal conductivity; Heat transfer; Least squares method; Thermal conduction analysis; Thermal treatment of steel

*Corresponding Author. Email: rafal.wyczolkowski@pcz.pl

Nomenclature

A	–	cross-sectional area, m^2
k	–	thermal conductivity, W/(m·K)
k_{el}	–	electron thermal conductivity, W/(m·K)
k_{ph}	–	phonon thermal conductivity, W/(m·K)
L_0	–	Lorenz number, $\Omega \cdot W/K^2$
Q	–	heat flux rate, W
R^2	–	coefficient of deretmination
T	–	absolute temperature, K
t	–	temperature, °C
x_C	–	carbon content
m, n	–	constants form Eq. (4)
k_m	–	measured thermal conductivity, W/(m·K)
k_c	–	calculated thermal conductivity, W/(m·K)

Greek symbols

α', α'', β	–	constants form Eq. (4)
γ	–	parameter dependent on the atomic concentration of point defects
δ	–	constant form Eq. (6)
σ	–	electrical conductivity, $1/(\Omega \cdot m)$

1 Introduction

Reliable knowledge of thermo-physical properties of materials is essential for the interpretation of solidification behaviour, forming, heat treatment and joining of metallic systems. It is also a precondition for precise simulation calculations of technological processes. Numerical calculations usually require the knowledge of temperature dependencies of three basic thermo-physical properties: thermal conductivity, heat capacity, and density.

In many situations, in order to plan and control different production processes, it is necessary to know the thermal conductivity of different materials. This parameter is responsible for the magnitude of thermal loads which occur in components during production. One of such processes is the heat treatment of steel products [1, 2].

Thermal conductivity is the property that characterizes the ability of a material to conduct heat, and it is defined as the rate of heat transfer through a unit thickness of the material per unit area and per unit temperature gradient imposed normal to the unit area [3]

$$Q = -kA\frac{dT}{dx}\,, \tag{1}$$

where Q is the heat flux rate, k is the thermal conductivity, A is the cross-sectional area perpendicular to the direction of heat flow, and dT/dx is the temperature gradient.

Because this parameter pertains to the transfer of energy within the system, it is a nonequilibrium (transport) property, usually determined in experiments utilizing the Fourier law for unidirectional heat transfer [4]. Measuring thermal conductivity requires the development of experimental approximations of boundary value problems. Analytical solutions to many classical problems that can be useful in the measurements of thermal conductivity are described in [5]. Descriptions of methods used for thermal conductivity measurements can be found in [6, 7]. Direct measurements of the k coefficient have usually used steady-state methods. For materials like steel of moderate to high thermal conductivity ($k > 10$ W/(m·K)), axial heat flow, radial heat flow and direct electrical heating methods are used [8]. Detailed descriptions of measurements of steel and other alloys can be found in [9–13].

The results of experimental investigations show that the thermal conductivity is a function of the state, composition, purity, and physical structure of the material. But first of all this property depends strongly on the temperature. In the case of anisotropic materials, the thermal conductivity varies with the heat flow direction. Generally, it is also assumed that the conductivity is independent of the size and shape of the material. However, this assumption is not valid when the size of the conductor is comparable to the mean free path of the thermal energy carriers [14].

Metals, in which thermal conduction is caused mainly by the movement of free electrons and to a much lesser extent by atomic vibration of the crystal structure with energy quanta called phonons, are characterized by the biggest coefficients of thermal conductivity. Therefore, it is assumed that the metal's thermal conductivity is a sum of the electron thermal conductivity, k_{el}, and phonon thermal conductivity, k_{ph}, [15, 16]

$$k = k_{el} + k_{ph}\,. \tag{2}$$

In high-purity metals, the mechanism of heat transport by electrons is more efficient than the phonon contribution because electrons are not as easily scattered as phonons and have higher velocities [17]. Alloying metals with impurities result in a reduction in electron thermal conductivity because alloying additives reduce the mean free path of electrons.

Since the electron thermal conductivity is caused by the movement of the same energy carriers as electric conduction, the electrical conductivity

and electron thermal conductivity of metals can be related by equation [18]

$$L_0 T = \frac{k_{el}}{\sigma}\,, \tag{3}$$

where: k_{el} – electron thermal conductivity, σ – electrical conductivity, T – absolute temperature, L_0 – the Lorenz number – a constant for all metals with a theoretical value of 2.44×10^{-8} $\Omega{\cdot}\mathrm{W/K^2}$. Dependence (3) is a mathematical notation of the Wiedemann-Franz law, which says that the k_{el}/σ quotient is the same for different metals at a constant temperature. According to the theory of thermal conductivity of metallic materials, the electron thermal conductivity can be described by the following correlation [9]:

$$k_{el} = \frac{1}{\dfrac{\alpha' T^n + \beta}{T}}\,, \tag{4}$$

where

$$\alpha' = \alpha^n \left(\frac{\beta}{n\alpha''}\right)^{\frac{m-n}{m+1}}\,, \tag{5}$$

and α, m, n are constants for a given metal – for most metals, the value of n lies between 2.0 and 3.0. The β parameter is defined as the ratio between the residual electrical resistivity and the Lorenz number.

For pure metals, the phonon coefficient of thermal conductivity is much smaller than the electron coefficient. However, for alloys the above mentioned coefficients can be of the same order of magnitude. The number of phonons in a unit of volume is directly proportional to the temperature, so the length of the free path of phonons and the phonon thermal conductivity of pure metals is inversely proportional to the absolute temperature. With a big number of additives the coefficient k_{ph} practically does not depend on the temperature. For alloys with a lower quantity of additives, this coefficient can be described with the following equation [19]:

$$k_{ph} = \frac{\gamma}{T^{\delta}}\,, \tag{6}$$

where γ is the parameter dependent on the atomic concentration of point defects, and the exponent parameter δ has values between 0.5 (at high temperatures) and 2.0 (at low temperatures).

For alloys, the dependence of the k coefficient on its composition is not governed by general rules. The only thing that can be stated is that adding even a little amount of a poorly conductive metal to a metal with a high

thermal conductivity results in a dramatic decrease of the thermal conductivity of the whole alloy, while adding a small quantity of metal with a significant k value to a metal with a small k coefficient does not cause any substantial improvement of thermal conductivity. The thermal conductivity of steel depends on the temperature and its composition and generally is within the range from 14 W/(m·K) for highly alloyed steels to 75 W/(m·K) for Armco iron. The biggest differences in thermal conductivity among different steel grades exist at room temperature. For carbon steels, the k coefficient decreases with increasing temperature, but for high-alloy grades, it slightly increases with increasing temperature. At higher temperatures when austenite forms all the steel grades have a similar thermal conductivity in the range of 25–27 W/(m·K) [20]. The data on the thermal conductivity as a function of temperature for a number of different steel grades can be found in [21–24].

Although the process of thermal conduction in metal alloys is a very complex phenomenon, attempts have been made to develop methods of determining the value of the k coefficient analytically depending on the composition and temperature. There are different approaches that can be adopted. For example [25] presents a model using the neural network technique based upon the Bayesian statistic framework. In another approach to determining the thermal conductivity of metals as a function of temperature, the multilinear regression model has been found [18].

Current practice in the analysis of heat transfer problems requires the use of computer programs [3,26]. This applies also to the phenomena connected with heat treatment of metal products [27–30]. The algorithms which occur in such programs should take into account the changes of thermal conductivity of the analysed materials with temperature. The simplest way to achieve it is by implementing suitable mathematical dependencies which describe the changes of the k coefficient in the function of temperature with the required accuracy.

The aim of the present paper is to determine (based on the literature data) the mathematical dependence that will allow calculating the thermal conductivity of carbon steel depending on the temperature and carbon content. The method of least squares has been used for the analysis.

2 Method and results

As already mentioned, tabular data on the thermal conductivity of low-carbon steels can be found in many books [21–24]. However, the most consistent data in this respect, which were chosen for the present analysis,

have been shown in [31]. These values have been obtained on the basis of experimental research and have been collated in Table 1. They include five grades of steel with the carbon content from 0.1% to 0.6% for the temperature range 0–800°C.

Table 1: Thermal conductivity of carbon steel depending on temperature and carbon content [31].

Temperature, °C	Carbon content, %				
	0.1	0.2	0.4	0.5	0.6
0	59.5	51.3	48.0	45.2	49.2
100	57.6	50.8	47.2	44.7	46.4
200	53.4	48.3	46.5	42.6	43.8
300	49.3	44.6	43.8	40.2	40.6
400	43.7	42.6	41.0	37.2	37.6
500	40.2	39.2	38.4	34.3	34.9
600	36.0	35.4	36.0	31.9	32.2
700	31.8	31.8	31.4	28.2	29.1
800	28.5	27.4	26.7	23.7	24.2

For each grade of steel, a diagram showing changes in thermal conductivity within the analysed temperature range has been drawn (Figs. 1–5). The values of the k coefficient have been marked as black points. It has been

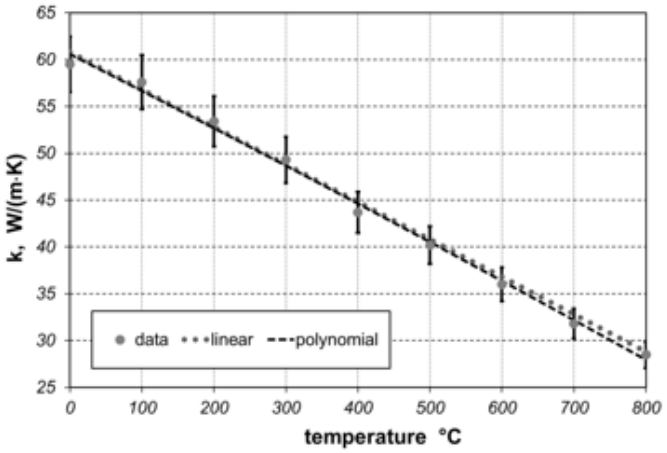

Figure 1: Thermal conductivity of steel with the carbon content of 0.1% (measurement data and fitting functions).

assumed that these values are burdened with a 5% uncertainty of measurement, which has been marked in the diagrams with error bars. Such a value is typical for thermal conductivity measurements [32–34].

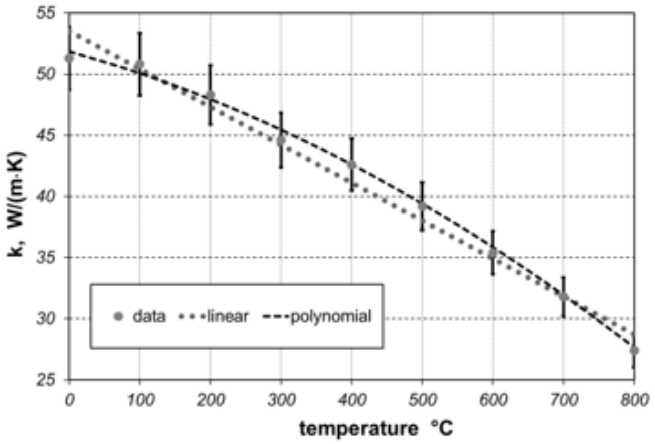

Figure 2: Thermal conductivity of steel with the carbon content of 0.2% (measurement data and fitting functions).

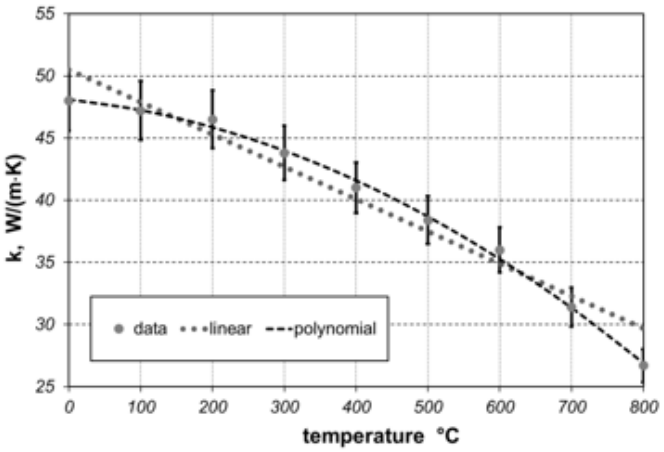

Figure 3: Thermal conductivity of steel with the carbon content of 0.4% (measurement data and fitting functions).

The next step was seeking a function that approximates changes of the k coefficient depending on the temperature, with the use of the method

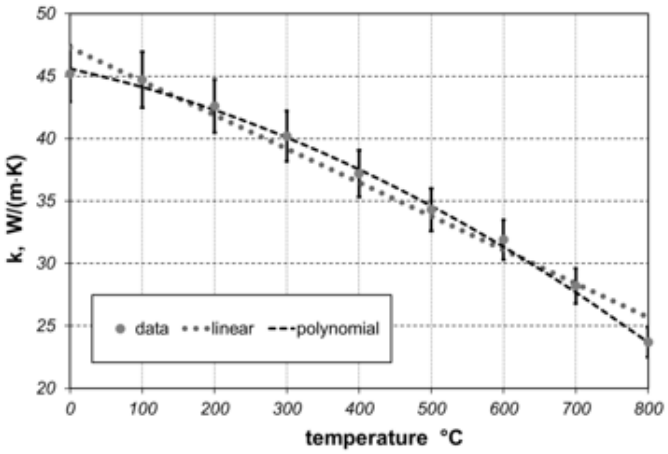

Figure 4: Thermal conductivity of steel with the carbon content of 0.5% (measurement data and fitting functions).

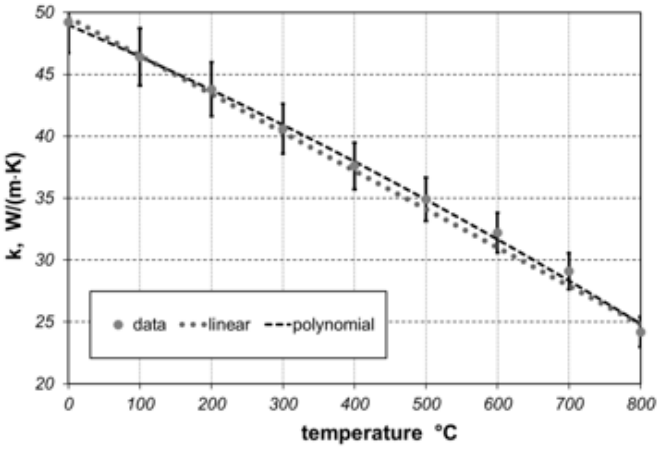

Figure 5: Thermal conductivity of steel with the carbon content of 0.6% (measurement data and fitting functions).

of least squares [35]. In the study, two fitting equations have been used, a linear equation

$$k(t) = A_0 + A_1 t \tag{7}$$

and a second degree polynomial

$$k(t) = B_0 + B_1 t + B_2 t^2. \tag{8}$$

Values of the A_i and B_i coefficients obtained for particular grades of steel as a result of the fitting have been collated in Table 2 (coefficients A_0 and A_1) and Table 3 (coefficients B_0, B_1 and B_2). The tables also present values of the coefficient of determination R^2. For both of the analysed functions, the obtained values were close to 1 (the smallest value of R^2 was 0.95), which shows that they fit the calculated values to the measured values well. Nevertheless, a polynomial provides a better fit, which is confirmed by Figs. 1–5.

Table 2: The values of A_0, A_1, and R^2 coefficients obtained for particular steel grades.

Carbon content, %	A_0	A_1	R^2
0.2	60.81	–0.040	0.955
0.2	53.52	–0.031	0.980
0.4	50.49	–0.026	0.950
0.5	47.29	–0.027	0.975
0.6	49.61	–0.030	0.995

Table 3: The values of B_0, B_1, B_2, and R^2 coefficients obtained for particular steel grades.

Carbon content, %	B_0	B_1	B_2	R^2
0.2	60.59	–0.039	-2.26×10^{-6}	0.995
0.2	51.86	–0.016	-17.8×10^{-6}	0.997
0.4	48.11	–0.006	-25.5×10^{-6}	0.997
0.5	45.61	–0.013	-18.0×10^{-6}	0.997
0.6	49.00	–0.025	-6.47×10^{-6}	0.997

The next step of the analysis was to determine the equation which would allow calculating thermal conductivity both in the function of temperature and carbon content x_C. In order to do that it was necessary to determine the dependences which describe the changes of coefficients A_i (Eq. (7), $i = 0, 1$) and B_i (Eq. (8), $i = 0, 1, 2$) in the function of the x_C parameter. The values of coefficients A_i and B_i from Tables 2 and 3 in the function of carbon content have been presented in individual diagrams (Figs. 6–10). On the basis of these diagrams, again using the least squares method, the functions which best approximate the changes of these coefficients have been determined. For all of the five coefficients, the best function turned out to be a second degree polynomial. The forms of these polynomials for

individual coefficients are described by Eqs. (9)–(13):

$$A_0(x_C) = 80.54x_C^2 - 77.88\,x_C + 67.17\,, \tag{9}$$

$$A_1(x_C) = -0.14x_C^2 + 0.116\,x_C - 0.049\,, \tag{10}$$

$$B_0(x_C) = 112.51x_C^2 - 100.85\,x_C + 68.89\,, \tag{11}$$

$$B_1(x_C) = -0.42x_C^2 + 0.318\,x_C - 0.065\,, \tag{12}$$

$$B_2(x_C) = 334.05x_C^2 - 247.27\,x_C + 18.59\,. \tag{13}$$

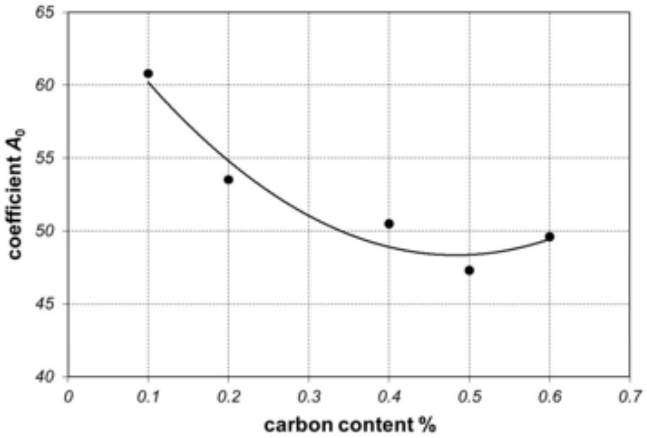

Figure 6: The values of A_0 coefficient from Eq. (7) depending on the carbon content x_C and the approximating function.

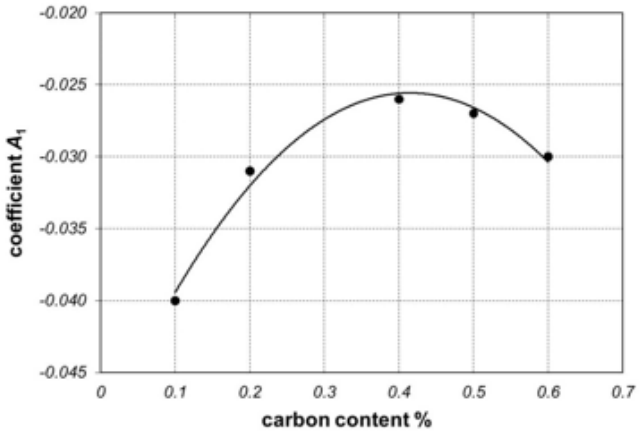

Figure 7: The values of A_1 coefficient from Eq. (7) depending on the carbon content x_C and the approximating function.

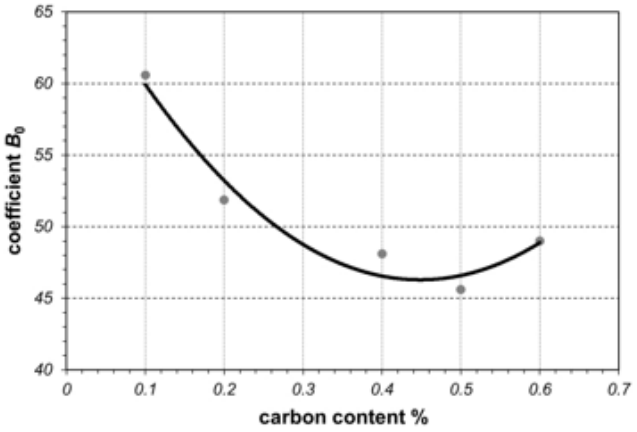

Figure 8: The values of B_0 coefficient from Eq. (8) depending on the carbon content x_C and the approximating function.

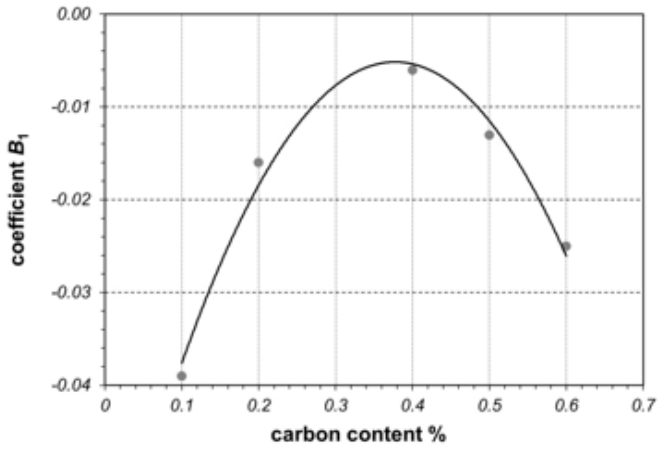

Figure 9: The values of B_1 coefficient from Eq. (8) depending on the carbon content x_C and the approximating function.

It must be emphasized that Eqs. (9)–(13) describe the changes of the given coefficient within the whole range of temperature, i.e. in 0–800°C and therefore they do not refer to a specific value of temperature.

The last stage of developing the sought equation was replacing the A_i coefficients in Eq. (7) with expressions given in Eqs. (9) and (10) and the B_i coefficients in Eq. (8) with expressions given in Eqs. (11)–(13), respectively.

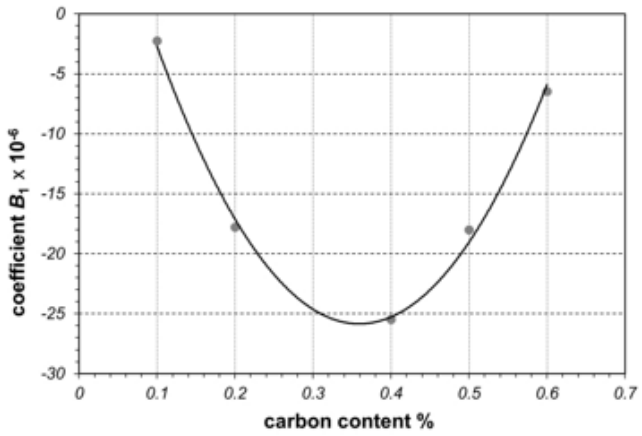

Figure 10: The values of B_2 coefficient from equation (8) depending on the carbon content x_C and the approximating function.

Thanks to this it was possible to obtain a function that allows calculating the thermal conductivity of carbon steel for any temperature (within the range 0–800°C) and any percentage value of carbon content (within the range 0.1–0.6%). In case the changes of the k coefficient are approximated by the linear function this equation has the following form:

$$k(x_C, t) = 80.54x_C^2 - 77.88x_C + 67.17 - \left(0.14x_C^2 - 0.116x_C + 0.049\right) t\,, \tag{14}$$

where t is the temperature of material in °C.

However, when the approximation is realized with the use of the second degree polynomial (the nonlinear model) the following dependence is obtained:

$$k(x_C, t) = 112.51x_C^2 - 100.85x_C + 68.89 - \left(0.42x_C^2 + 0.318\,x_C + 0.065\right) t + \left(334.05x_C^2 - 247.27x_C + 18.59\right) \cdot 10^{-6}\, t^2. \tag{15}$$

In order to determine the quality of results obtained with the use of Eqs. (14) and (15) they have been compared with the measured values from Table 1 with the use of the relative percentage difference

$$\Delta k = \frac{k_m - k_c}{k_m} \times 100\%\,, \tag{16}$$

where: k_m – the measured value of the thermal conductivity, k_c – the value of the thermal conductivity calculated with the use of Eqs (14) or (15).

The values of Δk obtained when the k_c coefficient was calculated with the use of Eq. (14) have been collated in Table 4. As can be seen, in the vast majority of cases the obtained values were lower than 5%. A value which exceeded 10% has been obtained for only one case, which concerns the temperature of 800°C and the carbon content of 0.5% for which the value of Δk was 16.27%. The last two rows present values of Δk averaged with regard to temperature. For three values of the x_C parameter (0.1, 0.2, and 0.6) the obtained results were below 2.8%, however for the other two cases (0.4 and 0.5) about 4.5%. Upon averaging the Δk parameter for the whole range of x_C the obtained value of Δk was 3.07%.

Table 4: Values of the Δk parameter depending on the temperature and carbon content calculated with the use of the linear model (Eq. (14)).

Temperature, °C	Δk, %				
	Carbon content, %				
	0.1	0.2	0.4	0.5	0.6
0	1.15	6.85	1.87	6.98	0.46
100	2.25	1.72	1.70	2.36	0.10
200	1.83	0.48	5.60	1.30	0.77
300	1.53	1.77	5.49	0.88	0.28
400	2.21	0.82	5.13	2.03	0.25
500	1.46	0.23	5.22	3.08	1.08
600	2.52	1.61	5.84	2.68	2.04
700	3.85	3.24	0.01	6.93	1.84
800	2.26	8.36	8.23	16.27	5.72
Mean	2.12	2.79	4.34	4.72	1.39
	3.07				

Table 5 presents values of Δk obtained when the k_c coefficient was calculated with the use of Eq. (15). Only one value exceeds 10%. For x_C equal to 0.1 and 0.2 the values of Δk are equal to 1.5%, for 0.5–1.9%, and for the two other cases (0.4 and 06) it is equal to 4.1%. The average value of Δk for this case in relation to x_C is equal 2.63%. It is about 0.5% less than for the linear model. Therefore, the nonlinear model is slightly more accurate than the linear model. However, for $x_C = 0.6\%$ the value of Δk is bigger than the value obtained for the linear model ($\Delta k = 1.39\%$). Therefore, for steel

with the biggest carbon content, the linear model provides more accurate values of the k coefficient.

Table 5: Values of the Δk parameter depending on the temperature and carbon content calculated with the use of the nonlinear model (Eq. (15)).

Temperature, °C	Δk, %				
	Carbon content, %				
	0.1	0.2	0.4	0.5	0.6
0	0.72	3.74	3.02	3.08	0.65
100	2.50	0.82	3.07	1.17	0.49
200	2.00	1.17	4.49	1.95	1.20
300	1.72	3.50	2.89	2.49	1.02
400	1.86	1.19	2.14	3.62	1.84
500	0.80	1.29	3.20	3.37	4.18
600	1.31	1.56	6.43	0.12	7.51
700	1.78	0.15	5.54	0.75	10.88
800	1.04	0.02	6.00	0.38	9.52
Mean	1.53	1.49	4.09	1.88	4.14
	2.63				

The values of Δk obtained both for the linear model (Table 4) and nonlinear model (Table 5) show that the results of thermal conductivity calculations performed with the use of both models are burdened with a smaller error than the uncertainty of the measured values from Table 1. Therefore, the developed equations can be successfully used in engineering calculations connected with transient thermal conductivity through steel elements within the temperature range 0–800°C. Moreover, the developed equation can be used in modelling the effective thermal conductivity of a steel porous charge. Such models have been described in [36, 37].

3 Summary

The regression equations which have been determined on the basis of the performed analyses provide an easy way to calculate the value of thermal conductivity for steel with the carbon content from 0.1 to 0.6% within the temperature range 0–800°C. The accuracy of the calculated values is absolutely sufficient from the point of view of engineering calculations. The determined functions can be used for numerical analysis of the heating of

a steel charge connected with heat treatment process optimization. The presented equations allow taking into account temperature changes of the thermal conductivity coefficient of the given steel grade in a numerical model of non-stationary heat transfer based on the energy balance method. The developed equations can be used in modelling the effective thermal conductivity of a steel porous charge.

Received 17 February 2022

References

[1] Sahay S.S., Kumar A.M.: *Application of integrated batch annealing furnace simulator*. Mater. Manuf. Process. **17**(2002), 4, 439–453.

[2] Sahay S.S., Krishnan K., Kuthle M., Chodha A., Bhattacharya A., Das A.K.: *Model-based optimization of a highly automated industrial batch annealing operation*. Eng. Mater. Sci. – Ironmak. Steelmak. **33**(2006), 306–314.

[3] Cengel Y.A.: *Heat Transfer and Mass Transfer – A Practical Approach* (3rd Edn.). McGraw Hill, New York 2002.

[4] Oleśkiewicz-Popiel Cz., Wojtkowiak J.: *Experiments in Heat Transfer*. Wydawn. Politechniki Poznańskiej, Poznań 2007 (in Polish).

[5] Carslaw H.S., Jaeger J.C.: *Conduction of Heat in Solids*. Oxford Univ. Press, London 1959.

[6] Maglic K.D., Cezailliyan A., Peletsky V.E. (Eds.): *Compendium of Thermophysical Property Measurements Methods: Recommended Measurement Techniques and Practices*. Pleneum Press, New York 1992.

[7] Shirtliffe C.J., Tye R.P.: *Guarded Hot Plate and Heat Flow Meter Methodology*. ASTM, Philadelphia 1985.

[8] Beyan A., Kraus A.D. (Eds.): *Heat Transfer Handbook*, Chap. 2.3.4 *Measuring Thermophysical Properties in Solids*. Wiley, New Jersey 2003.

[9] Koltys J.: *Determination of thermal conductivity of steel 1H18N9T*. Prace Naukowe Politechniki Warszawskiej, ser. Konferencje, **26**(2009), 219–226.

[10] Koltys J., Furmanski P., Domanski R.: *Determination of thermal conductivity of highly conducting materials by comparative method using parameter estimation technique*. In: Proc. 26th Int. Conf. Thermal Conductivity of Highly Conducting Conference, Boston, 6-8 August 2001.

[11] Domański R., Gogół E., Gogół W.: *Measurements of the thermal conductivity of steel*. J. Power Technol. **39**(2011), 3–21.

[12] Bode K.H.: *45 – measurements of the thermal conductivity of metals at high temperatures*. Prog. Int. Res. Thermodyn. Transport Prop. (1962), 81–499.

[13] In-Rak Ch., Kyung-Soo Ch., Do-Hwan K.: *Thermal and mechanical properties of high-strength structural steel HSA800 at elevated temperatures*. Mater. Des. **63**(2014), 544–551.

[14] Bodzenta J.: *Heat Transport in Thin Layers and Measurements of Their Thermal Properties*. Wydawn. Politechniki Śląskiej, Gliwice 2006 (in Polish).

[15] Incropera F.P., DeWitt D.P.: *Fundamentals of Heat and Mass Transfer* (4th Edn.). Wiley, New York 1996.

[16] Wiśniewski S., Wiśniewski S.T.: *Heat Transfer*. WNT, Warszawa 2012 (in Polish).

[17] Callister W.D.: *Materials Science and Engineering: An Introduction* (5th Edn.). Wiley, New York 2000.

[18] Abu-Eishah S.I.: *Correlations for the thermal conductivity of metals as a function of temperature*. Int. J. Thermophys. **22**(2001), 1855–1868.

[19] Touloukian Y.S., Powell R.W., Ho C.Y., Klemens P. (Eds.): *Thermophysical Properties of Matter* Vol. 1. *Thermal Conductivity of Metallic Solids*. Plenum Press, New York 1972.

[20] Sverdlin A.V., Ness A.R.: *Fundamental concepts in steel heat treatment*. In: Steel Heat Treatment – Metallurgy and Technologies Chap. 2. CRC Taylor & Francis Group, Boca Raton Londyn New York 2006.

[21] Powell, R.W. Hickman, M.J.: *Thermal conductivity of a 0.8% carbon steel*. J. Iron Steel Inst. **154**(1945), 112–116.

[22] Richter F.: *Die Wichtigsten Physikalischen Eigenschaften von 52 Eisenwerkstoffen*. Verlag Stahleisen, Dusseldorf 1973.

[23] Krivandin V., Markov B.: *Metallurgical Furnaces*. MIR, Moscow 1980.

[24] Raźnievié K.: *Thermal Tables with Graphs*. WNT, Warszawa 1966 (in Polish).

[25] Peet M.J., Hasan H.S., Bhadeshia H.K.D.H.: *Prediction of thermal conductivity of steel*. Int. J. Heat Mass Tran. **54**(2011), 2602–2608.

[26] Nellys G., Klein S.: *Heat Transfer*. Cambridge Univ. Press, 2012.

[27] Sahay S.S., Krishman K.: *Model based optimization of continuous annealing operation for bundle of packed rods*. Iron Steelmak. **34**(2007), 1, 89–94.

[28] Jaluria Y.: *Numerical simulation of the transport process in a heat treatment furnace*. Int. J. Numer. Meth. Eng. **25**(1988), 387–399.

[29] Rao T.R., Barth G.J., Miller J.R.: *Computer model prediction of heating, soaking and cooking times in batch coil annealing*. Iron Steel Eng. **60**(1983), 9, 22–33.

[30] Laden B.: *A control system for fuel optimization reheating furnaces*. Scand. J. Metall. **15**(1986), 16–24.

[31] Senkara T.: *Thermal Calculations of Heating Furnaces in Methalurgy*. Wydawn. Śląsk, Katowice 1981 (in Polish).

[32] Kulesza J.: *Measurements of thermal conductivity*. In: Thermal measurements. Part I, (T. Fodemski, Ed.). WNT, Warszawa 2001 (in Polish).

[33] Nowak A.: *Selected methods of measuring the heat conductivity coefficient*. In: Proc. Sem. on Heat Flow in the Processes of Heating and Cooling Metals, Ustroń-Jaszowiec, 1986, 94–102 (in Polish).

[34] Gogół W.: *Measurements of Thermal Conductivity*. Zeszyty Naukowe Politechniki Łódzkiej **606**, 1991 (in Polish).

[35] Majchrzak E., Mochnacki B.: Numerical Methods – Theoretical Foundations, Practical Aspects and Algorithms. Wydawnictwo Politechniki Śląskiej, Gliwice 2004 (in Polish).

[36] Wyczółkowski R., Gała M., Bagdasaryan W.: *Model of complex heat transfer in the package of rectangular steel sections*. Appl. Sci. **10**(2020), 9044.

[37] Wyczółkowski R., Bagdasaryan V., Szwaja S.: *On determination of the effective thermal conductivity of a bundle of steel bars using the Krischer model and considering thermal radiation*. Materials **14**(2021), 4378.