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Study on the braking process of the hydraulic actuator with discrete control

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The study is aimed at improving the application effectiveness of the hydraulic actuator with discrete control, which is widely used in various fields. The goal is to improve the actuator performance by increasing its working speed while satisfying the braking characteristics. The study is based on a developed mathematical model of the hydraulic actuator, which includes the models of the discrete spool valve and the asymmetric hydraulic cylinder, and takes into account bi-directional movement of the rod, compressibility of the working fluid and nonlinear friction. Accuracy of the mathematical model is verified by comparing the simulation and experimental data. The study makes it possible to determine, for the specified parameters of the hydraulic actuator and the variation ranges of the working speed and inertial load, the quantitative relationship between working speed, braking time and maximum pressure. The study also reveals a new phenomenon of nonlinear influence of the spool response time on the braking time, which can be used to minimize the braking time. The obtained results can be used in the design of an actuator with discrete control, parameter setting of throttle elements, and development of control programs.

1. Introduction

Discrete hydraulic actuators are most widely used in hydraulic equipment and machines for various purposes. A typical discrete hydraulic actuator consists of a hydraulic cylinder and a discrete-acting spool valve that are connected by hydraulic pipelines. After the spool valve receives the control signal, the spool is switched to one of the fixed positions. At this time, the working fluid is supplied to one

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chamber of the hydraulic cylinder, and from the other chamber the working fluid flows into the tank. The result is that the hydraulic cylinder rod moves in a direction corresponding to the position of the spool, or stops in case the spool is set to the middle position. The movement speed of the rod can be controlled by means of an additional throttle valve or a flow regulator. In the case of using additional means for controlling the current position of the rod, such actuators enable positioning in a wide range of displacements and velocities.

The actuator with the position sensor and pulse width modulation (PWM) control is one type of discrete hydraulic actuator. It is controlled by adjusting the pulse frequency, which causes the spool to oscillate at the corresponding frequency. The position sensor feedbacks the signal as the target position is reached, and the movement speed of the rod is determined by the pulse frequency. Such actuators are used, for example, in belt coal conveyor systems [1]. Actuators with such control methods have the frequency limitation. For example, the oscillation frequency of the spool does not exceed 200 Hz [1], which is due to the mass of the spool and the magnitude of the electromagnetic drive force. For the valve with this oscillation frequency, the working fluid pressure at the inlet is 70 bar [1]. The mass of hydraulic cylinder rod load is 20 000 kg, the positioning distance is 3 mm, the maximum speed of the spool is 0.24 m/s and the acceleration is 800 m/s^2 [1]. It takes about 1 s for the rod to reach the target position. It should be noted that increasing the working frequency of the valve can improve performance of the actuator, but it is limited by factors such as the friction between the spool and the valve sleeve, the preload of the spring and the magnitude of the electromagnetic drive force. And these factors are closely related to the dimensions of the spool. For example, the 2/2 ball valve mentioned in the literature [2] have the on-off frequency of up to 1 kHz, but the valve stroke is limited to 0.05 mm and the flow rate is only 0.2 l/min, so the actuators with such valves operate at low speeds.

Discrete actuators with high-speed on/off valves are also used as a pilot stage for a two-stage proportional valve [3]. It was confirmed by simulation and experiment that increasing the frequency of the pilot spool can increase the speed of the main spool. In addition, a higher frequency of the pilot spool not only reduces the oscillation of the main spool but also improves the positioning accuracy of the actuator. This is due to the fact that the pilot valve with higher switching frequency can output smaller portions of liquid, i.e., higher resolution of flow. Therefore, it is clear from the study [3] that the characteristics of this type of discrete actuator, such as motion speed and positioning accuracy, are limited by the frequency and duty cycle of the pilot spool.

Discrete actuators based on frequency control of discrete valves are also employed in precision tables for micromachining [4]. The displacement resolution of such tables can be as high as 1.2 μm, thanks to the spool with the response time of 1.6 ms [4]. In the work [4], a method is also proposed to weaken the step oscillation during the table movement by adjusting the phase difference between the input signals of the parallel valves. This ensures the required accuracy at small displacement, but the problem of low speed of the actuator cannot be solved, especially in certain critical parts of the micromachining process.

There is also the discrete hydraulic actuator with low pressure [5]. The actuator uses water as the working fluid and is controlled by the combination of a 4/3 directional valve and 2/2 on/off valves mounted in parallel. The experimental results show that under the load of 100 kg, the positioning speed is up to 0.4 m/s, and the positioning accuracy is 0.3 mm [5]. However, the study [5] was mainly limited to the investigating pressure variation in the cylinder chamber under some specific conditions of spool response. Meanwhile, the influence of the rod motion speed and its load on the actuator's operating process were neglected.

Another application case for discrete actuators is the motion control of the strip steel in a production line for rolling strip steel [6]. The hydraulic cylinder as the executing component is controlled by a 4/3 directional valve and two high-speed on/off valves. The actuator's feature is the two-stage control: the directional valve, controlled by pulse signals, is used to correct the direction and rough position of the strip steel, while the on/off valves, controlled by high frequency, allow for precise position correction of the strip steel. This feature allows increasing the achievable movement speed of the rod, but also complicates the structural design of the actuator.

Analyzing the known studies on discrete actuators with high-frequency pulse control of the valve spool one can notice that their application conditions are limited to low speeds and small displacement range. This is due to the fact that the size and mass of the spool limits the working frequency as well as the output flow rate of the valve, which leads to low energy efficiency of the method of pulse frequency control on the spool. In addition, actuators with such control method face a common problem, which is that the positioning accuracy of the rod during the positioning process is also decreased by the influence of external mechanical vibrations [7]. An effective way to reduce the vibration was proposed in [7].

Discrete actuators, which are widely applied to industrial, road, construction and agricultural machines, make use of a variety of positioning methods. Positioning of the discrete actuators' rod can also be achieved manually, by stoppers, or by limit switches [8, 9]. Due to the small number of stoppers or limit switches set along the travel of the rod, the actuators have a limited number of working positions. If the current position of the rod is monitored using a high-resolution position sensor and the spool displacement is controlled by the constant high-amplitude step signal, then the frequency of the control signal only needs to be very low, while the movement speed of the rod can be significantly increased. However, the problem is that in this control solution, when the movement speed of the rod is high and the inertial load is large, the time for the braking process and the pressure in the pipelines can sharply increase. This problem should be solved in order to increase the working speed of discrete actuators, thereby improving efficiency and expanding the range of applications.

The problem of improving the dynamic characteristics of actuators is usually solved by applying mathematical models and simulation of dynamic processes.

1.1. Overview of mathematical models of hydraulic actuators with discrete control

The key task in the design of hydraulic actuator with discrete control is to provide the desired dynamic characteristics in the braking process. In order to solve the problem, a mathematical model with sufficient accuracy and universality is needed to describe the dynamic process. The analysis of the literature shows that there is a variety of mathematical models for process modeling of hydraulic actuators with discrete control. They are suitable for modelling the structural features of the specific hydraulic actuator and for solving particular problems.

Hydraulic actuators based on high-speed switching valves are the most representative type of hydraulic actuator with discrete control. Mathematical models of such actuators [10–12] are based on linear equations and transfer functions. This reduces accuracy of the models and complicates evaluation of the influence of the actuator's parameters on its characteristics. In addition, these models are created for symmetrical hydraulic cylinders, which limits their universality.

The mathematical models of discrete actuators [13–15], although based on asymmetric hydraulic cylinders, do not take into account friction, which significantly affects the dynamic characteristics of the actuators. Consequently, their accuracy is not sufficient.

In the discrete hydraulic actuators' structures proposed in the literature [16–18], asymmetric hydraulic cylinders, 4/3 directional valves and high-speed switching valves are used. Mathematical models of these actuators are based on the bulk-cavity-node method. Using this method, one can take into account the topology of the working process, however, the influence of nonlinear friction is still not included in these models.

Mathematical models of discrete actuators that take into account the nonlinear friction forces have also been proposed [4, 19]. However, these mathematical models are only applicable to the actuators with unique structures that are adapted to some specific application purposes, such as braking of aircraft wheels [19] and precision movement of a table for micro-processing [4]. Consequently, the application range of these models is limited.

In summary, it can be stated that the known hydraulic actuators with discrete control have limitations in terms of their application areas and achievable effectiveness. Moreover, mathematical models for the known actuators cannot be used to study the dynamic processes during braking due to insufficient accuracy or specificity.

The aim of this work is to improve the application effectiveness of the hydraulic actuator with discrete control by studying the factors affecting the braking process.

2. Research methods

The tasks of studying the braking process of the hydraulic actuator with discrete control are solved by establishing a mathematical model, simulating the working process of the actuator in a given range of parameters and presenting the results in the form of relationships between the parameters and the characteristics of the actuator. Depending on the type of task to be solved, different software packages are used in modeling the process of hydraulic actuators. In order to study the local phenomena of hydraulic components, it is recommended to use a modeling software with distributed parameters, such as Computational Fluid Dynamics (CFD). Studying the braking process of a hydraulic actuator requires modeling the actuator as a whole. Therefore, it is necessary to model the process in the form of lumped parameters. For this purpose, the MATLAB/Simulink software package is the preferred choice. A variable-step solver is selected in the Simulink setup, which allows controlling the relative tolerance up to 1e-3 and the absolute tolerance up to 1e-6, and performing zero-crossing detection.

3. Mathematical model of the hydraulic actuator with discrete control

The hydraulic actuator with discrete control (Fig. 1a) contains a double-acting single-rod hydraulic cylinder which is controlled by a 4-way 3-position valve. Two working ports of the 4/3 valve are connected to the hydraulic cylinder and the other two ports are connected to the hydraulic source and the tank, respectively.

The mathematical model of the hydraulic actuator with discrete control was established based on the modularity principle (Fig. 1b). The general mathematical model contains models of the hydraulic cylinder and the 4/3 discrete valve, which interact with each other by means of the corresponding variable parameters.

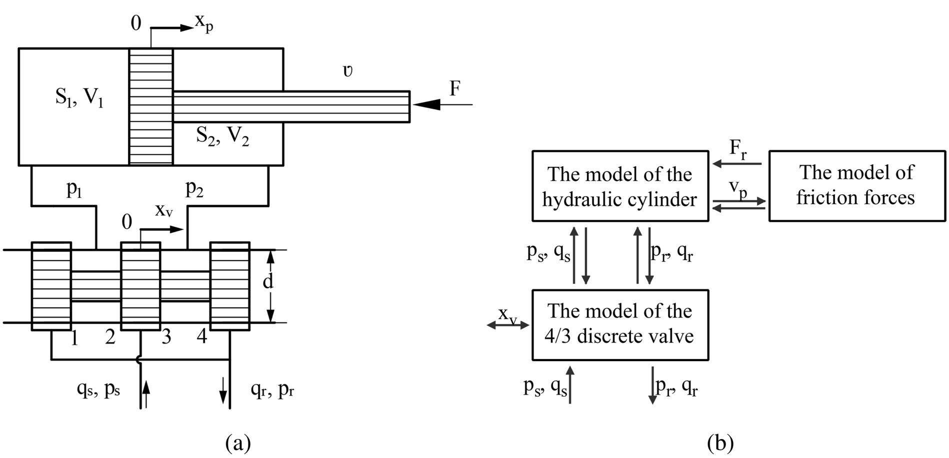


Fig. 1. Schematic of the hydraulic actuator (a) and modular diagram of its mathematical model (b)

Considering the significant influence of friction on the actuator's working process, the LuGre nonlinear friction sub-model [20, 21] is introduced in the general mathematical model. This structure of the general mathematical model facilitates the simplification of its configuration as well as the understanding of the working process. This general mathematical model takes into account the asymmetry of the hydraulic cylinder, the elastic properties of the working fluid, and the temperature consistent with the stable operating state of the actuator. The following assumptions are also made. (1) Since modern hydraulic pump stations are usually equipped with pressure compensation or pressure stabilizers and the outlet pressure is sufficiently stable, it is assumed that the outlet pressure of the hydraulic source is constant. (2) It is assumed that there are no leakages and overflows, considering that leakages and overflows from the 4/3 discrete valve and the hydraulic cylinder have a relatively small influence on the obtained results. (3) No influence of the heat exchange between the actuator and the external environment is assumed due to the fact that the dynamic process occurs very quickly. (4) No internal wave process is assumed, because in many compact or integrated hydraulic actuators the connecting pipelines are short, and the propagation time of the pressure wave is too short to cause significant pressure fluctuations.

Considering the asymmetry of the hydraulic cylinder, a mathematical model was established for the two operating modes of the actuator.

3.1. The model of the 4/3 discrete valve

The actual 4/3 discrete valve typically has a positive overlap, whereas in this model an ideal zero overlap is assumed (i.e., $x_v > 0$ or $x_v < 0$). This is due to the fact that such an assumption makes it possible to increase the sensitivity and response speed, which reduces the stability of the actuator and exposes it to the most demanding operating conditions, especially during the braking process. Therefore, this study allows us to determine the characteristic changes of the braking process under the most influential parameters and the assumption of the most demanding operating conditions.

In addition, the friction and hydrodynamic forces within the 4/3 discrete valve are simplified. It can be done because such forces in this type of valve are controlled by an electromechanical converter, and in actual standard valves the electromechanical converter has a sufficient power to move the valve spool and ensure the expected response time. Besides, the load on the electromechanical converter is not the subject of study in this model.

Flow equations for the 4/3 discrete valve:

The spool moves to the right ($x_v > 0$):

$$q_s = \operatorname{sign}(p_s - p_1) C_d x_v \pi d \sqrt{\frac{2|p_s - p_1|}{\varrho}}, \tag{1}$$

$$q_r = \text{sign}(p_2 - p_r) C_d x_v \pi d \sqrt{\frac{2|p_2 - p_r|}{\varrho}} \,. \tag{2}$$

The spool moves to the left ($x_v < 0$):

$$q_s = \text{sign}(p_s - p_2) C_d x_v \pi d \sqrt{\frac{2|p_s - p_2|}{\varrho}} \,, \tag{3}$$

$$q_r = \text{sign}(p_1 - p_r) C_d x_v \pi d \sqrt{\frac{2|p_1 - p_r|}{\varrho}} \,, \tag{4}$$

where q_s is the flow of working fluid supplied from the pump, (m^3/s); q_r is the flow of working fluid into the tank, (m^3/s); C_d is the flow coefficient; p_s represents the pressure of working fluid at the pump outlet, (Pa); p_r represents the pressure of the working fluid in the return pipeline, (Pa); x_v is the displacement of the valve spool, (m); p_1, p_2 represent the pressure of the left and right chambers of the hydraulic cylinder, (Pa); d is the spool diameter, (m); ϱ is the density of the working fluid (kg/m^3).

3.2. The model of the hydraulic cylinder

Flow continuity equations for the asymmetric hydraulic cylinder.

The spool moves to the right ($x_v > 0$):

$$q_s = S_1 \frac{\mathrm{d}x_p}{\mathrm{d}t} + \frac{(V_1 + S_1 x_p)}{E} \frac{\mathrm{d}p_1}{\mathrm{d}t} \,, \tag{5}$$

$$q_r = S_2 \frac{\mathrm{d}x_p}{\mathrm{d}t} - \frac{(V_2 - S_2 x_p)}{E} \frac{\mathrm{d}p_2}{\mathrm{d}t} \,. \tag{6}$$

The spool moves to the left ($x_v < 0$):

$$q_s = S_2 \frac{\mathrm{d}x_p}{\mathrm{d}t} - \frac{(V_2 - S_2 x_p)}{E} \frac{\mathrm{d}p_2}{\mathrm{d}t} \,, \tag{7}$$

$$q_r = S_1 \frac{\mathrm{d}x_p}{\mathrm{d}t} + \frac{(V_1 + S_1 x_p)}{E} \frac{\mathrm{d}p_1}{\mathrm{d}t} \,. \tag{8}$$

The force balance equation on the rod of the asymmetric hydraulic cylinder

$$p_1 S_1 - p_2 S_2 = m \frac{\mathrm{d}^2 x_p}{\mathrm{d}t^2} + F + F_r \,, \tag{9}$$

where S_1, S_2 represent the left and right effective area of the cylinder piston (m^2); x_p is the displacement of the cylinder rod, (m); V_1, V_2 represent the initial volume of the working fluid in the left and right chambers of the hydraulic cylinder (m^3); E is the elastic modulus of the working fluid, (Pa); m is the inertial mass of the moving parts of the hydraulic cylinder, including the piston, the rod, and the load on the rod (kg); F represents the force acting on the rod, (N); F_r represents the total friction force (N).

3.3. The model of friction forces

Friction effect is observed in all mechanical systems [22, 23] and has a significant influence on the braking process in hydraulic actuators. The influence of friction on the operation of hydraulic actuators can lead to control errors, low-speed creep, etc. [24, 25].

Mathematical description of friction is based on the LuGre nonlinear friction model [24, 25]. The model can simulate the static properties of friction and almost all dynamic behavior of friction [26]. The mathematical description of the LuGre model is as follows:

$$g(v) = F_C \left(F_S - F_C\right) e^{-|v/v_s|^n}, \tag{10}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = v - \sigma_0 \frac{|v|}{g(v)} z, \tag{11}$$

$$F_r = \sigma_0 z + \sigma_1 \frac{\mathrm{d}z}{\mathrm{d}t} + \sigma_2 v, \tag{12}$$

where F_C is the Columb friction, (N); F_S is the stiction friction, (N); $g(v)$ is the function of the Stribeck friction curve; v_s refers to Stribeck velocity, (m/s); v refers to relative velocity of the two contacting surfaces (velocity of the rod v_p relative to the cylinder), (m/s); z is average deflection of the bristle, (m); σ_0 is bristle stiffness, (N/m); σ_1 is bristle damping coefficient, (N s/m); σ_2 represents viscous friction coefficient, (N s/m); n is a constant.

The above group of equations (1)–(12) represents the mathematical description of the working process for the hydraulic actuator with discrete control. The general mathematical model based on this description is created in the Simulink software environment.

The main parameters and its values used in the simulation experiments are shown in Table 1.

Table 1. Parameters of the hydraulic actuator

Parameters	Symbols (unit)	Value	Parameters	Symbols (unit)	Value
Pump pressure	P_s (MPa)	6.3	Density of the working fluid	ϱ (kg/m^3)	880
Tank pressure	P_r (MPa)	0	Columb friction	F_c (N)	60
Spool diameter	d (mm)	16	Stiction friction	F_s (N)	500
Piston diameter	D_r (mm)	40	Stribeck velocity	v_s (m/s)	0.025
Rod diameter	d_r (mm)	25	Bristle stiffness	σ_0 (N/m)	1×10^8
The maximum stroke of the rod	L_r (mm)	800	Bristle damping coefficient	σ_1 (N s/m)	1000
Flow coefficient	C_d	0.62	Viscous friction coefficient	σ_2 (N s/m)	1395
Elastic modulus of the working fluid	E (MPa)	1740	Constant	n	1

The parameters of the pump station, the dimensions of the valve and cylinder, and the characteristic parameters of the working fluid in Table 1 are taken from physical experiments [27]. The parameters related to the LuGre friction model are based on empirical values and confirmed by physical experimental data. The flow coefficient C_d is a dimensionless constant with a recommended value of 0.62 according to [28, 29]. The value of C_d corresponds to the working fluid temperature of 50°C.

4. Validation of the mathematical model

The adequacy and accuracy of the mathematical model were confirmed by comparing the results of the simulation experiment with the physical experiment.

A total of 12 data samples were extracted from the physical experimental data. The sampling time interval between the experimental and model data points is fixed at 0.02 s (Fig. 3). Due to the small sample size and manual selection, no time lag and shifts occurred. Comparison results were evaluated using root mean square error (RMSE) and relative root mean square error (RRMSE) [30]. RMSE is a non-negative value, with smaller values indicating that the simulated and experimental results for the same data type are closer [30]. RRMSE is the result of dividing the RMSE by the average value of the measured data, which indicates the overall relative accuracy of the model [31]. The model accuracy is considered "excellent" when RRMSE < 10%, "good" when 10% < RRMSE < 20%, "fair" when 20% < RRMSE < 30%, and "poor" when 30% ≤ RRMSE [32]. The mathematical expressions for root mean square error (RMSE) and relative root mean square error (RRMSE) are given below [31].

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (Y_{\mathrm{exp},i} - Y_{\mathrm{model},i})^2}{n}}, \tag{13}$$

$$\mathrm{RRMSE} = 100 \frac{\sqrt{\dfrac{\sum_{i=1}^{n} (Y_{\mathrm{exp},i} - Y_{\mathrm{model},i})^2}{n}}}{\dfrac{1}{n} \sum_{i=1}^{n} Y_{\mathrm{exp},i}}, \tag{14}$$

where $Y_{\mathrm{exp},i}$ and $Y_{\mathrm{model},i}$ represent respectively the experimental and simulated values for the i-th data from the total number of n data. Due to the small sample size of the data used to calculate the RRMSE, no data filtering or smoothing was performed. This is because a conventional filtering tool (such as a low-pass filter) cannot accurately model frequency components, and excessive smoothing may lead to the loss of real dynamic characteristics. In this study, in the case of an obvious outlier point that appeared in the sample of the actuator's rod velocity, we directly manually replaced the outlier point with a neighboring one.

4.1. Comparison of simulation and experimental results and assessment of model accuracy

In order to ensure the consistency of the conditions of simulation and experimentation, the structure of the mathematical model built in Simulink must be adapted to the scheme of the actuator in the experiment [27]. Consequently, a feedback to the spool displacement was added to the model, i.e., the displacement signal of the rod with appropriate coefficients was added to the spool displacement control signal. In addition, the parameters and operating conditions of the actuator corresponding to the physical experiments were set in the model (Table 1). The spool displacement value was set to 0.52 mm and the braking start time to 0.31 s.

The simulation results for the braking process are shown in Fig. 2. The comparison shows that the change features of the spool displacement, the pressure in the cylinder chambers, the velocity and displacement of the rod are similar to their change's features on the oscillograms of the physics experiment [27]. This confirms the correctness of the mathematical model. In order to assess the model's accuracy, 12 data samples on the rod displacement and velocity were taken from the physical experiment results by systematic random sampling and compared with the simulation results (Fig. 3). Comparisons from the rod displacement graphs showed RMSE = 0.694 mm and RRMSE = 0.19% (Fig. 3a). These values correspond to the model's accuracy level "excellent". Comparison of the rod velocity graphs resulted in an RMSE = 0.069 m/s and RRMSE = 17.49% (Fig. 3b). Therefore, the model's accuracy reaches a "good" level from the point of view of the rod velocity comparison. It is worth noting that the relatively large RRMSE value of the rod velocity is due to the presence of an elastic element in the feedback link in the physical experiment, which negatively affects the velocity measurements. This is also confirmed by the relatively smooth shape of the rod displacement curve

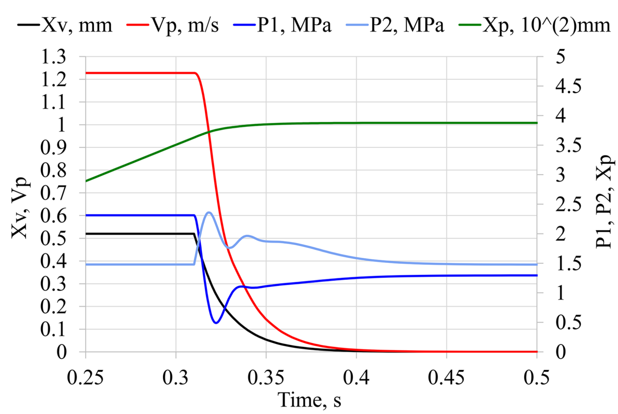
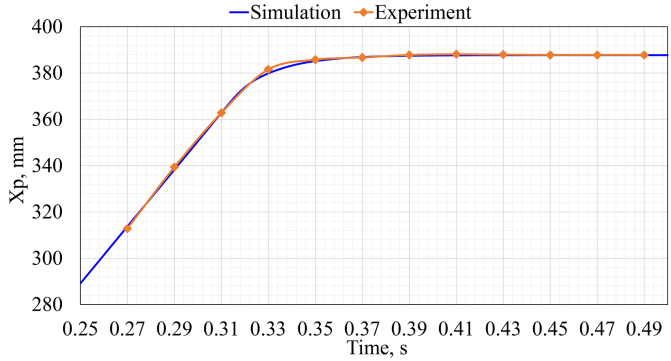
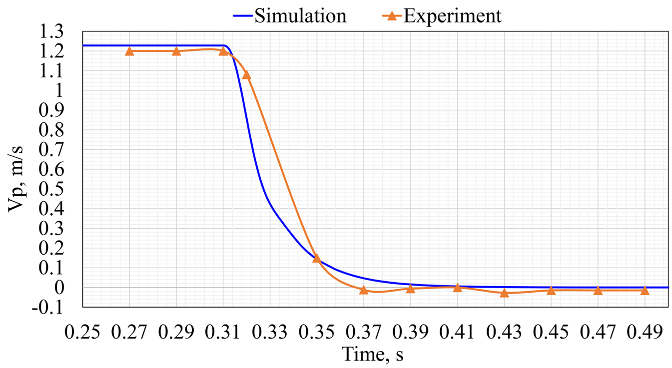


Fig. 2. Simulation results of braking process (x_v – the spool displacement; x_p – the rod displacement; v_p – the rod velocity; P_1 – the pressure in the left chamber; P_2 – the pressure in the right chamber)

obtained from the physical experiments. The obtained comparison results indicate the adequacy and accuracy of the developed mathematical model, which allows for its application in the further study.



(a)



(b)

Fig. 3. Comparison of simulation and physical experiment results: (a) the rod displacement (RMSE = 0.694 mm, RRMSE = 0.19%); (b) the rod velocity (RMSE = 0.069 m/s, RRMSE = 17.49%)

5. Justification and implementation methods for the study work

In order to improve the application effectiveness of the hydraulic actuator with discrete control, it is necessary to determine the relationship between the operating conditions of the actuator and its characteristics. This makes it possible to rationally choose the characteristics of the actuator in accordance with the specific task and the existing operating conditions. For example, it is possible to increase effectiveness by ensuring high performance of the actuator. For this purpose, it is reasonable to significantly reduce the time of the work process, which includes the acceleration and braking time of the rod. The dynamic braking process is the most complex and can prolong the working process. A significant reduction of braking time can lead to a significant increase of pressure in the cylinder chambers, which can damage the actuator. In addition, the values of the inertial load and braking velocity of the rod also significantly affect the characteristics and duration of the braking process. Therefore, by considering specific factors when setting up and controlling the actuator, one can achieve better characteristics of the working process.

Increasing the rod's maximum speed is also considered to be a factor in improving the performance of the actuator. To a large extent, the maximum speed of the rod is governed by the braking process, because the accumulated kinetic energy of the moving parts needs to be dissipated during braking. Therefore, the study aimed at increasing speed should also focus on the braking process itself. The knowledge of the factors that affect the braking process can help finding ways of controlling actuator's operation and facilitate the process of design.

The braking process of hydraulic actuators is characterized by the braking time, the number of speed fluctuations, the maximum pressure in the hydraulic cylinder and the time when the pressure exceeds the permissible value.

The research method consists in setting the actuator's parameters and conducting simulation experiments based on the validated mathematical model. It is necessary to set the inertial mass of the moving part of the actuator and the spool displacement value. In order to study the braking process of the actuator, a discrete pulse signal of a given duration is applied to the 4/3 valve's spool. The rising edge of the pulse signal causes the valve to open and then the rod begins to accelerate. The duration of the pulse signal should be sufficient for the rod to reach a steady speed. At the falling edge of the pulse signal, the rod with the steady speed starts braking, and the braking process is recorded at this point. At the same time, the spool displacement, the velocity and displacement of the rod, as well as the pressure in the cylinder chambers are monitored.

6. Results and analysis

Simulation experiments were performed to determine the influence of operating conditions and the actuator's parameters on its characteristics. Two simulation studies were conducted. In the first one, an extreme operating condition of the actuator was adopted: the spool response time to the control signal was assumed equal to zero. In the second one, the influence of the spool response time on the braking process was considered.

6.1. Research results on braking process with zero spool response time

The braking time and the maximum pressure in the cylinder chambers are determined from the braking process graphs obtained from the simulation experiments. For example, under the conditions of inertial mass $m = 37$ kg, spool discrete displacement value $x_v = 0.32$ mm, when the corresponding steady speed of the rod reaches $v_p = 0.792$ m/s, the velocity change of the rod is shown in Fig. 4a, and the pressure change in the cylinder chambers is shown in Fig. 4b. The rising edge of the pulse signal controlling the spool displacement appears at 0.01 seconds and the falling edge is at 0.21 seconds. It can be seen from Fig. 4a that when the spool displacement value reaches 0.32 mm, the valve is opened, and the velocity v_p of the rod increases and then stabilizes at 0.792 m/s after a slight vibration. Initially, a certain volume the working fluid suddenly enters the cylinder chamber, therefore, there is a sudden increase of the pressure in the chamber due to the elasticity of the fluid (Fig. 4b). The pressure in the chamber drops significantly after the rod starts moving, and then the forces on the rod reach equilibrium. At the falling edge of the pulse signal, the spool returns to the neutral position, so the valve is closed, then the rod immediately starts braking. After a significant fluctuation, the rod stops within 0.128 s due to the stopping of the working fluid supply and the action of

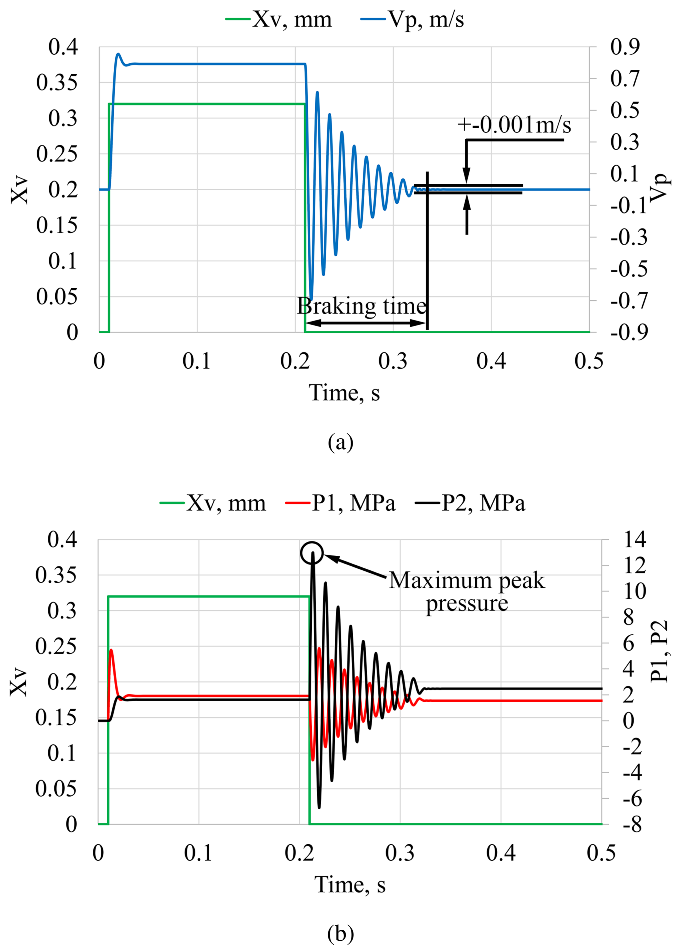
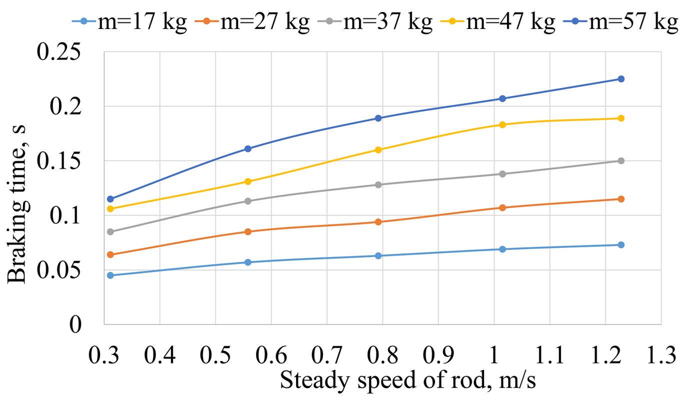


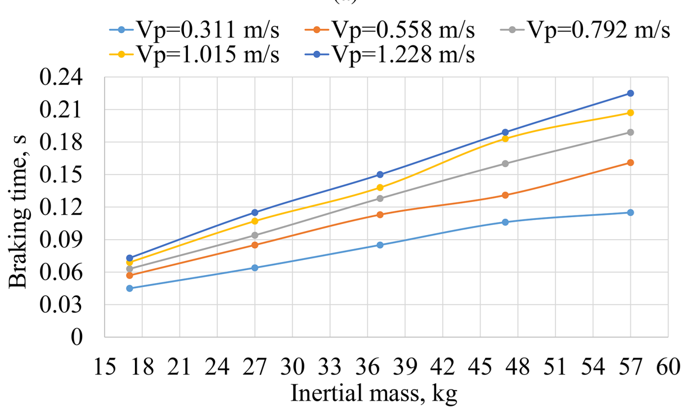
Fig. 4. Changes of parameters with time during braking process: (a) spool displacement x_v and rod velocity v_p; (b) spool displacement x_v and pressure P_1, P_2 in the cylinder chambers

friction resistance (Fig. 4a). Meanwhile, during the braking process, the pressures in the left and right chambers of the cylinder fluctuate noticeably before reaching equilibrium (Fig. 4b). The maximum peak pressure of 12.99 MPa is observed in the right chamber of the hydraulic cylinder (Fig. 4b), which is attributed to the greater compression in the right chamber due to the inertia of the moving part.

A total of five series of simulation experiments were conducted corresponding to inertial masses of 17 kg, 27 kg, 37 kg, 47 kg and 57 kg respectively. For each series of a given inertial mass, five groups of simulation experiments were performed for different displacement values of the spool (0.12, 0.22, 0.32, 0.42 and 0.52 mm). For each simulation experiment, the steady speed of the rod, braking time and maximum pressure were recorded. A total of 25 groups of simulation experiment data were obtained. These data were processed and the results are shown in Figs. 5, 6, 7.



(a)



(b)

Fig. 5. Dependence of rod braking time with different inertial mass and rod speed: (a) variation of braking time with steady speed of the rod under different inertial mass; (b) variation of braking time with inertial mass under different steady speed

The dependencies presented in Fig. 5 allow us to determine the braking time according to the different rod speed under different inertial mass. The simulation results are also presented in the form of three-dimensional plots (Fig. 6 and Fig. 7), taking into account the interdependence of braking time, maximum pressure in the cylinder chambers, steady speed and inertial mass.

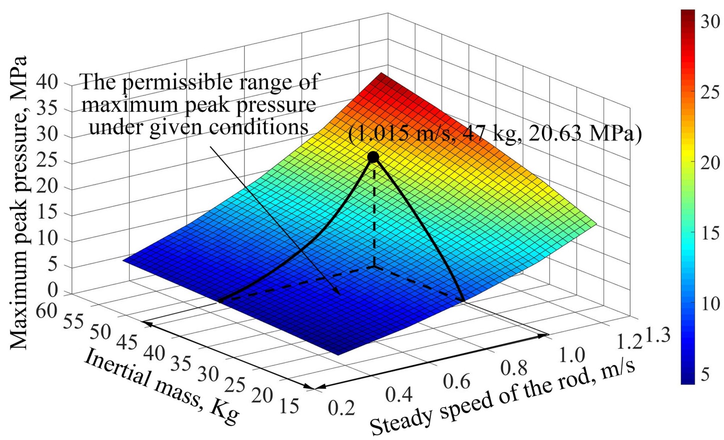


Fig. 6. Dependence of the maximum peak pressure on the rod steady speed and inertial mass

The dependencies in Fig. 6 indicate the quantitative relationship between the steady speed of the rod, the inertial mass of the moving part, and the maximum peak pressure in the cylinder chambers. The relationship (Fig. 6) allows for determining the range of working velocity and inertial load of the actuator under the condition that the set pressure value is kept during the braking process. The dependencies in Fig. 7 reflect the quantitative relationship between the braking time, the inertial mass of the moving part and the steady speed of the rod. The relationship (Fig. 7)

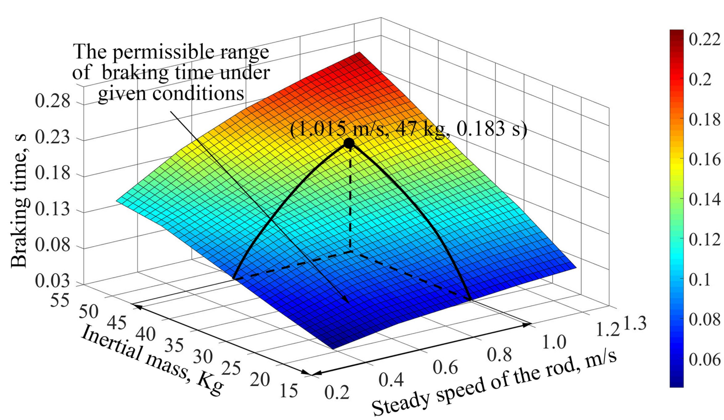


Fig. 7. Dependence of the braking time on the rod steady speed and inertial mass

makes it possible to determine the braking time for a given inertial load and working velocity. On that basis, the permissible velocity and inertial load can also be determined while satisfying the braking time requirement.

For example, with the maximum peak pressure of the hydraulic cylinders limited to 20.63 MPa, one can determine from Fig. 6 that the speed of the rod should be less than 1.015 m/s and the inertial mass should not exceed 47 kg. Similarly, with a given braking time of 0.183 s, it can be determined that the maximum steady speed of the rod is 1.015 m/s and the maximum inertial mass is 47 kg according to Fig. 7.

Therefore, the study results make it possible to establish the quantitative relationship between the operating conditions of the hydraulic actuator and its characteristics. The simulation results were obtained with the 4/3 discrete valve in an extreme operating condition, i.e., with zero response time of the spool. However, in the practical application of hydraulic actuators, the spool response time is not zero and may affect the dynamic characteristics of the actuator, so the next study aims at accurately exploring this effect.

6.2. Research results on braking process considering the spool response time

The spool response time depends on the specific valve parameters and characteristics. In the present research work, the spool response times were assumed from 0 to 0.05 seconds, which was a time range applicable to discrete valves used in hydraulic actuators for various purposes [33–37]. In the second study, all the parameters of the actuator, the velocity of the rod and the inertial mass range are the same as those in the first one study.

The results in Fig. 8 show the dependence of the braking time on the spool response time obtained under different inertial masses and two different steady speeds of the rod: 0.311 m/s (Fig. 8a) and 1.228 m/s (Fig. 8b).

The dependence in Fig. 8 reflects a certain contradiction, which is that slower movement of the spool leads to the increase of the actuator response speed. In order to explain this dependence, the simulation results for three characteristic values of the spool response time (0.005 s, 0.02 s and 0.05 s) were compared in detail. The simulation experiments were carried out under the same conditions: inertial mass $m = 57$ kg, steady speed of the rod $v_p = 0.311$ m/s, spool displacement value $x_v = 0.12$ mm, and the rising edge of the impulse signal for the discrete valve occurs at the moment of 0 s, and the falling edge at the moment of 0.25 s. The obtained results are presented in Figs. 9, 10, 11.

The working process of the hydraulic actuator in Fig. 9 illustrates that there is a sufficiently rigid interaction between the mechanical and fluid components. This is caused by the rapid opening of the 4/3 discrete valve, whereby the fluid rapidly and impactfully enters the cylinder chamber, resulting in a sharp pressure change in the cylinder chamber. Consequently, the velocity of the rod fluctuates, which leads to a fairly long braking time.

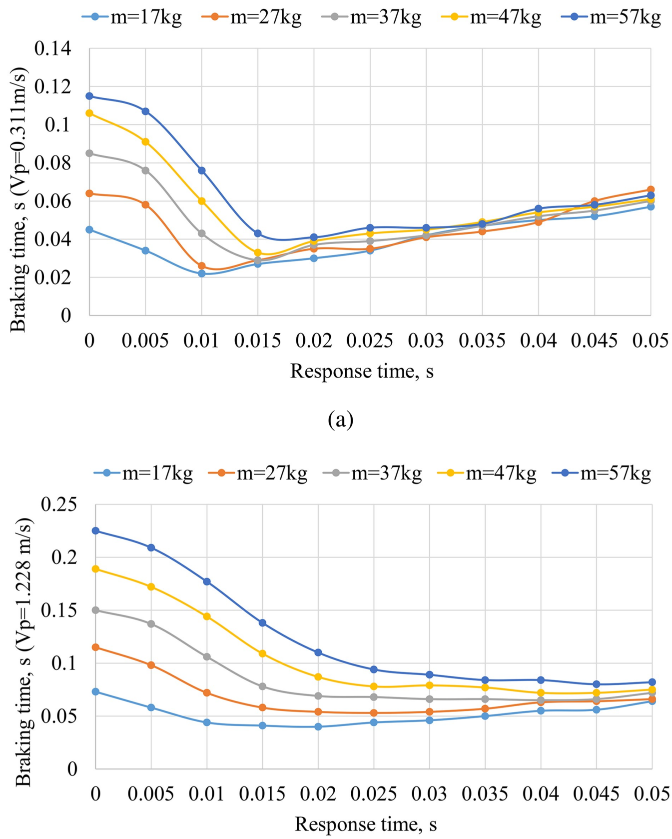


Fig. 8. Dependence of rod braking time on spool response time under different inertial mass: (a) steady speed of the rod $v_p = 0.311$ m/s; (b) steady speed of the rod $v_p = 1.228$ m/s

The comparison of the simulation results in Fig. 9 and Fig. 10 shows that with the increase of spool response time, the working process of the hydraulic actuator becomes smoother. The peak pressure in both cylinder chambers is reduced and the number of pressure fluctuations is also reduced. Accordingly, the number of rod velocity fluctuations is reduced and the braking process duration is also reduced.

In the case of a significant increase in the spool response time in Fig. 11, the working process is further smoothed, and the pressure amplitude in the cylinder chambers is reduced, but the duration of the pressure fluctuations is prolonged. Furthermore, as the fluid pressure acting on the piston of the hydraulic cylinder decreases, the braking efficiency of the rod decreases too, thereby increasing the braking time.

Therefore, the comparison results from Figs. 9, 10, 11 confirm the dependence features in Fig. 8, and reveal that the spool response time is one of the key influencing factors on the braking characteristics of the hydraulic actuator.

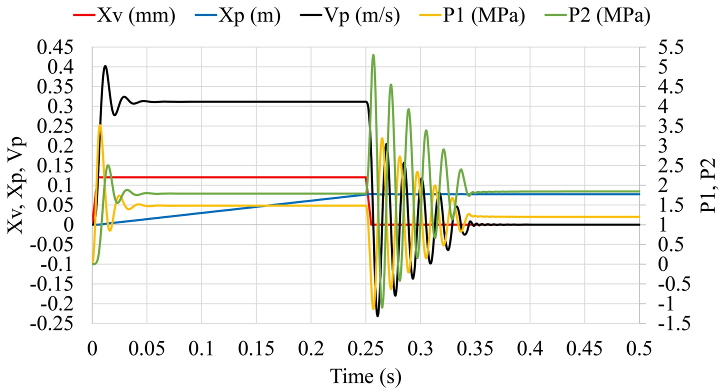


Fig. 9. The working process of the hydraulic actuator with spool response time $t_{sp} = 0.005$ s

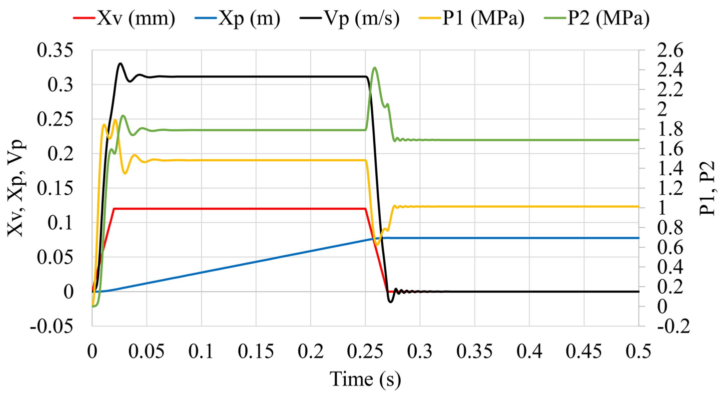


Fig. 10. The working process of the hydraulic actuator with spool response time $t_{sp} = 0.02$ s

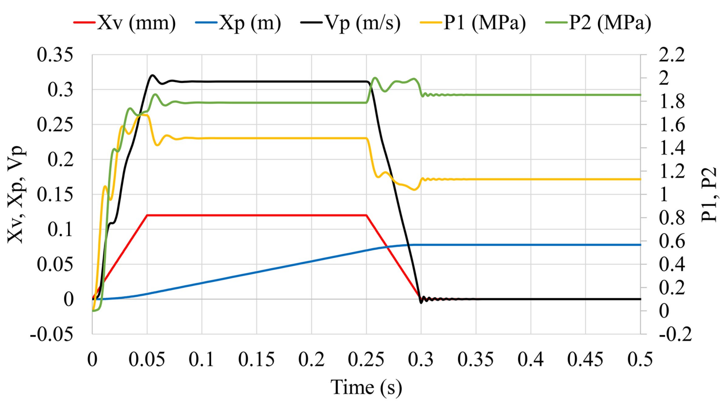


Fig. 11. The working process of the hydraulic actuator with spool response time $t_{sp} = 0.05$ s

Considering the fact that the factors affecting the braking process may also include other actuator parameters and operating conditions, several additional series of simulation experiments were conducted. The dependence of the braking time on the spool response time was determined under different spool displacement values (Fig. 12), pump pressure (Fig. 13) and the ratio of the piston effective area in the left and right chambers (Fig. 14).

The analysis of the obtained results shows that the dependencies under these different conditions are similar to the dependence of the braking time on the spool response time under different inertial masses reflected in Fig. 8. This can be explained by the reasons given above.

The study results on the braking process of the hydraulic actuator with discrete control show that: (1) the braking time and the maximum pressure in the hydraulic

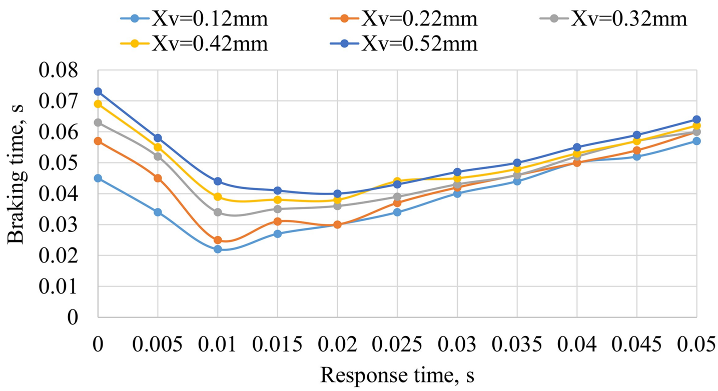


Fig. 12. Dependence of the rod braking time on the spool response time under different values of spool displacement (m = 17 kg, P_1 = 6.3 MPa, S_1/S_2 = 1.64)

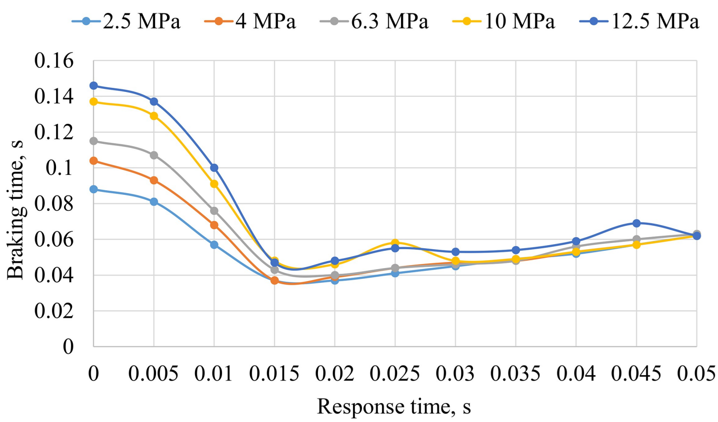


Fig. 13. Dependence of the rod braking time on the spool response time under different pump pressures (x_v = 0.12 mm; m = 57 kg, S_1/S_2 = 1.64)

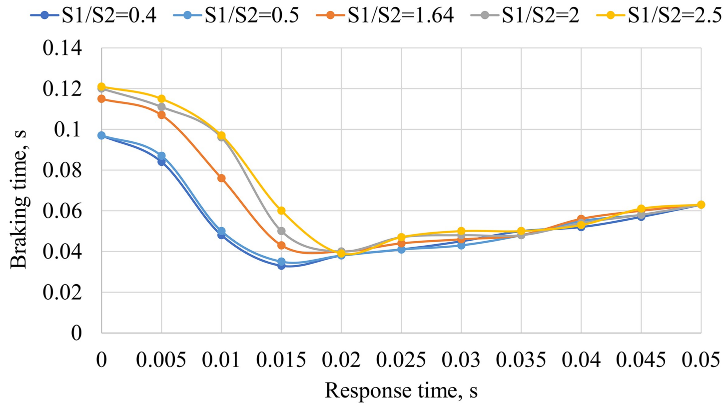


Fig. 14. Dependence of the rod braking time on the spool response time for different ratios of the piston effective area in left and right chambers (x_v = 0.12 mm, m = 57 kg, P_1 = 6.3 MPa)

cylinder chambers strongly depend on the velocity of the rod and the inertial load; (2) the increase of the spool response time leads to a significant decrease of the braking time and the maximum pressure and its fluctuation, while with a continuous increase of the spool response time leads to an increase of the braking time (Fig. 12–Fig. 14). The spool displacement values, pump pressure, and hydraulic cylinder parameters do not affect the nature of the above dependence.

Consequently, the study of the actuator with discrete control allowed us to determine the dependence and quantitative relationship of the braking time and the maximum pressure in the cylinder chambers for the following parameter ranges: the steady speed of the rod v_p from 0.311 m/s to 1.228 m/s; the inertial mass m from 17 kg to 57 kg; the spool response time t_{sp} from 0 s to 0.05 s, the ratio of the piston left and right effective areas S_1/S_2 from 0.5 to 2.5; pump pressure P_1 from 2.5 MPa to 12.5 MPa. The established relationships between the parameters, characteristics and operating conditions of the actuator allow for selecting their ratios rationally to ensure maximum application effectiveness.

7. Discussion

The results obtained in Fig. 7 confirm the conclusion given in the study of the discrete hydraulic actuator [38] that the braking time increases with the inertial load. Furthermore, the conclusion applies to a wider range of inertial loads and the relevant quantitative relationship is determined. It is pointed out in the articles [39, 40] that the instability of rod vibration during braking can be diminished or eliminated by reducing the inertial mass of the moving part. This is partly confirmed by the research results presented in Fig. 5. The result in Fig. 4 closely corresponds to that of the study [41], where it is shown that when the valve is closed, the pressures P_1 and P_2 jump in the two chambers of the hydraulic cylinder and the pressure ratio changes. In addition, it is also pointed out in the article [41] that the opening size of the valve and the inertial load have analogous influence on the working process of the hydraulic actuator, that is, the larger their values, the worse the characteristics of the hydraulic actuator. Similarly, it is observed in the results of Fig. 6 and Fig. 7 that a larger value of spool displacement (i.e., larger opening size of the valve) increases the steady speed of the rod, which, accordingly, leads to an increase in the braking time and peak pressure, thus the characteristics of the actuator are worsened. Moreover, the specific characteristics of the actuator for the given steady speed and the inertial mass can be predicted from Fig. 6 and Fig. 7.

In research works [42–44], in order to reduce the peak pressure during the braking process of the hydraulic actuator, it is respectively proposed to use an additional mounted safety valve, a combination of electromagnetic valves, and a variable damping check valve. The results from our study (Fig. 9, Fig. 10) show that the peak pressure during the braking process can be significantly reduced by increasing the spool response time. This solution is easier to implement and has better effects on the improvement of braking characteristics. It reduces not only the peak pressure and the time of its action, but also reduces the braking time.

Therefore, the results we obtained not only confirm the known studies on discrete hydraulic actuators, but also complement the quantitative relationship between the actuator's parameters, operating conditions and braking characteristics. Moreover, it is also revealed that the spool response time has a significant effect on the braking process. This provides an additional opportunity to improve the application effectiveness of the hydraulic actuator with discrete control.

8. Conclusions

In this paper, we developed a mathematical model of the hydraulic actuator with discrete control. The model takes into account the bi-directional asymmetry of the single-rod double-acting hydraulic cylinder, the compressibility of the working fluid, and the nonlinear friction based on the LuGre model. The accuracy of the mathematical model was verified by comparing the simulation and experimental data, and it was shown that the relative root-mean-square error (RRMSE) was 0.19% for rod displacement and 17.49% for rod velocity.

1. The quantitative relationship between the characteristics, parameters and operating conditions of the actuator was determined.
2. Under the most severe operating conditions of the actuator, which assume zero response time of the spool, the quantitative relationships between the braking time and the maximum pressure in the cylinder chamber were determined for rod velocity ranging from 0.311 m/s to 1.228 m/s and inertial mass ranging from 37 kg to 57 kg.
3. It can be shown that the spool response time has a significant effect on the braking time of the rod, and there is a clear minimum in the braking time versus response time function. This form of dependence is common

to different actuator's parameters and operating conditions, so the spool response time must be considered as a key factor in minimizing braking time for hydraulic actuators.

4. The application effectiveness of the hydraulic actuator with discrete control can be improved by rationally choosing the rod working velocity, the inertial mass of the moving part and the spool response time.
5. The obtained results can be applied to the development of hydraulic actuators for machinery manufacturing equipment, mobile machinery, and agricultural machinery, where the primary requirements focus on increasing productivity while maintaining low cost and relatively modest requirements for the purity of the working fluid.

References

[1] M. Yang. Study on the Digital hydraulic driving system of the belt conveyor. *Machines*, 10(6):417, 2022. doi: 10.3390/machines10060417.

[2] F. Messner and R. Scheidl. Development and experimental results of a small fast switching valve derived from fuel injection technology. In *Proceedings of the eighth workshop on digital fluid power*, pages 9–25, Tampere, Finland, 24-25 May, 2016.

[3] Q. Gao, Y. Zhu, C. Wu, and Y. Jiang. Development of a novel two-stage proportional valve with a pilot digital flow distribution. *Frontiers of Mechanical Engineering*, 16:420–434, 2021. doi: 10.1007/s11465-020-0622-2.

[4] S. Tsuchiya, H. Yamada, and T. Muto. A precision driving system composed of a hydraulic cylinder and high-speed on/off valves. *International Journal of Fluid Power*, 2(1):7–16, 2001. doi: 10.1080/14399776.2001.10781097.

[5] M. Linjama, H. Sairiala, KT. Koskinen, and M. Vilenius. High-speed on/off position control of a low-pressure water hydraulic cylinder drive. *SAE Technical Paper*, 2002(1):1378–1387, 2002. doi: 10.4271/2002-01-1378.

[6] L.Y. Gui, Y.X. Ming, H.Q. Xue, and L.T. Cai. The application of high speed on/off valve in the rectify a deviation system for high speed strip steel. *Shanxi Machinery*, 2000(2):12–14, 2000.

[7] M. Stosiak, M. Karpenko, A. Deptuła, K. Urbanowicz, P. Skačkauskas, A.M. Deptuła, A. Danilevičius, Š. Šukevičius, and M. Łapka. Research of vibration effects on a hydraulic valve in the pressure pulsation spectrum analysis. *Journal of Marine Science and Engineering*, 11(2):301, 2023. doi: 10.3390/jmse11020301.

[8] Q. Li and O. Uzunov. Quantitative evaluation of properties of structural solutions of electro-hydraulic positioning actuators. *Mechanics and Advanced Technologies*, 6(3):254–261, 2022. doi: 10.20535/2521-1943.2022.6.3.267700.

[9] Q. Li, and O. Uzunov. Study on the relationship between structure and properties of electro-hydraulic positioning actuators. *Archive of Mechanical Engineering*, 71(1):87–107, 2024. doi: 10.24425/ame.2024.149186.

[10] Y. Juntao, ZH. Hao, W. Li, and J. Zongxia. Research on cylinder position control system with high-speed on-off valve drived by piezoelectric. *Journal of Beijing University of Aeronautics and Astronautics*, 47(04):706–714, 2021. doi: 10.13700/j.bh.1001-5965.2020.0345 (in Chinese).

[11] H. Juan and Y. SongYue. H_∞ control for hydraulic cylinder position control system based on high-speed on/off value. *Control Engineering of China*, 2008(15):79–84, 2008. doi: 10.14107/j.cnki.kzgc.2008.s1.026 (in Chinese).

[12] J. Haiyu. *Research of Electro-hydraulic Position Control System Based on High Speed On-off Valve*. Master's thesis, Northeastern University, Shenyang, China, 2011.

[13] H. Juan and Y. Song-Yue. High-speed on/off valve hydraulic position control system based on DSP. *China Mechanical Engineering*, 21(24):2929–2931, 2010.

[14] L. Zhong, L. Deng, J. Yao, and Z. Hao. Research and simulation of a new hydraulic drive technology for engineering mechanical arm. *China Mechanical Engineering*, 23(23):2830–2835, 2012. doi: 10.3969/j.issn.1004-132X.2012.23.012 (in Chinese).

[15] Y. Jianbin. Feasibility of high-speed switch valve used for position control system. *Shandong Metallurgy*, 37(6):60–62, 2015 (in Chinese).

[16] Q. Gao, Z. Liu, H. Niu, and Y. Tang. Position control of hydraulic cylinder controlled by high-speed on-off valve. *China Mechanical Engineering*, 25(20):2775–2871, 2014. doi: 10.3969/j.issn.1004-132X.2014.20.014 (in Chinese).

[17] G. Qinhe, L. Zhihao, and N. Hailong. Controlling position of hydraulic cylinder with high-speed on-off valve parameters optimized. *Mechanical Science and Technology for Aerospace Engineering*, 34(6):903–909, 2015. doi: 10.13433/j.cnki.1003-8728.2015.0617 (in Chinese).

[18] Z.H. Liu, Q.H. Gao, G. Deng, H.L. Niu, and J.Y. Li. The position control of hydraulic cylinder based on high-speed on-off valve. *International Journal of Modelling, Identification and Control*, 22(1):54–67, 2014. doi: 10.1504/IJMIC.2014.063876.

[19] Z. Jiao, H. Zhang, Y. Shang, X. Liu and S. Wu. A power-by-wire aircraft brake system based on high-speed on-off valves. *Aerospace Science and Technology*, 106:106177, 2020. doi: 10.1016/j.ast.2020.106177.

[20] K. Johanastrom and C. Canudas-de-Wit. Revisiting the LuGre friction model. *IEEE Control Systems Magazine*, 28(6):101–114, 2008. doi: 10.1109/MCS.2008.929425.

[21] C. Canudas-de-Wit, H. Olsson, K.J. Astrom and P. Lischinsky. A new model for control of systems with friction. *IEEE Transactions on Automatic Control*, 40(3):419–425, 1995. doi: 10.1109/9.376053.

[22] L. Gaul and R. Nitsche. The Role of Friction in Mechanical Joints. *Applied Mechanics Reviews*, 54(2):93–106, 2001. doi: 10.1115/1.3097294.

[23] X. Nguyen, Y. Arai, and W. Araki. Effect of friction on the buckling behavior of shallow spherical shells contacting with rigid walls. *Archive of Mechanical Engineering*, 71(1):1–26, 2024. doi: 10.24425/ame.2024.149187.

[24] G.A. Sohl and J.E. Bobrow. Experiments and simulations on the nonlinear control of a hydraulic servosystem. *IEEE Transactions on Control Systems Technology*, 7(2):238–247, 1999. doi: 10.1109/87.748150.

[25] X.B. Tran, W.H. Khaing, H. Endo, and H. Yanada. Effect of friction model on simulation of hydraulic actuator. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 228(9):690–698, 2014. doi: 10.1177/0959651814539476.

[26] H. Yanada and Y. Sekikawa. Modeling of dynamic behaviors of friction. *Mechatronics*, 18(7):330–339, 2008. doi: 10.1016/j.mechatronics.2008.02.002.

[27] V.A. Baikalov. *Development and research of a high-speed electrohydraulic actuator for a manipulator with program control*. Ph.D. Thesis, Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine, 1978. (in Russian).

[28] N.D. Manring and R.C. Fales. *Hydraulic Control Systems*. John Wiley & Sons, New York, 2019.

[29] E. Lisowski and G. Filo. Analysis of a proportional control valve flow coefficient with the usage of a CFD method. *Flow Measurement and Instrumentation*, 53(B):269–278, 2017. doi: 10.1016/j.flowmeasinst.2016.12.009.

[30] T.O. Hodson. Root-mean-square error (RMSE) or mean absolute error (MAE): When to use them or not. *Geoscientific Model Development*, 15(14):5481–5487, 2022. doi: 10.5194/gmd-15-5481-2022.

[31] M. Ahmad, M.A. Al-Zubi, E. Kubińska-Jabcoń, A. Majdi, R.A. Al-Mansob, M.M.S. Sabri, and B. Zamin. Predicting California bearing ratio of HARHA-treated expansive soils using Gaussian process regression. *Scientific Reports*, 13:14376, 2023. doi: 10.1038/s41598-023-41737-7.

[32] M. Despotovic, V. Nedic, D. Despotovic, and S. Cvetanovic. Evaluation of empirical models for predicting monthly mean horizontal diffuse solar radiation. *Renewable and Sustainable Energy Reviews*, 56:246–260, 2016. doi: 10.1016/j.rser.2015.11.058.

[33] Q. Zhang, X. Kong, B. Yu, K. Ba, Z. Jin, and Y. Kang. Review and development trend of digital hydraulic technology. *Applied Sciences*, 10(2):579, 2020. doi: 10.3390/app10020579.

[34] K. Meena, M. Dhavalikar, and S. Thorat. A review on the position control of hydraulic cylinder for accuracy and response time. *Materials Today: Proceedings*, 72(3):995–999, 2023. doi: 10.1016/j.matpr.2022.09.114.

[35] J. Zhang, M. Yang, and B. Xu. Design and experimental research of a miniature digital hydraulic valve. *Micromachines*, 9(6):283, 2018. doi: 10.3390/mi9060283.

[36] Y. Li, R. Li, J. Yang, X. Yu, and J. Xu. Review of recent advances in the drive method of hydraulic control valve. *Processes*, 11(9):2537, 2023. doi: 10.3390/pr11092537.

[37] V.T. Nguyen, K.A. Nguyen, and V.L. Nguyen. An improvement of a hydraulic self-climbing formwork. *Archive of Mechanical Engineering*, 66(4):495–507, 2019. doi: 10.24425/ame.2019.131419.

[38] Y. Xia, Y. Shi, Y. Yuan, Y. Zhang, and Z. Yao. Analyzing of influencing factors on dynamic response characteristics of double closed-loop control digital hydraulic cylinder. *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, 13(3):48, 2019. doi: 10.1299/jamdsm.2019jamdsm0048.

[39] J. Zhe, K. Jian, and Y. Lan-ying. Study on low speed creeping dynamics of hydraulic cylinder. *Journal of Machine Design*, 23:39–42, 2006.

[40] Y. Zhu, W.L. Jiang, and M. Wang. Creeping mechanism and suppression methods of hydraulic cylinder under nonlinear time-varying force. *Transactions of the Chinese Society for Agricultural Machinery*, 45(3):305–313, 2014. doi: 10.6041/j.issn.1000-1298.2014.03.050 (in Chinese).

[41] K. Takahashi and Y. Takahashi. Dynamic Characteristics of a Spool-valve-controlled Servo-motor with a Non-symmetrical Cylinder. *Bulletin of JSME*, 23(181):1155-1162, 1980. doi: 10.1299/jsme1958.23.1155.

[42] P. Rudzki and P. Krot. Dynamics control of powered hydraulic roof supports in the underground longwall mining complex. In *IOP Conference Series: Earth and Environmental Science*, 942(1):012014, 2021. doi: 10.1088/1755-1315/942/1/012014.

[43] C. Derong, L. Xuzhi, and D. Shiping. Analysis of the hydraulic impact of the wind turbine hydraulic yaw brake system. *International Journal of Multimedia and Ubiquitous Engineering*, 11(3):369–378, 2016. doi: 10.14257/ijmue.2016.11.3.35.

[44] W.F. Huang and W. Li. Study on influence of hydraulic control check valve on dynamic characteristics of hydraulic support prop. *Coal Mine Machinery*, 42(9):41–45, 2021. doi: 10.13436/j.mkjx.202109014 (in Chinese).