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DIMENSIONAL SYNTHESIS OF A FIVE-ROD GUIDING MECHANISM FOR CAR FRONT WHEELS

The paper presents a method for dimensional synthesis of a five-rod guiding mechanism for the front wheels of a car. The goal is to find some unknown coordinates of joint centers of three suspension rods on the basis of given input data describing selected coordinates of other points, rods lengths and desired suspension characteristics for jounce-rebound and steering displacements. The synthesis problem is formulated as a single-criteria optimization procedure with a few substages solved in hierarchical order. The procedures for position and displacement analysis of the spatial multilink mechanism and for determination of the screw axes are also described. A numerical example for the AUDI A4 front axle is given.

NOMENCLATURE

Reference frames:

W – $(0xyz)$ attached to the car body,

W^s – $(0^s x^s y^s z^s)$ attached to the suspension subframe,

W^k – $(0^k x^k y^k z^k)$ attached to the wheel knuckle,

Indices:

$(^s)$, $(^k)$, $(\ )$ – (upper right-hand) coordinates described with respect to W^s, W^k, and W respectively,

$(_i)$ – (bottom right-hand) suspension links indexing, $i = 1, 2...5$,

$(^j)$ – (upper left-hand) design points indexing,

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Points:

A_i – center of the joint linking the i-th ($i = 1, 2...5$) rod with the car body or with the subframe,

B_i – center of the joint linking the i-th ($i = 1, 2...5$) rod with the wheel knuckle,

B_6 – wheel center,

B_7 – center of the tire contact patch of the wheel fixed to the knuckle,

P – center of the tire contact patch of the free rotating wheel,

Tensors:

$\boldsymbol{a}_i$ – position vector of the point Ai described in the assigned reference frame,

$\boldsymbol{b}_i$ – position vector of the point Bi described in the assigned reference frame,

$\boldsymbol{o}$ – position vector of the origin of an assigned reference system with respect to W,

$\boldsymbol{u}_{sa}$ – steering axis versor,

$\boldsymbol{q}_{sa}$ – position vector of intersection point of the ground plane and the virtual steering axis,

$\boldsymbol{r}_{sa}$ – position vector of the steering axis $\boldsymbol{r}_{sa} = \boldsymbol{q}_{sa} - \boldsymbol{b}_7 = [r_\tau, r_\sigma, 0]^T$

$\boldsymbol{R}$ – orientation matrix $\boldsymbol{R} = [\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}]$ of an assigned reference system with respect to W,

$\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}$ – unit vectors of the x, y, z-axes of an assigned reference system with respect to W,

Scalars:

d_i – i-th rod length $i = 1, 2...5$,

s_s – main spring length,

s_w – vertical displacement of the point B_6 (wheel center),

p – lateral displacement of the steering rack,

Suspension and steering system geometry:

γ – wheel camber angle,

δ – wheel steer angle,

δ_{dif} – difference of steer angles determined for outer and inner wheels relative to a turn,

δ_m – mean steer angle,

σ – inclination angle of the steering axis,

τ – caster angle of the steering axis,

ε – pitch angle of the wheel knuckle,

r_τ – caster offset,

r_σ – brake radius (roll radius),

r_w – loaded wheel radius,

h_{rc} – instantaneous roll center height.

1. Introduction

Recently, much effort has been focused on the improvement of the ride/handling compromise of the car by using multi-rod suspension systems. The *5S-S* spatial mechanism (five spherical joints at the base and five spherical joints at the platform) is a general realization of an independent wheel guidance mechanism, Fig. 1 and 2. A parallel structure of the wheel guiding mechanism provides the separation of the wheel-suspension parameters determining travel comfort from the parameters responsible for handling. However, the difficult problems associated with the design and the resulting high price make this suspension system feasible mainly in high-performance automobiles [8].

Position and displacement analysis of a multi-rod guiding mechanism can be performed only if its kinematical scheme is fully and correctly described. The coordinates of selected points belonging to the steering knuckle, subframe, and disassembled rods of the considered mechanism were measured directly by using a three-coordinate measuring machine. However, the coordinates of the joint centers linking the upper rods to the car body, due to their spatial arrangement and limiting overall dimensions of a car, are troublesome to determine by direct measurements without a specialized apparatus. They can be indirectly deduced from feasible measurements of actual dependencies of the steering knuckle position and orientation on the corresponding variables.

The considered problem can be solved by methods of the dimensional synthesis used to find the significant dimensions and the starting position of a mechanism for a specified task by using exact or approximate methods [3]. Depending on the size of the considered design space, different configurations of the mechanism can be found. For a design space narrowed around initial values of the wanted geometrical parameters, the stated problem can be referred to the parametric identification. In robotics, the parametric identification is used for example in the kinematic calibration as a procedure for determining the actual kinematic parameters of a mechanism, based on the knowledge of the equations that describe its kinematics, the set of initial values for kinematic parameters, and measurement results obtained at various poses of the mechanism [3].

The design procedure for the *5S-S* mechanism can be based on the solution of constraint equations [12] or it can be considered as a spatial Burmester problem [7]. Although both methods belong to the exact methods of the synthesis, they usually require approximate numerical procedures. When a set of arbitrarily chosen poses of the guided link with respect to the base is imposed, the dimensional synthesis of this guidance linkage provides a finite number of solutions in terms of possible rod lengths and coordinates of the joint centers [7]. Solution of the dimensional synthesis problem is equivalent to searching for these points of the guided body which, at the specified poses, lie on the same spherical surface fixed to the base. These points, also called the sphere-points, represent possible joint centers located on the guided body. For every sphere point, the center of the corresponding sphere (called the center-point) provides the joint center of the rod which is linked to the base, while the sphere radius matches the rod length.

Another approach to the synthesis problem for a five-rod mechanism uses the approximate methods based on optimization procedures. A general objective function of 64 decision variables used for the synthesis of a five-rod rear suspension system satisfying three desired poses of the wheel carrier is given in [4]. A similar method but with a simpler formulation of the objective function, described by 30 design variables, is shown in [11]. The wheel carrier, displaced in successive poses along the desired trajectory, is considered as freed of all joints. The objective function is assumed as a sum of squares of differences between the rod lengths and the distances of the corresponding joints.

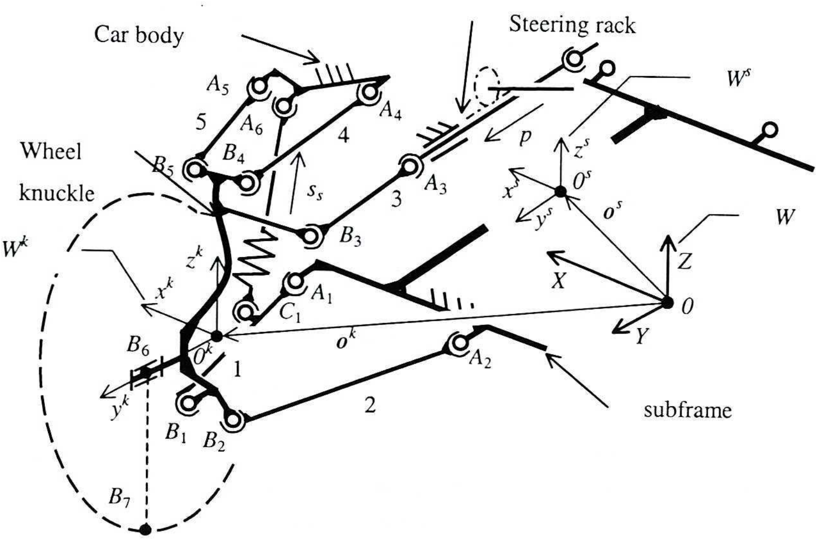

Fig. 1. Kinematic scheme of the five-rod front wheel guiding mechanism (left side)

The effective solution of the synthesis problem, with so many decision variables and possible configurations of the mechanism, is still a formidable task even for today's computers. One way to increase reliability of the design-synthesis process is to make the problem well formulated, bounded and conditioned. In [4] and [11], only changes of the wheel toe angle, camber angle, and the wheel track for a suspension jounce-rebound displacements were set as the basic objectives for a designed suspension system. The five-rod suspension, considered as a spatial parallel mechanism, is able to meet more conditions simultaneously, for example the preferable roll center height, anti-dive geometry, and proper elastokinematic characteristics [5], [8]. When synthesis of a wheel guiding mechanism deals with front wheels, additional conditions can be imposed by the designer, like for instance the position and orientation of the instantaneous steering axis (or more precisely the screw axis of the spatial displacement of the wheel knuckle in steering motion), steering linkage geometry allowing for achieving maximum steer angles, the desired Ackermann errors, and others.

The location of the virtual steering axis is also considered for rear wheels, because it gives the possibility to analyze the compliance steer angle of the wheel under the acting load (drive and brake forces, lateral force, and self-aligning torque).

The dimensional synthesis is often treated as an initial stage of the design process of a new mechanism. The final solution of a car suspension system is usually completed by additional design requirements concerning force transmission by the linkage [6], pressure angles in the joints, limits of their relative displacements [5], mechanism compliance properties [5], [8], friction and damping forces, avoidance of singular positions, and others. Synthesis for motion generation can be also supplemented by velocity and acceleration specifications.

This paper presents a new algorithm for determining unknown dimensions of the spatial five-rod mechanism that should guide the wheel knuckle (platform) through a set of given positions and orientations described in the form of geometrical characteristics, while some parameters are given. An approximate approach is used, formulated as an optimization task divided into small-size substages of hierarchical structure. Taking into account additional parameters, for example the location of steering axis, steering linkage geometry, and the roll center height, a few types of objective functions are used. A numerical example of the dimensional synthesis is given.

2. Problem formulation

A kinematic scheme of the five-rod wheel guiding mechanism used as a front axle in AUDI A4, A6, and A8 [10] is shown in Fig. 1. The wheel knuckle is joined by two lower rods (1 and 2) with the suspension subframe (compliantly mounted to the car chassis), by two upper rods (4 and 5) – to the car body, and by a tie rod (3) – to a steering rack of the steering system with trailing geometry. The main spring-damper unit is supported on the rod 1 at point C_1 and in the car body at point A_6. Some joints (B_1, B_2, B_3, B_4, B_5, A_3) are made as steel ball-joints, the other joints (A_1, A_2, A_4, A_5, A_6, C_1) are made as revolute joints with elastomeric bushes, which for kinematical analysis can be substituted by equivalent spherical joints.

Three reference systems are distinguished in Fig. 1. The reference system W is attached to the car body, which is assumed as space-fixed in the analysis. Its XY plane is parallel to the road plane and X-axis is directed towards front of the car. The Z-axis is vertical with the positive sense upwards. The reference system W^s is attached to the suspension subframe and W^k is attached to the wheel knuckle, where the y^k-axis coincides with the wheel axis (Fig. 2). Positions and orientations of the reference systems W^k and W^s relative to W are described by position vectors ($\boldsymbol{o}^k$ and $\boldsymbol{o}^s$) and orientation matrices ($\boldsymbol{R}^k$ and $\boldsymbol{R}^s$) for the wheel knuckle and subframe respectively.

The orientation of the wheel knuckle system is described by three angles defined according to the convention used in [2], [5], [6], [8]. The definitions of these angles are unique in the angle range of $\pm\pi/2$ what is entirely sufficient for elastokinematic analysis of a suspension system.

Having an orientation matrix of the wheel knuckle given in the form $\boldsymbol{R}^k = [\boldsymbol{l}^k, \boldsymbol{m}^k, \boldsymbol{n}^k]$, the wheel steer angle δ and the camber angle γ can be determined on the basis of the following formulas:

$$tg\,(\delta) = \frac{-\boldsymbol{m}^k \cdot \boldsymbol{l}}{\boldsymbol{m}^k \cdot \boldsymbol{m}}$$

$$tg\,(\gamma) = \frac{\boldsymbol{m}^k \cdot \boldsymbol{n}}{\sqrt{(\boldsymbol{m}^k \cdot \boldsymbol{l})^2 + (\boldsymbol{m}^k \cdot \boldsymbol{m})^2}}$$

The third angle noted as the wheel knuckle pitch angle ε is defined as the rotation angle of the knuckle around the wheel axis described by a unit vector $\boldsymbol{m}^k$. To derive this angle from the given rotation matrix, the Rodrigues formula [1] that describes a rotation of the unit vector $\boldsymbol{l}^k$ about $\boldsymbol{m}^k$ can be used in the following form

$$\boldsymbol{l}^k = \frac{[\boldsymbol{l}^{k,0}(1-t^2) + 2\boldsymbol{m}^k(\boldsymbol{m}^k \cdot \boldsymbol{l}^{k,0})\,t^2 + 2(\boldsymbol{m}^k \times \boldsymbol{l}^{k,0})\,t]}{1+t^2}$$

where

$t = \mathrm{tg}\,(\varepsilon/2)$,
$\boldsymbol{l}^{k,0} = [\cos(\delta), \sin(\delta), 0]^{\mathrm{T}}$ – initial-reference orientation of the $\boldsymbol{l}^k$ unit vector.

The wheel knuckle pitch angle ε can be determined as a solution of a quadratic equation obtained from the Rodrigues formula.

An inverse problem can also be considered when, for given values of three angles (δ, γ, and ε), a corresponding rotation matrix in the form $\boldsymbol{R}^k = [\boldsymbol{l}^k, \boldsymbol{m}^k, \boldsymbol{n}^k]$ is looked for. On the basis of equations for δ and γ (and remembering that $\boldsymbol{m}^k \cdot \boldsymbol{m}^k = 1$, all components of the unit vector $\boldsymbol{m}^k$ can be derived. The second unit vector $\boldsymbol{l}^k$ is determined directly by the Rodrigues formula. To complete an orthonormal coordinate system, the third unit vector $\boldsymbol{n}^k$ is derived as a proper cross product of the previous two.

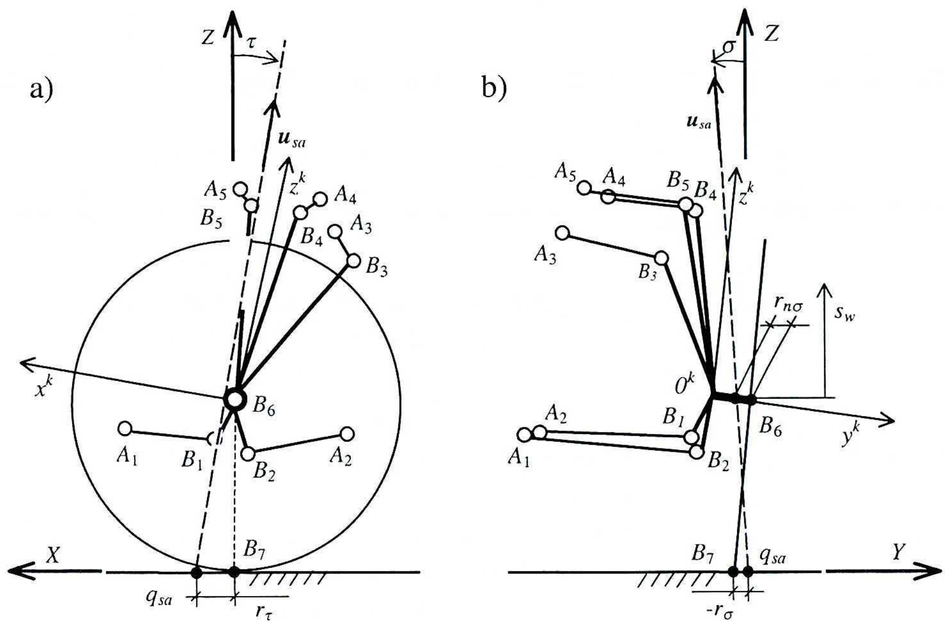

Fig. 2. Parameters of the suspension/steering system: a) side view, b) front view

The orientation of the suspension subframe with respect to the space-fixed reference system W is described in accordance to the roll-pitch-yaw convention [11], [12].

A series of geometrical measurements were carried out for the considered wheel guiding mechanism. Disassembled suspension components were measured by using a three-coordinate NC measurement machine, which makes it possible to achieve accuracy of 0.001 mm for linear dimensions.

The set of the obtained coordinates was properly transformed into desired geometrical parameters of the real mechanism, for example the rod lengths were determined as the distances between suitable joints centers. The centers of the steel ball-joints were determined on the basis of the measured points coordinates lying on the face of the spherical pins, which were disassembled. Points lying on sleeve faces were used for determination of rotation axes of the elastomeric bushes.

The values of these geometrical parameters were rounded to 0.1 mm, what was caused by using an indirect method for their determination from the measurment data including shape errors and dimensional tolerances of the real mechanism.

The measured quantities, including center coordinates of certain joints, selected geometrical characteristics or individual points taken from these characteristics, form a set of input data to the synthesis problem. These are the following:

- $\boldsymbol{a}_i^s$ – position vector of the joint center of i-th rod (i = 1, 2) linked to the subframe, described relative to W^s;
- $\boldsymbol{R}^s$ – orientation matrix of W^s reference system relative to W;
- d_i (i = 1,2...5) – length of i-th suspension rod;
- $\boldsymbol{b}_i^k$ (i = 1,2...5) – position vector of the joint center, where i-th rod is linked to the steering knuckle, described in W^k;
- 0^k and B_6^k – points that determine the wheel spin axis and the rim mounting plane at the steering knuckle, described in W^k;
- r_w – loaded wheel radius.

Target properties of the considered suspension, what means the characteristics that the real assembled mechanism performs under given conditions, were taken from [10]:

- δ, γ, o_x^k (as the functions of o_z^k) – selected coordinates of the wheel knuckle position and orientation described with respect to W;
- $\boldsymbol{u}_{sa}$, $\boldsymbol{q}_{sa}$ – orientation and position of the so called virtual steering axis described with respect to W and determined for given position of the mechanism;
- h_{rc} – instantaneous roll center height with respect to W for given position of the mechanism;
- δ_{dif} – steer angle difference for given mean steer angle.

The goal of the synthesis is to determine non-measured geometrical parameters that form the following set of 12 design variables:

- o_z^s – z-component of the position vector of the reference frame W^s with respect to the frame W;
- $\boldsymbol{a}_i$ – position vectors of the point A_i (i = 3, 4, 5) described in the reference frame W;
- Δd_3 – the change of the tie rod length,
- Δb_{3z}^k – z-coordinate of the point B_3 in the reference frame W^k.

The last two parameters are responsible for the alignment and adjustment of the wheel toe angles [10].

3. Mechanism position analysis

Position and displacement analysis of the considered wheel guiding mechanism is performed under assumptions that the mechanism is composed of rigid bodies connected via joints with ideal rigid elements and without clearances. This kind of approach is acceptable when the suspension characteristics taken into account are only slightly contingent upon properties of the compliant joints.

The spatial displacement of the wheel knuckle is described by two independent variables: s_s – the main spring length and p – the steering rack displacement (Fig. 1). Variable s_w is chosen to represent the wheel-suspension vertical displacements (Fig. 2).

There are two ways often used to solve the direct position problem of a spatial mechanism described by highly nonlinear constraint equations. The first approach is to apply some algebraic methods to eliminate unknowns from the set of constraint equations and to form one polynomial with a single unknown but in complicated form. The second approach is to apply iterative numerical methods to solve the set of nonlinear constraint equations in their primary form [2], [10], [11].

The proposed vector-algebraic approach [5], [6] combines some advantages of these methods. A modified mechanism is considered with two of the guiding links (4 and 5 on Fig. 1) disassembled from the wheel knuckle. The displacement equations for the knuckle with 3 degrees of freedom are derived from solutions of the vector tetrahedrons described by the unit vectors of rods and lines connecting joint centers of a spatial multi-loop mechanism. The condition that the distances between the released joints remain equal to the respective lengths of the disconnected rods is described by two nonlinear constraint equations. These equations are solved iteratively for given input variables describing the suspension bounce or steering displacements.

Any general spatial displacement of a rigid body can by considered as a screw displacement, a combination of a rotation about the screw axis and a translation along the same axis. The screw representation of the spatial displacements of a wheel guiding mechanism has found a number of applications in vehicle dynamics. For example, it may be used to determine an equivalent swing arm of a suspension, body roll and pitch axes, suspension compliance axes [5], virtual steering axis [2], [8], parameters of the steering system like castor offset, brake radius, shock radius [8], and others.

For a finite body displacement, the screw parameters can be determined on the basis of some initial and final position coordinates of three non-collinear points fixed to a rigid body [1]. When the screw pitch of a rigid body displacement is equal zero or is small enough to be neglected, then the screw axis is reduced to the rotation axis describing a pure spherical motion. The screw axis of the finite displacement of the steering knuckle between its two positions (with the upper left index ^{n} and $^{n+1}$) can be determined as [1], [5]

$$e^{0}_{n,n+1}\,\mathrm{tg}\,\frac{\theta_{n,n+1}}{2}=\frac{({}^{n+1}\boldsymbol{b}_{jk}-{}^{n}\boldsymbol{b}_{jk})\times({}^{n+1}\boldsymbol{b}_{ji}-{}^{n}\boldsymbol{b}_{ji})}{({}^{n+1}\boldsymbol{b}_{jk}-{}^{n}\boldsymbol{b}_{jk})\cdot({}^{n+1}\boldsymbol{b}_{ji}+{}^{n}\boldsymbol{b}_{ji})}$$

$$\boldsymbol{p}_{n,n+1}=\frac{1}{2}\left[{}^{n}\boldsymbol{b}_{i}+{}^{n+1}\boldsymbol{b}_{i}+\boldsymbol{e}^{0}_{n,n+1}\times({}^{n+1}\boldsymbol{b}_{i}-{}^{n}\boldsymbol{b}_{i})\,\mathrm{ctg}\,\frac{\theta_{n,n+1}}{2}-\boldsymbol{e}^{0}_{n,n+1}\cdot({}^{n+1}\boldsymbol{b}_{ji}+{}^{n}\boldsymbol{b}_{ji})\,\boldsymbol{e}^{0}_{n,n+1}\right] \tag{1}$$

where: $\theta_{n,n+1}$ – angular displacement of the wheel knuckle from position n to $n+1$;

${}^{n}\boldsymbol{b}_{jk}={}^{n}\boldsymbol{b}_{k}-{}^{n}\boldsymbol{b}_{j}$ – position vector of point B_j relative to B_k, corresponding to n-th position of the knuckle;

$\boldsymbol{e}^{0}_{n,n+1}, \boldsymbol{p}_{n,n+1}$ – unit vector of the screw displacement axis and position vector of the axis point.

4. Synthesis algorithm

4.1. Objectives and assumptions

The synthesized mechanism must satisfy a finite number of constraint equations at a prescribed set of poses. The difference between the specified and generated poses is described as a structural error and in precision synthesis the objective is to reduce the structural error as much as possible. A different approach to the problem is to establish a function of the structural error integrated over the full range of displacement. A specific set of mechanism parameters that would lead to a minimum of the error function could then be considered as an optimal design. There is no requirement that the structural error is equal to zero at all design positions, only that the sum of the squares of the structural errors achieves a minimum.

Many design poses may be specified, although experience has shown that numbers of these poses assumed over three or four times greater than the maximum number of design variables do not lead to any reduction of the maximum error.

The optimal synthesis problem can be formulated as a multi-criteria problem of the nonlinear programming with constraints. The problem can often be reduced to a single-criteria by using programming in the sense of weighted sum of the least squares. In this case the computational procedure can be formulated in a simpler form, but some criteria used as components of the objective function are not pronounced well.

The objective function should have fully continuous feasible domain and smooth variation with non-linearity reduced to a minimum. It should not produce extraneous or multiplied solutions, solutions requiring the mechanism disassembling to perform the desired sequence of displacements, or solutions for a degenerated mechanism.

Two types of the objective function used in the synthesis can be formulated.

A. The structural error to be minimized is derived at each pose of the mechanism by using a solution to the direct position analysis problem, which for a spatial mechanism with many kinematic closed-loops is often cumbersome and time consuming.

B. Making use of the mechanism particular properties, one can substitute the structural error by some simpler relationships, which can give an optimal solution to the synthesis. The mechanism direct position problem is solved after optimization to check fitness of the obtained solution.

The results of sensitivity analysis concerning the influence of the mechanism specific dimensions on its geometrical characteristics, allow us to select the most responsible parameters for the considered properties. The knowledge about these special relationships between selected parameters and desired properties of the designed mechanism should be included in formulation of an objective function improving the problem conditioning.

Analysis of the multi-rod suspension mechanism shows that in the established set of design variables the coordinates of points A_4 and A_5 (Fig. 1) first of all exert an influence on the following characteristics:

- trajectory of the wheel center point (B_6 in Fig. 1) and variations of camber wheel angle and knuckle pitch angle, corresponding to vertical displacement of the wheel knuckle,
- position and orientation of the virtual steering axis.

The rest of the design variables describing the tie rod (3 in Fig. 1) strongly influence on:

- steer angle (toe angle) changes corresponding to the suspension vertical motion,
- relation between the steer angle of inner and outer wheel (the instantaneous kinematic ratio of the steering linkage).

Different influences of the design variables on the particular properties of the mechanism allow for dividing the optimal synthesis problem into two stages.

The first stage of synthesis, thanks to a suitable formulation of the objective function of the type B, can be further divided into three independent optimization procedures for a separate determination of the coordinates of points A_3, A_4 and A_5. The following vector of decision variables is defined for each substage (totally 9 decision variables)

$$\xi_{I,i} = [a_{ix}, a_{iy}, a_{iz}]^T_{1\times 3} \qquad \text{for } i = 3, 4, 5 \tag{2}$$

The optimization criteria used in this part describe desired variation of the wheel track, camber angle, instantaneous roll center height and location of a virtual steering axis.

In the second stage of synthesis, the coordinates of the points A_4 and A_5 are assumed to be ultimately determined after the first stage. Next, minimizing a new formulated objective function of the type A, we determine the following vector of five decision variables

$$\xi_{II} = [a_{3x}, a_{3y}, a_{3z}, \Delta d_3, \Delta b^k_{3z}]^T_{1\times 5} \tag{3}$$

The optimization criteria used in this part describe the structural error of a particular characteristic, which the suspension system should fulfill accurately i.e. variations of the wheel toe angle corresponding to the suspension jounce-rebound displacements. The vector of design variables (3) comprises parameters the most responsible for that property. Coordinates of the point A_3 are again determined from the design space reduced in the first stage of synthesis. A detailed description of both synthesis stages is given below.

4.2. Input data preparation

The set of input data should be properly prepared before entering the synthesis procedure. The synthesis problem can be formulated in a different manner depending on the given set of input data and unknown parameters.

When all the suspension links are disassembled, then the steering knuckle can be considered as a free body with 6 DOF (Fig. 1). If m coordinates describing its position and orientation are imposed, then m DOF is restricted. The geometrical characteristics of the designed mechanism are given in the form of a set of discrete values: ${}^j\gamma$ (wheel camber angle), ${}^j\delta$ (wheel steer angle) and ${}^{j}o_{x}^{k}$ (longitudinal component of the wheel center position) as the functions of the vertical position ${}^{j}o_{z}^{k}$, where:

$$ {}^{j}o_{z}^{k} = {}^{j}s_{w} + r_{w} \qquad \text{for } j = 1, 2...\text{n} \tag{4} $$

Considering ${}^{j}o_{z}^{k}$ as a parameter, we get $m = 4$ coordinates. If a spatial displacement of the steering knuckle have to be uniquely described with respect to the frame W, then additional constraint equations described for two lower rods (1 and 2 on Fig. 1) should be used. The coordinates of points A_1 and A_2 (joints at the subframe) are calculated by using the formulas

$$ \boldsymbol{a}_{i} = \boldsymbol{R}^{s}\boldsymbol{a}_{i}^{s} + \boldsymbol{o}^{s}, \qquad \text{for } i = 1, 2 \tag{5} $$

In equations (5) only z-component of vector $\boldsymbol{o}^{s}$, describing the height of the origin of W^{s} (the subframe reference system) above the road plane is not directly given in the set of input data. Its value can be evaluated on the basis of information about an instantaneous roll center height at the prescribed suspension vertical position. The procedure for finding this parameter will be explained later. In the first iteration loop, an initial arbitrary value of the wanted parameter needs to be selected.

If we appropriately select $j = 1, 2...n$ design points from the suspension workspace, then the coordinates of the joints centers at the wheel knuckle can be determined for each design points as follows

$$ {}^{j}\boldsymbol{b}_{i} = {}^{j}\boldsymbol{R}^{k}\boldsymbol{b}_{i}^{k} + {}^{j}\boldsymbol{o}^{k} \qquad \text{for } i = 1, 2 \text{ and } j = 1, 2...n \tag{6} $$

The rotation matrix ${}^{j}\boldsymbol{R}^{k}$ is a function of ${}^{j}\gamma$, ${}^{j}\varepsilon$ and ${}^{j}\delta$, where ${}^{j}\gamma$ and ${}^{j}\delta$ are given and ${}^{j}\varepsilon$ is unknown variable. The position vector ${}^{j}\boldsymbol{o}^{k}$ $= {}^{j}[\boldsymbol{o}_{x}^{k}, \boldsymbol{o}_{y}^{k}, \boldsymbol{o}_{z}^{k}]^{T}$ of W^{k} relative to W is a function of given coordinates ${}^{j}\boldsymbol{o}_{x}^{k}$ and ${}^{j}\boldsymbol{o}_{z}^{k}$, however, ${}^{j}\boldsymbol{o}_{y}^{k}$ have to be found.

In order to determine the unknown values of ${}^{j}\varepsilon$ and ${}^{j}\boldsymbol{o}_{y}^{k}$ for each design point $j = 1, 2...n$, a set of two nonlinear constraint equations needs to be solved

$$ d_{i} - {}^{j}|\boldsymbol{b}_{i} - \boldsymbol{a}_{i}| = 0 \qquad \text{for } i = 1, 2 \text{ and } j = 1, 2...n \tag{7} $$

where: ${}^{j}|\boldsymbol{b}_{i} - \boldsymbol{a}_{i}| = {}^{j}\left[\sqrt{(b_{ix}-a_{ix})^{2}+(b_{iy}-a_{iy})^{2}+(b_{iz}-a_{iz})^{2}}\right]$

The set of equations (7), including expressions (5) and (6), can be solved by using one of the numerical procedures with analytical gradient which is easy to derive in this case.

The wheel-knuckle spatial positions and orientations, uniquely described by ${}^j\gamma, {}^j\varepsilon, {}^j\delta, {}^j\boldsymbol{o}_x^k, {}^j\boldsymbol{o}_y^k$ (considered as the functions of ${}^j\boldsymbol{o}_z^k$ or s_w), represent a desired trajectory for the synthesis of the rigid body guidance mechanism, Fig. 3.

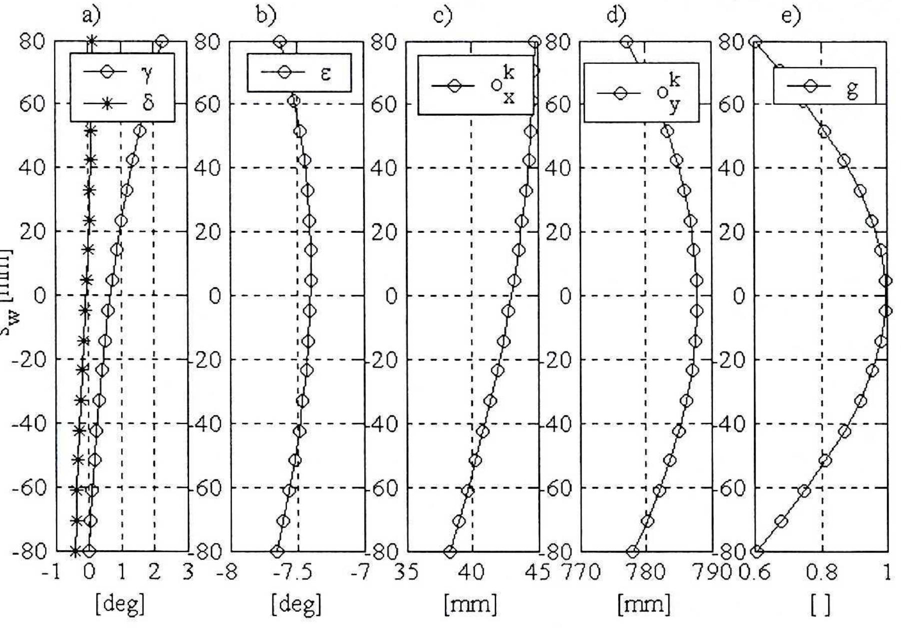

Fig. 3. Desired characteristics for jounce-rebound displacement of the wheel knuckle (a,b,c,d) and (e) weighting factor used in the optimization objective function, $p = 0$ (this corresponds to the straight-ahead position of the steering mechanism)

The position vectors of the other points (centers of the joints at the steering knuckle) can be calculated by using formula (6) with the index changed respectively.

The above considerations were performed for an initial guess of the unknown z-component of the vector $\boldsymbol{o}^s$ described by (5). With the variation of the height of the suspension subframe with mounting points of the bottom rods (1 and 2 on Fig. 1) the wheel track changes are effected, which in turn determine the suspension roll center [8]. Having the trajectory of the wheel knuckle (Fig. 3), we can determine the instantaneous roll center height according to a method presented in [8]. The roll center height corresponding to the design position of the suspension is given in the set of input data [10]. The missing z-component of $\boldsymbol{o}^s$ has been selected by an trial and error method to obtain a roll center characteristic fulfilling input data for the given

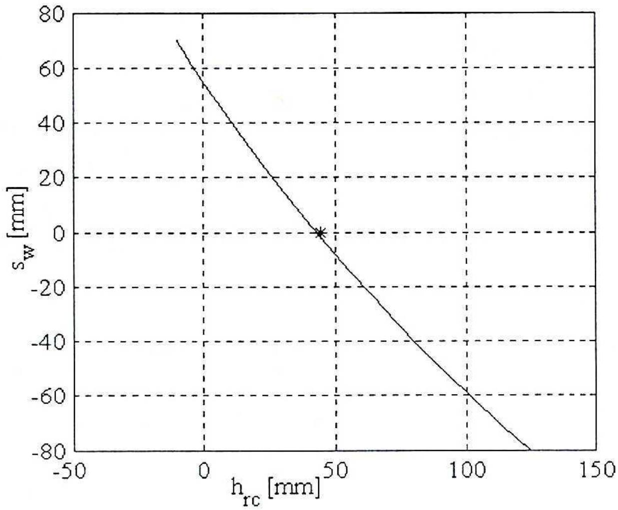

Fig. 4. Instantaneous roll center height h_{rc} for the suspension jounce-rebound motion, $p = 0$ (this corresponds to the straight-ahead position of the steering mechanism). Asterisk shows the desired value of h_{rc}

suspension vertical position ($s_w = 0$ and $p = 0$). This procedure, due to simple relationships, can be easily automated. Fig. 4 shows an ultimate characteristic of falling down roll center with the suspension vertical displacement, which makes it possible to avoid the car body jacking up effect during the body roll motion [8].

4.3. First stage of synthesis

The coordinates of the joint centers A_i ($i = 3, 4, 5$), where the three suspension rods (3, 4 and 5) are linked to the car body, were assumed as design variables in the first stage of synthesis. A modified mechanism with these three rods of variable lengths is considered. An optimization procedure, formulated below, is used to find the coordinates of points A_3, A_4 and A_5 that give the smallest changes of their distances from the respective points B_3, B_4 and B_5 for a defined sequence of the wheel knuckle displacements.

In the used method, the coordinates of each joint center can be found apart of each other. For example, if coordinates of the point A_5 are found, that guarantee the smallest changes of its distance from B_5, then the mechanism with the rod of constant primary length (d_5) assembled between A_5 and B_5 performs its displacement very close to the target one. If the considered point A_5 is close to the exact solution of the synthesis, i.e. to the so called center-point, then a deviation of the actual trajectory from that imposed as the desired one is small. Under this assumption, the same geometrical requirements can be utilized for independent searching for a successive point, for instance A_4, and ultimately A_3.

Making use of the mentioned feature, we divided the first synthesis stage into three substages for determination of suitable design variable vectors of the form (2).

For each vector of the decision variables (2), the following objective function was defined

$$^{j}f_{I,i}(\xi_{I,i}) = d_i - {}^{j}|\boldsymbol{b}_i - \boldsymbol{a}_i|, \qquad i = 3, 4, 5; \qquad j = 1, 2...n \tag{8}$$

which is minimized as a sum of weighted least squares at each design point

$$\min F_{I,i}(\xi_{I,i}) = \frac{1}{2}\sum_{j=1}^{n}[{}^{j}g\,{}^{j}f_{I,i}\,(\xi_{I,i})]^2 \tag{9}$$

under variable constraints of the form

$$\xi_{\min} \leq \xi \leq \xi_{\max} \tag{10}$$

where ${}^{j}g$ – weighting factors for j-th position.

This task can be considered as a reduced Burmester problem with geometrical interpretation of searching for sphere centers, with radius equal to the length of the appropriate suspension rod, which have the sphere sectors close to the path of the considered point of the knuckle displaced between desired locations.

Taking the advantage of a simple form of the objective function (8), we can derive its gradient analytically, what improves effectiveness of the optimization procedure.

For any optimization task, the goal is to find the global minimum of the objective function (8) in a feasible region, where the constraints (10) for the design variables are satisfied. The method used in this paper is based on a union of some deterministic optimization algorithm [9] (trust-region reflective Newton method) with exploration of the allowable design space by random selection (with uniform distribution) of starting points for each substage of the synthesis.

Exemplary results obtained by using the described algorithm with starting points randomly distributed are shown in Fig. 5. Different solutions are achieved depending on the initial guess to the starting value. The family of solutions with different values of the objective function is located along

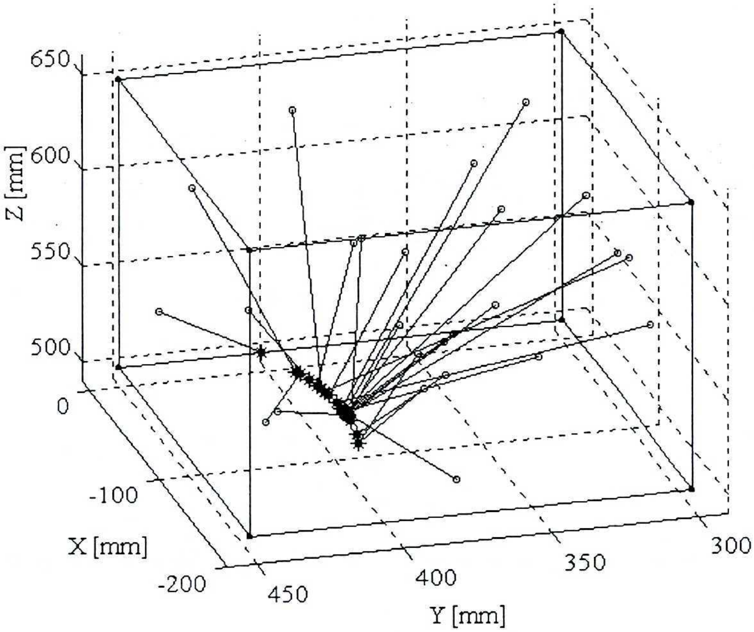

Fig. 5. Convergence of the optimization solution for randomly selected initial design variables from design space bounded by a cube marked in the Figure. Example for the A_3 point synthesis

characteristic curves, which are visualized in Fig. 6 for the set of considered points. Each curve (from the family of solutions in Fig. 6) can be considered as an arc with the center located at the point B_i ($i = 3, 4, 5$) – the joint center connecting the i-th rod with the steering knuckle.

The given trajectory (Fig. 3), used for the synthesis of the rigid body guidance mechanism, describes poses of the steering knuckle in the range of jounce-rebound vertical displacement. This makes the problem well conditioned correspondingly to the z-coordinates of the obtained family of solutions. However, the orientation of the respective rod (linking points A_i and B_i) in the horizontal plane is not sufficiently conditioned by the imposed displacements of the mechanism, what results in many solutions-rods satisfying not unique conditions.

The selection of the best solution should be based on additional criteria described by the steering knuckle angular displacement caused by the steering rack linear displacement (p – input variable).

The coordinates of points A_4 and A_5 should satisfy some additional conditions for the position and orientation of the virtual steering axis for a given spatial displacement of the steering knuckle determined by two input variables (s and p). The location of the steering axis corresponding to a straight ahead running of the unloaded car are given in [10]. This axis is determined by two closed kinematic chains Fig. 2, one formed by two lower rods (1 and 2) and the second – by two upper rods (4 and 5) which are to be found.

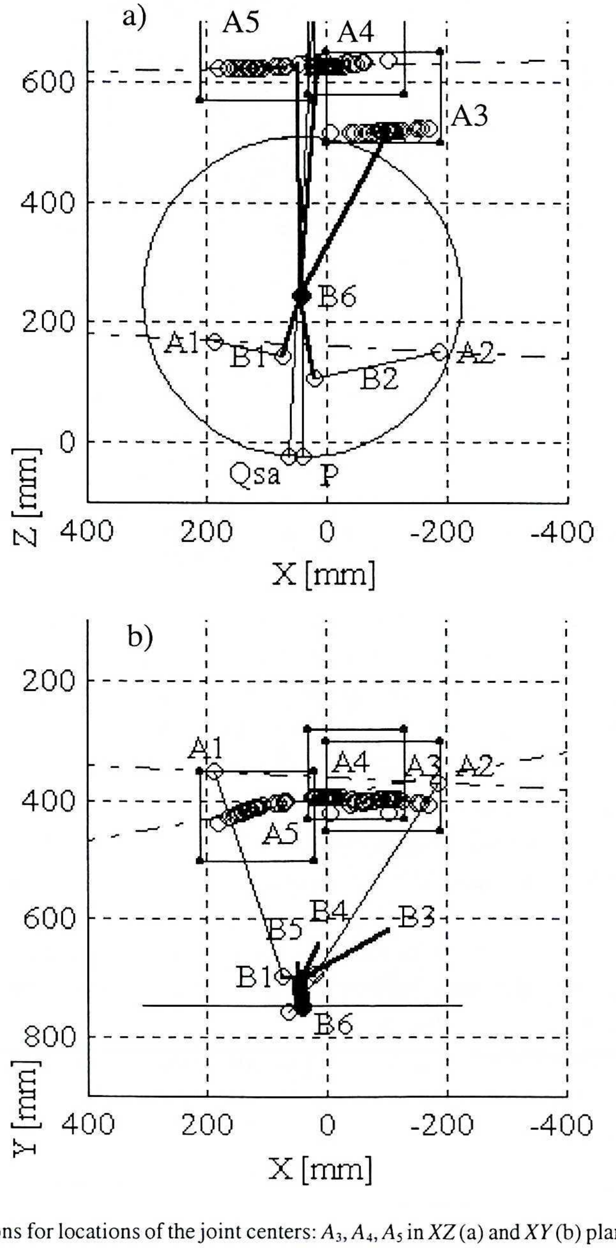

Fig. 6. Set of solutions for locations of the joint centers: A_3, A_4, A_5 in XZ (a) and XY (b) planes. There are also shown: cubic bounds of the design variables, a scheme of the wheel-knuckle system with rods 1 and 2, a desired virtual steering axis for $s_w = -20.0$, $p = 0$

If the rods 4 and 5 are located in the planes Π_{B4} and Π_{B5}, described by the imposed steering axis and by respective points of the wheel knuckle (point B_4 for the rod 4, and point B_5 for the rod 5), then the wheel guiding mechanism is characterized by the same steering axis as the one desired (Fig. 2). This condition makes it possible to choose among solutions of the joint centers A_4 and A_5, already obtained in minimization of the objective function (8), Fig. 6, the one located in the vicinity of the respective plane (Π_{B4} for A_4 and Π_{B5} for A_5) and having simultaneously the lowest value of the objective function (8). The position and orientation of the steering axis, described by $\boldsymbol{q}_{sa}$ and $\boldsymbol{u}_{sa}$, and the coordinates of the points B_4 and B_5 for given position of the mechanism ($s_w = -20.0$ mm and $p = 0$) are used to determine the planes Π_{B4} and Π_{B5}.

Finally, the set of design variables obtained from the optimization procedure in the first stage is used as the input data for a search of the solutions satisfying simultaneously the basic optimization criteria (8) and additional criteria describing the steering axis position and orientation. The additional objective function, described by l_{Ai} a distance between the point A_i and the plane Π_{Bi} ($i = 4, 5$), should be minimized.

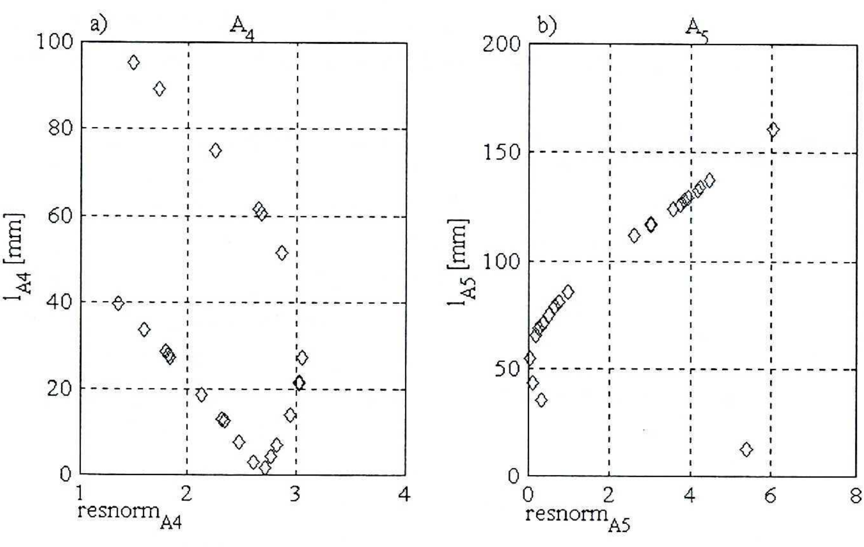

Fig. 7. Distance of the point A_i with respect to the plane Π_{Bi} (l_{Ai}) as a function of an fitness parameter ($resnorm_{Ai}$) of the objective function used in the first stage of the synthesis, (a) for $i = 4$ and (b) for $i = 5$

This problem can be considered as a two-criteria optimization, where the Pareto plane is described in Fig. 7 by $resnorm_{Ai}$ – norm of objective function (8) and l_{Ai} – distance of the point A_i with respect to the plane Π_{Bi} ($i = 4, 5$). The solution for the both joints centers has to be chosen so that it represents the assumed compromise between these criteria.

The family of solutions for the point A_3, obtained in the first stage of the optimization, is constrained by the following condition describing the selected characteristic of the steering linkage. The relationship between the steer angles of a left and right wheel depends on dimensions of the steering linkage. In the set of input data, the difference between the steer angles of both wheels is given for the mean value of the steer angle of 20 deg. The solution for the tie rod 3 (Fig. 1), linking the joint B_3 at the steering knuckle and A_3 at the steering rack, should be chosen to satisfy this condition [8]. In order to describe the relationship between the wanted parameter and the desired characteristic, a displacement analysis of the mechanism is performed taking the coordinates of points A_4 and A_5 as finally determined. The obtained difference between the steer angles of both wheels is compared in the analysis with that imposed. The trial and error method is used to select the best solution satisfying this condition.

The range of the wheel steer angle can be limited by possible wheel-axle or wheel-body interference or unacceptable pressure angle between the steering knuckle arm and the tie rod [8]. These conditions should also be verified during the synthesis process of a new suspension-steering system.

4.4. Second stage of synthesis

Second stage of the mechanism synthesis is conducted under the assumption that the joint centers A_4 and A_5 are finally determined. The coordinates of the joint center A_3 are searched for again in the reduced space of the allowable solutions to satisfy the new condition of a minimal deviation of the wheel toe angle from the one imposed. The requirements for proper variation of the toe angle, during jounce-rebound displacement of the suspension, are important in respect of the car handling and stability behavior [8]. The objective function of the type A is used in the optimization algorithm in order to accurately describe these properties of the real mechanism.

The variations of the wheel toe angle, determined by using the procedure for displacement analysis presented in [5], can be expressed as a function of the input variable s_w and the vector of design variables ξ_{II} (2) in the following form

$$^{j}\delta_{ob} = h\,(^{j}s_{w}, \xi_{II}),\; j = 1, 2 \ldots \mathrm{n} \tag{11}$$

New objective function is assumed in the form

$$^{j}f_{II}\,(\xi_{II}) = {}^{j}\,\delta_{des} - {}^{j}\,\delta_{ob},\; j = 1, 2...\mathrm{n} \tag{12}$$

and the weighted sum of least squares

$$\min F_{II}(\xi_{II}) = \frac{1}{2}\sum_{j=1}^{n}[{}^{j}g\,{}^{j}f_{II}\,(\xi_{II})]^{2} \tag{13}$$

is minimized under constraints of type (10), where ^{j}g – weighting factors for j-th position.

Similarly as in the first stage of the synthesis, the deterministic optimization algorithm is used with initial values of the design variables randomly sampled from the allowable design space. Effective solution obtained in the first stage of the synthesis gives the possibility to diminish the design space for the point A_3 that is wanted in the second stage.

5. Numerical example

The numerical example concerns the AUDI A4 front suspension system. The necessary input data to the problem are listed below. Linear dimensions are given in [mm].

- $\boldsymbol{a}_1^s = [187.0, 351.5, 94.0]^{\mathrm{T}}$, $\boldsymbol{a}_2^s = [-186.5, 370.0, 76.0]^{\mathrm{T}}$;
- $\boldsymbol{R}^s = \boldsymbol{I}$ (W^s has the same orientation as W);

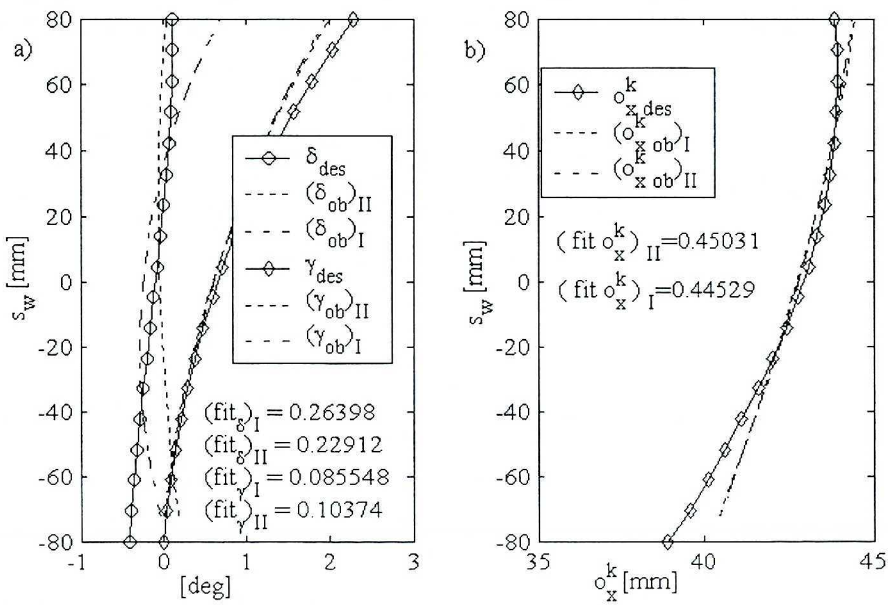

Fig. 8. Comparison of target and obtained coordinates of the wheel knuckle position and orientation ($p = 0$) after first and second synthesis stage. For each characteristic the parameter *fit* is shown

- $d_1 = 364.0$, $d_2 = 385.0$, $d_3 = 221.5$, $d_4 = 248.5$, $d_5 = 273.0$;
- $\boldsymbol{o}^k = [0, 0, 0]^T$, $\boldsymbol{b}_6^k = [0, -40.0, 0]^T$;
- $\boldsymbol{b}_1^k = [17.5, -91.6, -101.7]^T$, $\boldsymbol{b}_2^k = [-39.1, -95.5, -130.1]^T$,
- $\boldsymbol{b}_3^k = [-107.0, -166.2, 289.1]^T$,
  $\boldsymbol{b}_4^k = [20.6, -142.9, 387.1]^T$, $\boldsymbol{b}_5^k = [54.3, -109.5, 384.4]^T$;
- $r_w = 270.0$ (for a tire 195/65 R15 ET45);
- $\boldsymbol{u}_{sa} = [-0.0545, -0.0625, 0.9966]^T$ – unit vector of the steering axis determined on the basis of values of σ and τ given for $s_w = -20.0$ and $p = 0$;
- $\boldsymbol{r}_{sa} = [21.6, 6.9, 0]^T$ for $s_w = -20.0$ and $p = 0$;
- $h_{rc} = 44.9$ for $s_w = 0$ and $p = 0$;
- $\delta_{dif} = 1.6$ deg for $s_w = 0$ and p corresponding to $\delta_m = 20.0$ deg.

For both stages of the synthesis $n = 18$ design points were selected on the target characteristics [10], equally spaced in the range of s_w from 80 mm to 80 mm and for $p = 0$, Fig. 3. The number of design points was limited from the one side by the number k of design variables in order to achieve an over-determined system, and from the other side by an acceptable computation time.

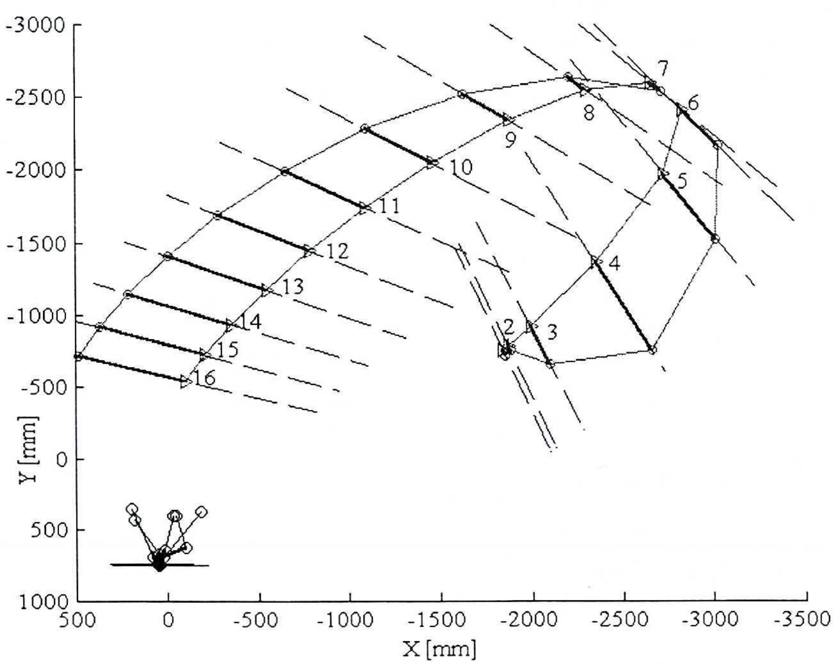

Fig. 9. Screw axes with screw pitches (for better visibility magnitudes of screw pitches are multiplied by 100) for 16 wheel knuckle positions in the range of $s_w = \pm 80$ mm, and for $p = 0$. Location of the suspension with respect to screw axes is also shown

In order to increase the algorithm sensitivity in the range of the suspension vertical displacements, close to the most important design position, the objective functions (8) and (11) are multiplied by weighting factors ${}^{j}g$ (j = 1, 2...n). The values of these factors are described by a probability distribution function of normal distribution with the following parameters: mean value equal to 0 and standard deviation equal to 80 mm, Fig. 3e.

The suspension characteristics analyzed in the paper were evaluated regarding their fitness to the target-desired characteristics by means of a parameter *fit*, which is defined below as a standard deviation of their residuals R:

$fit_F = \sqrt{\frac{1}{n}\sum_{j=1}^{n}({}^{j}R - \bar{R})^2}$, where ${}^{j}R = {}^{j}F_{target} - {}^{j}F_{obtained}$, j = 1, 2...n, F – selected characteristic.

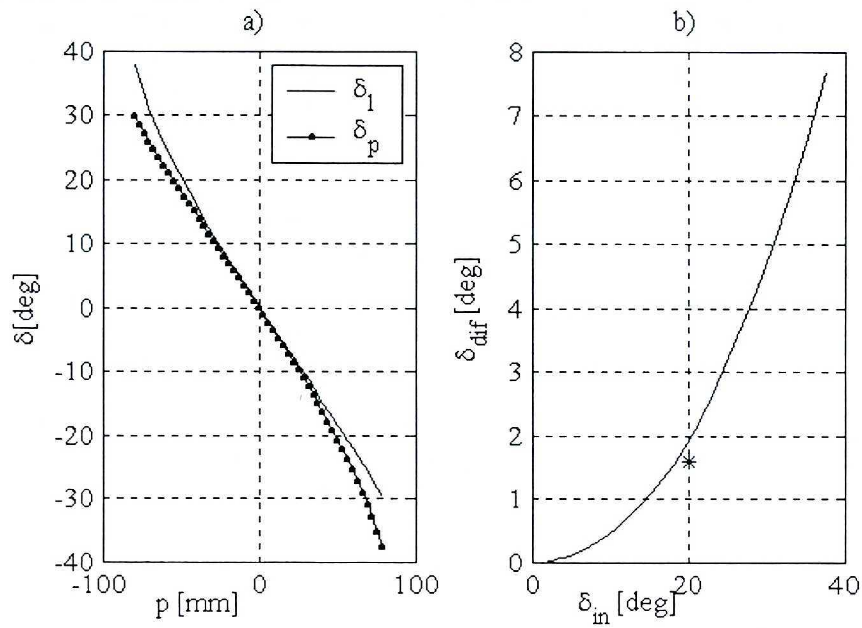

Fig. 10. a) Steer angles of left and right wheels for steering rack displacement, b) steer angles difference of inner and outer wheel as a function of mean steer angle at s_w = 0 mm. Asterisk shows the desired value of δ_{dif}

Components of the design variable vector (2), ultimately determined in the second stage of the mechanism synthesis, are the following (given in [mm]):

$\boldsymbol{o}^s = [0, 0, 75.0]^T$, $\boldsymbol{a}_3 = [-54.0, 385.5, 527.0]^T$, $\boldsymbol{a}_4 = [-37.0, 398.0, 629.0]^T$,

$\boldsymbol{a}_5 = [173.0, 433.5, 638.0]^T$, $\Delta d_3 = 17.5$, $\Delta b_{3z}^k = 2.2$.

Selected coordinates of the wheel knuckle position and orientation obtained in the first and second synthesis stages, are compared with the desired ones in Fig. 8. For each characteristic, the parameter *fit* is evaluated. The variations of the wheel camber angle and the x-component of the wheel center $\boldsymbol{o}^k$, determined in the first stage of the mechanism synthesis, are very close to the target characteristics. The second stage influences them only slightly. Comparison of the toe angle variations demonstrates differences in terms of quantity and quality. An improvement of their fitness after the second stage is noticeable but still unsatisfactory.

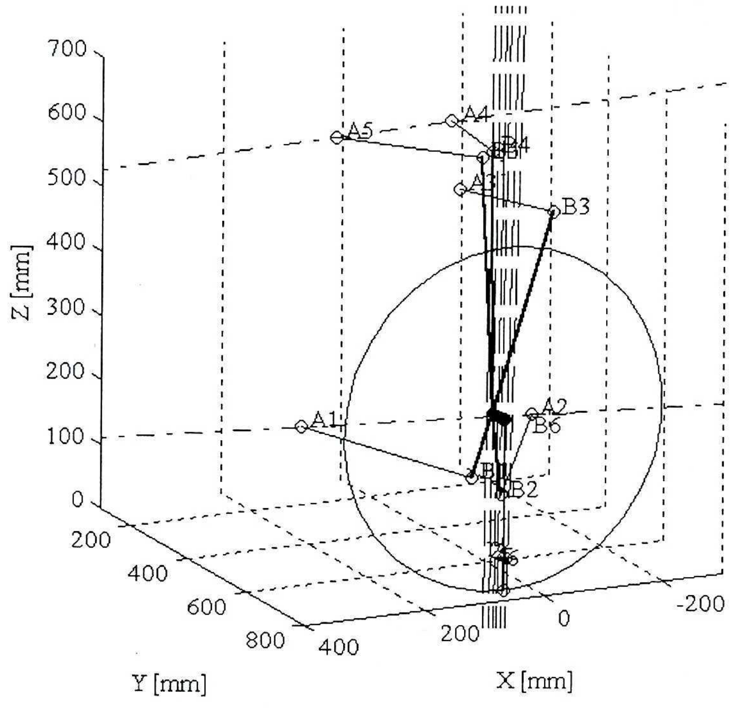

Fig. 11. Variation of position and orientation of the virtual steering axis (steering screw axis) for steering motion induced by the steering rack displacement in the range of $p = \pm 50$ mm at $s_w = 0$ mm

The variation of the screw motion axes correspondingly to the vertical suspension displacements, obtained for the finally designed mechanism, is presented in Fig. 9. An instantaneous equivalent suspension arm, also called swing arm radius, is quite long, what corresponds to small changes of the wheel rack and base. In the considered range of the wheel knuckle displacements, the screw pitch demonstrates changes of magnitude and sense what corresponds to a fully spatial motion.

The rest of parameters imposed on the requested solution are associated with the suspension-steering system. Varying the steering rack displacement p for given suspension position s_w, one can analyze a relationships between steer angles of both wheels (Fig. 10a) and location of the so called virtual steering axis. The difference between the steer angle values, determined for the both wheels (Fig. 10b), becomes more pronounced as the steering angle becomes greater, reaching a value close to the desired value at $\delta_m = 20$ deg.

The virtual steering axis slightly migrates and changes its orientation corresponding to variation of the steer angle, Fig. 11. As a consequence the parameters describing the steering axis orientation (angles: σ and τ in Fig. 12 a) and location (parameters: castor offset r_τ, and brake radius r_σ in Fig. 12 b) change their values. Because the screw pitches are quite small, the steering displacements of the wheel-knuckle can be treated as a purely spherical motion. A comparison of the determined and target parameters for the considered front wheel guiding mechanism is summarized in Table 1.

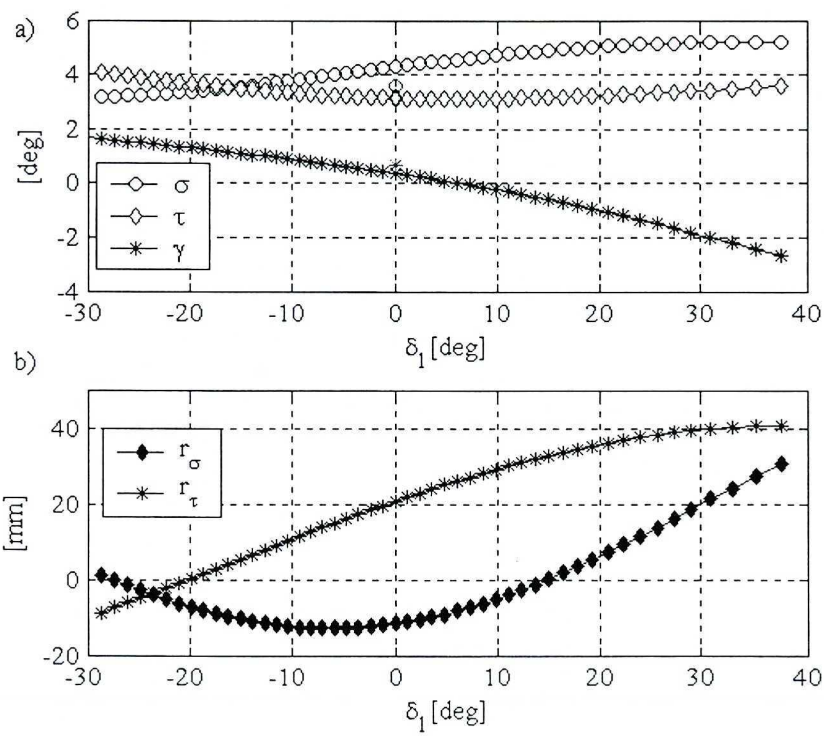

Fig. 12. a) Wheel camber angle γ, steering axis inclination σ, and castor angles τ, and b) castor offset r_τ, and brake radius r_σ as the functions of the left wheel steer angle, $s_w = -20$ mm. Suitable points for $\delta_m = 0$ show the desired values

Table 1.

Front suspension data

Curb weight conditions, $s_w = -20.0$ mm (according to DIN)		
	Data from [10]	Synthesis results
Track base [mm]	1495	1495
Wheel toe angle (total) [deg]	+0.3	+0.1
Wheel camber angle [deg]	−0.7	−0.6
Steering axis inclination angle [deg]	+3.6	+4.5
Steering axis castor angle [deg]	+3.1	+3.4
Castor offset [mm]	+21.6	+20.8
Brake radius (roll radius) [mm]	−6.9	−11.0
Design position, $s_w = 0$ mm (according to DIN 3 persons per 68 kg)		
Roll center height [mm]	+44.9	+43.0
Toe angle difference by 20 deg steer angle [deg]	+1.6	+1.9

6. Conclusions

The optimization-based algorithm can be used for the successful dimensional synthesis of the multi-rod suspension of the front wheel of a car. The applied methods of synthesis and analysis are also suitable for other simpler types of the wheel guiding mechanisms.

The proposed approach is based on a partitioning of one complex problem into smaller substages solved in hierarchical order. This approach can be used in the case where some geometrical characteristics of the mechanism are exploited, estimated on the grounds of the system sensitivity analysis or an experience and intuition of the designer. A great computational effectiveness can be achieved in comparison to the methods where design variables in huge numbers are treated simultaneously or where a complicated direct position problem of a mechanism has to be solved as an integral part of an optimization routine.

The proper choice of the target conditions that designed mechanism should satisfy is crucial to the optimal synthesis problem. This makes the problem well conditioned and permits avoiding or at least reduce multiple and ambiguous solutions. Besides the fundamental suspension characteristics, like variations of the wheel toe and camber angles or the wheel track as functions of the suspension vertical displacement, additional parameters concerning the instantaneous roll center height, position and orientation of the so called virtual steering axis, realization of Ackermann errors by the steering linkage are taken into account.

Slight deviations of the synthesis results from the ones treated as desired, can be the consequence of finding only a local minimum of the formulated objective functions, invalid input data or disregarding compliance characteristics of the suspension bushes. Comparison of the mechanism characteristics found in the first and the second stage of the synthesis justify goals of the problem division.

Further investigations into the considered problem are planned taking into account the influence of elastic joints on the elastokinematic characteristics of the real mechanism.

Optimization methods based on a genetic structure can be considered as a competitive approach to that presented in the paper for searching a globally optimal solution or at least for a reliable reducing of allowable design variables space.

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Synteza wymiarowa pięciowahaczowego mechanizmu prowadzenia kół przednich samochodu

Streszczenie

Przedstawiono metodę syntezy wymiarowej mechanizmu prowadzenia kół przednich w postaci układu 5-wahaczowego. Wyznaczono współrzędne trzech środków przegubów kulowych łączących wahacze (dwa górne wahacze i drążek kierowniczy) do nadwozia na podstawie posiadanego zbioru danych dotyczących współrzędnych pozostałych punktów mechanizmu, długości łączników, charakterystyk kinematycznych zawieszenia przy ruchach resorowania i zataczania. Zadanie syntezy sformułowano w postaci dwu etapowej procedury optymalizacyjnej o hierarchicznej strukturze, wykorzystując różnego typu funkcję celu. Przedstawiono ponadto metodę analizy przemieszczeń i położeń przestrzennego mechanizmu wielołącznikowego wraz z opisem parametrów ruchu śrubowego. Podany przykład liczbowy dotyczy zawieszenia przedniego samochodu AUDI A4.