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EVALUATION OF THE TYPE A UNCERTAINTY CAUSED BY SIMULTANEOUS INFLUENCE OF RANDOM NOISE AND POWER LINE INTERFERENCE

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Abstract

The paper presents the type A evaluation of standard uncertainty when the result of measurement is determined by digital averaging of the input signal which is distorted by simultaneous influence of the random uncorrelated noise and power line interference. It was shown that the classical evaluation of uncertainty based on determining of the standard deviation of input observations is not sufficient, because it does not take into account the effect of suppression of the interference by averaging. To correctly evaluate uncertainty, both the amplitude of the interference component and the standard deviation of the random component should be estimated separately. Simple methods of separate estimation of these components are proposed and analysed in detail. The proposed solutions to the uncertainty evaluation were studied when uniform and triangle averaging were used and verified both by Monte Carlo simulations and by experimental tests. The simulation and test results obtained showed very good accordance with theoretical results.

Keywords: uncertainty, averaging, power line, interference, noise.

1. Introduction

The classical approach [1] to the evaluation the Type A uncertainty takes into account the influence of random noise (usually uncorrelated). The standard uncertainty is determined by the well-known expression [1]:

$$u_A(X) = \frac{s_x}{\sqrt{n}}, \quad (1)$$

where $s_x = \sqrt{(1/(n-1))\sum_{i=1}^{n}(x_i - \overline{x})^2}$ is estimator of the standard deviation calculated from n observations of input signal, $\overline{x}$ is a signal mean value.

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However, in measurement practice, especially in industry, in addition to random noise, periodic interference from the industrial power line is very often present. The nominal frequency of such interferences is 50 Hz (60 Hz in North America) [2, 3]. Power line interference penetrates the measurement chain through different parasitic connections: inductances, capacitances, insulation resistances, and also by common wires, the grounding, *etc.* [4–6]. The level of interference depends on the power level, the configuration of the parasitic connection and distance between the measuring circuit and power line, *etc.* That is, the level of power line interference may be tens of times, or even sometimes more, higher than the level of random noise [2].

The power line interferences parameters are not stable. Namely, the relative frequency variations are allowed within $\delta f = \pm 1\%$ (that is from 49.5 Hz up to 50.5 Hz) for 99.5% of each one-year period, and variations may be as high as from –6% to +4%, that is from 47 Hz up to 52 Hz for the whole period (European standard EN 50160 [7]). Significantly higher frequency instabilities may occur in power systems for mobile vehicles (ships, aircraft, *etc.*). In such power systems, the frequency instability may be ±5% and for transient frequency components may be as high as ±10% [8].

Measurement problems related to these interferences and their suppression, mainly by filtering and averaging, have been studied, analysed, and discussed in various sources [9–16]. Namely, to reduce the impact of such interference, software filters can be used which are available, for example, in the Advanced Analysis Toolkit for LabVIEW [13]. The problems of reducing the effects of interference and suppressing noise by filtering during biomedical signal processing are described in [14–16].

However, from a metrological point of view, it is not enough to ensure the required suppression of power line interference and random noise, but it is necessary to estimate the uncertainty of such measurements [1].

The aims of the following research are: (i) to clarify the problems of correct uncertainty evaluation while measuring a signal distorted by periodic interference and random noise, (ii) to propose the methods for separate estimation of parameters of these components to correctly evaluate the uncertainty, and (iii) to test the effectiveness of the proposed methods using Monte Carlo simulation and by measurement experiments.

2. Problems of standard uncertainty evaluation caused by power line interference and random noise

Evaluation of the uncertainty in the case of simultaneous influence of both components on the measured signal involves a significant problem. In the present study, the measured signal model $v_{\text{in}}(t)$ comprises, in addition to the so-called information component V_x, the power line interference component $v_{\text{int}}(t)$ with amplitude V_m, frequency f and initial phase φ, and the random noise component $v_n(t)$ of a standard deviation σ_n, *i.e.*:

$$v_{\text{in}}(t) = V_x + v_n(t) + v_{\text{int}}(t) = V_x + V_m \cos(2\pi f t + \varphi) + v_n(t)\,. \tag{2}$$

To reduce the influence of the both components, the averaging of the n input observations $v_i = v_{\text{in}}(t_i)$ at sampling time moments $t_i = iT_s$ (T_s is a sampling period (Fig. 1a)) is usually used. The averaging interval is $T_{\text{av}} = mT_n$, where m is a number of the interference nominal period: $T_n = 1/f_n$ [13]. Then the average value can be expressed as:

$$\overline{V} = V_s + \frac{V_m}{n}\sum_{i=1}^{n}\cos(2\pi f T_s i + \varphi) + \frac{1}{n}\sum_{i=1}^{n} v_n(t_i) = V_x + \Delta_{\text{av,int}} + \Delta_{av,rnd}, \tag{3}$$

where $\Delta_{\text{av,int}}(V_m)$ is the error caused by the averaging of the power line interference and $\Delta_{av,rnd}$ is the error caused by the averaging of the random noise. When the origin of the time coordinate is located in the middle of the averaging time interval T_{av}, *i.e.* the averaging interval is between $-T_{\text{av}}/2$ and $+T_{\text{av}}/2$, the power line component after simplification can be presented as:

$$\Delta_{\text{av,int}} = \Delta_{\text{av,int}}(V_m) = \frac{V_m}{n}\sum_{i=1}^{n}\cos(2\pi f T_s i + \varphi) = V_m\cos(\varphi)\cdot\frac{\sin(\pi v)/n}{\sin(\pi v/n)}, \tag{4}$$

where $v = fT_{\text{av}}$ is normalized (to the averaging time T_{av} interval) interference frequency f.

The influence of such interference on the measured signal is quantified usually by the NMRR (*Normal Mode Rejection Ratio)* [2]. This value is determined on a decibel scale as the ratio of the interference amplitude V_m and the maximal value of the error module $\left|\Delta_{\text{av,int}}(V_m)\right|_{\max}$: NMRR = $20\cdot\log\left[\mathrm{V_m}\middle/\left|\Delta_{\text{av,int}}(\mathrm{V_m})\right|_{\max}\right]$. The maximum effect of cosine interference (4) occurs at the initial phase $\varphi = 0$, therefore the NMRR of the uniform averaging is:

$$\text{NMRR} = -20\log\left(\left|\frac{\sin(\pi \mathrm{v})/\mathrm{n}}{\sin(\pi\mathrm{v}/\mathrm{n})}\right|\right). \tag{5}$$

From (5), it follows that all harmonics of the power line interference with numbers $k = 1, 2, \ldots, n-1$ are completely rejected. Contrarily, when the interference frequency differs from the nominal f_n by a relative value $\delta_f = (f - f_n)/f_n = (v - v_n)/v_n$, the error is $\left|\Delta_{\text{av,int}}(V_m)\right|_{\max} \approx V_m\left|\delta_f\right|$. Thus, the NMRR of uniform averaging is limited by $1/\delta_f$, and the NMRR $\approx -20\log(|\delta_\mathrm{f}|)$ (Fig. 1b). When a maximum frequency deviation of the electrical power system is up to $\delta_f = \pm 1\%$, the usual averaging provides about 100-time interference reduction or the NMRR is limited at a level of 40 dB (Fig. 1b). When $\delta_f = \pm 5\%$ (±2.5 Hz) the NMRR decreases to ≈ 25.6 dB (Fig. 1b).

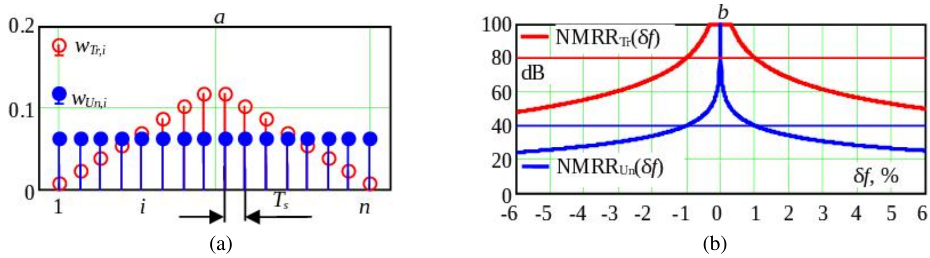

Fig. 1. Uniform (w_{Un}, blue) and triangle (w_{Tr}, red) weight functions for $n = 16$ (a); NMRR at frequency deviation δf (%) around $f_n = 50$ Hz when using averaging with uniform (blue) and triangle (red) weight functions.

To increase an NMRR, special weighting functions (windows) [17] can be used. The result of weight averaging (weighted mean value) of input observations is:

$$\overline{V} = \sum_{i=1}^{n} w_i v_{\text{in}}(t_i) = V_x + \sum_{i=1}^{n} w_i v_n(t_i) + V_m\cos(\varphi)\sum_{i=1}^{n} w_i\cos(2\pi f T_s i + \varphi), \tag{6}$$

where w_i $(i = 1, 2, \ldots, n)$ are the weighting coefficients. Here, the weight function used is symmetrically around its centre and normalized, *i.e.* $\sum_{i=1}^{n} w_i = 1$. The sampling period is $T_s = T_{\text{av}}/n$.

In (6), it can be seen that the last sum represents the *discrete Fourier transform of the weighting function* (DFTW) – spectral characteristic of the weighting averaging. Therefore, the NMRR is determined by the logarithm of the modulus of the DFTW:

$$G_{\mathrm{WF}}\left(fT_{\mathrm{WF}}\right)=G_{\mathrm{WF}}\left(\nu\right)=\sum_{i=1}^{n}w_i\cdot\cos\left(2\pi fT_s i\right),\quad \mathrm{NMRR}=-20\log\left|G_{\mathrm{WF}}\left(\nu\right)\right|. \tag{7}$$

From the point of view of high suppression of periodic interference, a triangle weight function (Fig. 1a) is very useful. For the even n, the DFTW of such function is:

$$G_{\mathrm{Tr}}\left(\nu\right)=\cos\left(\pi\nu/n\right)\left(\frac{2\sin\left(\pi\nu/2\right)/n}{\sin\left(\pi\nu/n\right)}\right)^2. \tag{8}$$

For the DFTW (8) of the triangle weight function the width of the main lobe is $\nu_{0\mathrm{Tr}}=2$, therefore, the width of triangle function must be twice that of the interference period: $T_{\mathrm{av}}=T_{\mathrm{Tr}}=2T$. Using the triangle weight function for a maximum frequency deviation $\delta_f=\pm 1\%$, the maximal interference averaging error is: $\left|\Delta_{\mathrm{av,int}}\left(V_m\right)\right|_{\max}\approx V_m\delta_f^2=10^{-4}V_m$. That is, this function provides a 10^4 – fold interference suppression (the NMRR is about 80 dB (Fig. 1b)), which is 100 times more than in uniform averaging.

Based on the general approach to determine the variance of the averaging in (6), there are two components of the variance: one is caused by the averaging of the power line interference ($\sigma^2_{av.int}$), and the other is caused by the averaging of the random noise ($\sigma^2_{\mathrm{av},n}$): $\sigma^2_{\mathrm{av}}=\sigma^2_{\mathrm{av,int}}+\sigma^2_{\mathrm{av},n}$. Because from (4), (6) and (7) $\left|\Delta_{\mathrm{av,int}}\left(V_m\right)\right|_{\max}\approx V_mG_{\mathrm{WF}}\left(\nu\right)$, assuming uniform distribution of the interference phase φ, the variance of the interference averaging can be presented as: $\sigma^2_{\mathrm{av,int}}=\left(V_m^2/2\right)G^2_{\mathrm{WF}}\left(\nu\right)$. For uncorrelated noise observations the variance of the second component is: $\sigma^2_{\mathrm{av},n}=C^2_{\mathrm{WF}}\sigma_n^2/n$, where $C^2_{\mathrm{WF}}=n\sum_{i=1}^{n}w_i^2$. Therefore, the variance of the signal averaging in (6) is:

$$\sigma^2_{\mathrm{av}}=C^2_{\mathrm{WF}}\sigma_n^2/n+\left(V_m^2/2\right)G^2_{\mathrm{WF}}\left(\nu\right). \tag{9}$$

The standard uncertainty of measurement is a root square of the variance (9):

$$u_A\left(V\right)_{\mathrm{theor}}=\sigma_{\mathrm{av}}=\sqrt{C^2_{\mathrm{WF}}\sigma_n^2/n+V_m^2/2G^2_{\mathrm{WF}}\left(\nu\right)}=u_A(V)_{n,\mathrm{theor}}\sqrt{C^2_{\mathrm{WF}}+\mathrm{HNR}^2nG^2_{\mathrm{WF}}\left(\nu\right)}, \tag{10}$$

where $u_A(V)_{n,\mathrm{theor}}=\sigma_n/\sqrt{n}$ is a Type A theoretical standard uncertainty, when only uncorrelated random noise distorts input signal. The quantity $\mathrm{HNR}=V_m/\left(\sigma_n\sqrt{2}\right)$ is the so-called harmonic-to-noise ratio, which reflects the influence of the power line interference.

From (10), we can see that for the averaging used (known $G_{\mathrm{WF}}(\nu)$ and C_{WF}) to determinate the standard uncertainty $u_A\left(V\right)$, the HNR ratio must also be known. In other words, the interference amplitude V_m and the standard deviation σ_n of random noise must be known separately. However, using the estimated standard deviation $s_{v,\mathrm{in}}$ of input observations:

$$s_{v,\mathrm{in}}=\sqrt{\frac{1}{n-1}\sum_{i=0}^{n-1}\left(v_{\mathrm{in},i}-\overline{V}\right)^2}\approx\sqrt{\sigma_n^2+\frac{V_m^2}{2}}=\sigma_n\cdot\sqrt{1+\mathrm{HNR}^2}, \tag{11}$$

the standard uncertainty following the GUM procedure [1] is:

$$u_{A,\mathrm{GUM}}\left(V\right)=\frac{s_{v,\mathrm{in}}}{\sqrt{n}}\approx\frac{\sigma_n}{\sqrt{n}}\sqrt{1+\mathrm{HNR}^2}=u_A(V)_{n,\mathrm{theor}}\sqrt{1+\mathrm{HNR}^2}. \tag{12}$$

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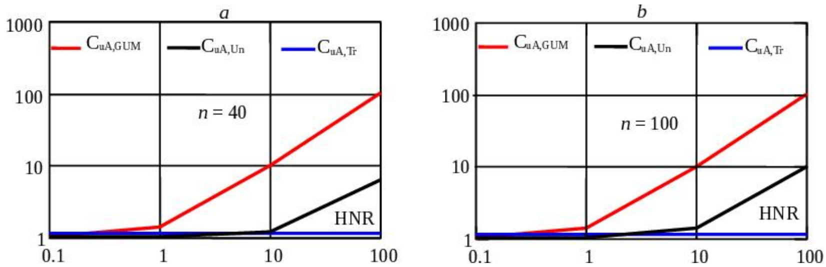
Comparing (10) and (12) and taking into account that $C_{\mathrm{WF}} \approx 1$ (for the uniform weight function $C_{\mathrm{WF}} = 1$ and for the triangle one $C_{\mathrm{WF}} \approx 1.14$), we can see that the impact of the standard deviation of random noise σ_n in both expressions is practically the same. But the impact of the interference amplitude $\left(V_m = \mathrm{HNR}\sigma_n\sqrt{2}\right)$ is quite different. In the correct expression (10) this component is: $\mathrm{HNR}^2 n G_{\mathrm{WF}}^2(v)$, *i.e.* it is proportional to the square of HNR multiplied by the square of the DFTW $G_{\mathrm{WF}}(v)$ of the weight function used and the number n of observations. On the other hand, in (12), the impact of the interference amplitude is proportional to the square of HNR only. Fig. 2 shows the dependence on the HNR of the coefficients $C_{uA,\mathrm{GUM}} = \sqrt{1+\mathrm{HNR}^2}$ (12), $C_{uA,\mathrm{Un}} = \sqrt{1+n\mathrm{HNR}^2 G_{Un}^2(v)}$ and $C_{uA,\mathrm{Tr}} = \sqrt{C_{\mathrm{Tr}}^2 + n\mathrm{HNR}^2 G_{\mathrm{Tr}}^2(v)}$ for the uniform and triangle averaging (10), on which the corresponding standard uncertainty values depend.



(a) (b)

Fig. 2. Dependences of the coefficients $C_{uA,\mathrm{GUM}}$, $C_{uA,\mathrm{Un}}$, and $C_{uA,\mathrm{Tr}}$ on the value of HNR for $n = 40$ (a) and $n = 100$ (b).

From the graphs in Fig. 2, it can be seen that even at a relatively low level of interference: HNR ≈ 1, the $C_{uA,\mathrm{GUM}}$ coefficient, which is used to calculate the uncertainty according to the (12), differs significantly from the true value. That is, when HNR = 1, the value of $C_{uA,\mathrm{GUM}}$ is approximately 1.4 times greater than the actual value. When the HNR > 1 the value of $C_{uA,\mathrm{GUM}}$ is approximately proportional to HNR. This causes a corresponding increase in the standard uncertainty $u_{A,\mathrm{GUM}}(V)$. For example, when HNR = 10, then the calculated standard uncertainty $u_{A,\mathrm{GUM}}(V)$ will be even more than 10 times larger than the real one.

From Fig. 2, it can also be seen that while using uniform averaging, due to the limitation of the interference suppression ($|G_{Un}(v)| \leq 10^{-2} = 0.01$), the significant influence of the interference on the standard uncertainty value starts at HNR $\approx >$ 10 for $n = 40$ and at HNR $\approx >$ 5 for $n = 100$. On the other hand, when triangular averaging is used, due to the high suppression of the interference up to $|G_{\mathrm{Tr}}(v)| \leq 10^{-4} = 0.0001$, the standard uncertainty value practically is independent of the interference level even for HNR < 100.

In order to demonstrate this important problem, the following numerical example is presented. The $n = 40$ observations (in mV) of the measured signal were registered using a DAQ card and are given in Table 1.

These observations are obtained by sampling (using the frequency $f_s = 1000$ Hz) of the signal $V_x = 175$ mV distorted by the random noise of the standard deviation $\sigma_n = 1.5$ mV and the power line interference of amplitude $V_m = 10$ mV (HNR ≈ 4.7) and frequency $f = 49.5$ Hz. Using uniform and triangular averaging (6), the measurement results are as follows: $V_{x,\mathrm{meas,Un}} = 174.627$ mV and $V_{x,\mathrm{meas,Tr}} = 174.909$ mV. The corresponding error values are: $\Delta_{Un} \approx -0.373$ mV and $\Delta_{\mathrm{Tr}} \approx 0.091$ mV. Estimated by the first part of (11), standard deviation of the input observations is $s_{v,\mathrm{in}} \approx 7.338$ mV. This value practically totally depends on the interference amplitude:

Table 1. The $n = 40$ observations (in mV) of the measured signal.

185.049	183.682	183.745	179.947	176.754	176.263	169.678	168.622	166.186	165.153
165.969	166.388	165.478	166.979	170.564	172.891	178.664	180.555	183.519	185.912
184.923	185.525	185.762	178.975	178.083	176.114	172.333	170.138	168.468	166.157
164.396	164.619	168.402	166.839	170.355	172.069	176.398	176.833	184.164	182.51

$s_{Vm} = V_m/\sqrt{2} \approx 7.07$ mV. Therefore, the standard uncertainty, calculated according to the generally accepted GUM [1] method (12), is: $u_A(V_x)_{\text{GUM}} = s_{v,\text{in}}/\sqrt{n} = 7.34\text{ mV}/\sqrt{40} \approx 1.2$ mV. For confidence level $p = 0.95$, the expanded uncertainty determined due to [1] is at least 2 times greater, that is about 2.32 mV.

Based on a comparison of the error values ($\Delta_{Un} \approx -0.373$ mV and $\Delta_{\text{Tr}} \approx -0.091$ mV) determined above with the expanded uncertainty (≈ 2.32 mV), it can be seen that the differences between them are very large (about ≈ 7 and ≈ 25 times). Therefore, the uncertainty determined according to the classical GUM method [1] (12) is highly inaccurate. This is quite consistent with the results of the theoretical analysis and Fig. 2 given above. The obvious reason for this is the failure to take into account the suppression of the interference component by means of appropriate averaging.

Theoretical values of the standard uncertainties are:

$$u_A(V)_{n,\text{theor}} = 1.5\text{ mV}/\sqrt{40} \approx 0.24\text{ mV};$$

$$u_A(V)_{\text{theor,GUM}} = 0.237\text{ mV} \cdot \sqrt{1 + 4.714^2} \approx 1.14\text{ mV};$$

$$u_A(V)_{\text{theor,Un}} = 0.237\text{ mV} \cdot \sqrt{1 + 4.714^2 \cdot 40 \cdot 0.01^2} \approx 0.25\text{ mV};$$

$$u_A(V)_{\text{theor,Tr}} = 0.237\text{ mV} \cdot \sqrt{1.141^2 + 4.714^2 \cdot 40 \cdot 0.0001^2} \approx 0.27\text{ mV}.$$

Therefore, the standard uncertainty $u_A(V)_{\text{theor,GUM}} \approx 1.14$ mV (and also expanded), determined according to the rule [1], *i.e.* by the estimated standard deviation of the input observations (11), (12), may differ significantly, even several times or more, from the correct value. Comparing the standard uncertainty values 0.25 mV (uniform averaging) and 0.27 mV (triangular averaging) with the standard uncertainty 0.24 mV determined by considering only the random component demonstrates their good agreement.

3. Separate estimation of power line interference and random components

It follows from the analysis of (10) and the example presented above that in order to correctly determine the standard uncertainty of measurement, the amplitude V_m of the harmonic component and standard deviation σ_n of the random component should be known. In signal analysis, a similar problem applies to determining the parameters of a harmonic signal distorted by random noise [18–20]. Since the results of the estimation of the standard deviation and amplitude are used for the evaluation of uncertainty, a high accuracy of the estimation of these parameters is not required. Uncertainty in parameter estimation of a few percent or even higher is acceptable. Therefore, simple methods can be used to estimate the parameters of these distortions.

It is clear that *a priori* information on the interference frequency (period) should be used to estimate the parameters of the two components. In general, two simple approaches are possible.

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In the first method, the periodic interference parameters are first estimated. Then the standard deviation of the noise component is estimated based on these parameters and the input samples. In the second method, the reverse order is used: the standard deviation of the noise component is estimated first, and then one calculates the interference amplitude.

Method 1. When knowing the nominal frequency f_n of the interference and also the sampling frequency f_s, the simplest method to estimate the amplitude $V_{m,k,\text{est}}$ and phase $\varphi_{k,\text{est}}$ of the k−th harmonics is calculating using the Fourier series [21] of the signal observations:

$$V_{k,\cos} = \frac{1}{n}\sum_{i=0}^{n-1}\left(v_{\text{in},i} - \overline{V}\right)\cdot\cos\left(2\pi f_n T_s k i\right);\quad V_{k,\sin} = \frac{1}{n}\sum_{i=0}^{n-1}\left(v_{\text{in},i} - \overline{V}\right)\cdot\sin\left(2\pi f_n T_s k i_n\right), \tag{13}$$

$$V_{m,k,\text{est}} = 2\sqrt{V_{k,\cos}^2 + V_{k,\sin}^2},\quad \varphi_{0,k} = \arctan\left(\frac{V_{k,\sin}}{V_{k,\cos}}\right). \tag{14}$$

After estimating the amplitudes and phases (14) of the harmonic component, using the average value $\overline{V}$ (6), the standard deviation s_n is determined making use of the differences $\Delta v_{\text{in},i} = v_{\text{in},i} - \overline{V} - \sum_{k=1}^{K} V_{m,k,\text{est}}\cos(2\pi k f_n t + \varphi_{k,\text{est}})$:

$$s_{n,1} \approx \sqrt{\frac{1}{n-1}\sum_{i=0}^{n-1}\left(\Delta v_{\text{in},i}\right)^2}. \tag{15}$$

Applying this method only to the main harmonic component ($K = 1$) for the data in Table 1, we obtain the following values: $V_{\cos} = 5.020$ mV; $V_{\sin} = 0.384$ mV; $V_{m,1} = 10.070$ mV; $\varphi_0 = 0.076$ rad; $s_{n,1} = 1.467$ mV. Assuming the maximum frequency deviation ±1%, the estimated standard uncertainties for the method GUM [1] (12) and for uniform and triangle averaging (10) using are as follows:

$$u_A(V_x)_{\text{GUM},1} \approx \frac{s_{n,1}^2}{n}\sqrt{1 + \frac{V_{m,1}^2}{2s_{n,1}^2}} = \frac{1.467}{\sqrt{40}}\sqrt{1 + \frac{10.070^2}{2\cdot 1.467^2}} \approx 1.15 \text{ mV},$$

$$u_A(V_x)_{\text{Un},1} \approx \frac{s_{n,1}^2}{n}\sqrt{1 + \frac{V_{m,1}^2}{2s_{n,1}^2} n G_{Un}^2(df)} = \frac{1.467}{\sqrt{40}}\sqrt{1 + \frac{10.070^2}{2\cdot 1.467^2} 40\cdot 0.01^2} \approx 0.24 \text{ mV},$$

$$u_A(V_x)_{\text{Tr},1} \approx \frac{s_{n,1}}{\sqrt{n}}\sqrt{C_{\text{Tr}}^2 + \frac{V_{m,1}^2}{2\cdot s_{n,1}^2} n G_{\text{Tr}}^2(df)} = \frac{1.467}{\sqrt{40}}\sqrt{1.141^2 + \frac{10.070^2}{2\cdot 1.467^2} 40\cdot 0.0001^2} \approx 0.26 \text{ mV}.$$

As can be seen, these estimated values are very close to the theoretical values (1.14 mV, 0.25 mV and 0.27 mV respectively) determined above.

Method 2. Another, simpler, method can also be used. In this method the random component is estimated directly by the differences $\Delta v_{n,i}$ of the input observations $v_{\text{in},i}$, $v_{\text{in},i+n_1}$ in adjacent periods of interference (n_1 is a number of observations in the interference period):

$$\Delta v_{n,i} = \left(v_{\text{in},i+n_1} - v_{\text{in},i}\right)/2 \approx \left(v_{n,i+n_1} - v_{n,i}\right)/2. \tag{16}$$

The variation $var(\Delta v)$ is: $\sigma_n^2/2$, therefore, the estimated value of the standard deviation of the random component is equal to:

$$s_{n,2} \approx \sqrt{2}\sqrt{\frac{1}{n_1 - 1}\sum_{k=0}^{n_1-1}\Delta v_{n,i}^2}. \tag{17}$$

Using the estimated value $s_{v,\text{in}}$ (11) of the input signal and standard deviation $s_{n,2}$ (17), the estimated value of the amplitude of the interference component is:

$$V_{m,2} \approx \sqrt{2}\sqrt{s_{v,\text{in}}^2 - s_{n,2}^2}. \tag{18}$$

Applying this method to the data in Table 1 gives the following results: $s_{n,2} = 1.373$ mV (17) and $V_{m,2} = 10.195$ mV (18). Therefore, the standard uncertainties estimated with Method 2 are as follows:

$$u_A(V_x)_{\text{GUM},2} \approx \frac{s_{n,2}^2}{n}\sqrt{1 + \frac{V_{m,2}^2}{2s_{n,2}^2}} = \frac{1.373}{\sqrt{40}}\sqrt{1 + \frac{10.195^2}{2 \cdot 1.373^2}} \approx 1.16 \text{ mV},$$

$$u_A(V_x)_{\text{Un},2} \approx \frac{s_{n,2}^2}{n}\sqrt{1 + \frac{V_{m,2}^2}{2s_{n,2}^2} nG_{\text{Un}}^2(df)} = \frac{1.373}{\sqrt{40}}\sqrt{1 + \frac{10.195^2}{2 \cdot 1.373^2} 40 \cdot 0.01^2} \approx 0.23 \text{ mV},$$

$$u_A(V_x)_{\text{Tr},2} \approx \frac{s_{n,2}}{\sqrt{n}}\sqrt{C_{\text{Tr}}^2 + \frac{V_{m,2}^2}{2 \cdot s_{n,2}^2} nG_{\text{Tr}}^2(df)} = \frac{1.373}{\sqrt{40}}\sqrt{1.141^2 + \frac{10.195^2}{2 \cdot 1.373^2} 40 \cdot 0.0001^2} \approx 0.25 \text{ mV}.$$

The estimated values obtained are also very close to the theoretical values determined above. It should be noted that the standard deviations $s_{v,\text{in}}$ (11) and $s_{n,2}$ (17) were always estimated inaccurately. As a result of these inaccuracies at the low level of interference amplitude, when HNR < 1, a situation may occur in which $s_{v,\text{in}} < s_{n,2}$. Therefore, the value of the interference amplitude according to (18) will not always be calculated correctly. That is why the use of this method is not recommended for a low level of interference (HNR ≈< 1).

4. Results of simulation and experimental studies

In order to check the correctness of the above results, two alternative methods were used: Monte Carlo simulations [22] and experimental tests.

4.1. Monte Carlo simulation results

Monte Carlo simulation involves two main steps. At the first step the start parameters of this study were established. The value of the informative voltage is $V_x = 500$ mV; the parameters of interference component are: nominal frequency f_n =50 Hz, the limits of instability $\delta_{f,lim} = \pm 1.0\%$, initial phase is changed in the range $\pm\pi$ and 7 values of interference amplitude were tested: V_m = 10 mV, 20 mV, 50mV, 100 mV, 200 mV, 500 mV, 1000 mV. The standard deviation of the random component is $\sigma_n = 7.071$ mV, for which the harmonic to noise ratios are as follows: HNR = 1, 2, 5, 10, 20, 50, 100.

For each amplitude V_m, the $M = 10^5$ random values of the interference frequency in the range 49.5 Hz ≤ f ≤ 50.5 Hz and uniformly distributed random phases in the range $-\pi \le \varphi \le +\pi$ are generated. For the random component, the 10^5 sets of the $n = 40$ normally distributed random values with zero expected value and standard deviation $\sigma_n = 7.071$ mV are generated. Therefore, the $M = 10^5$ $(j = 1, \ldots, n)$ tested signal realisations (2) of $n = 40$ observations are prepared.

In the second step the following values are determined:

1. average values $\overline{V}_{\text{Un},j}, \overline{V}_{\text{Tr},j}$ and corresponding standard deviations $s_{v,\text{in},j}$ (11) of input signal realizations;
2. standard uncertainties $u_A(V)_{j,\text{theor,GUM}}$ (12) following the GUM procedure [1] for known values of V_m and σ_n;

3. theoretical standard uncertainties $u_A(V)_{\text{theor,Un}}$, $u_A(V)_{\text{theor,Tr}}$ (10) for known values of V_m and σ_n and for uniform and triangle averaging;
4. estimators of the interference amplitude $V_{m,1,j}$ (14), $V_{m,2,j}$ (18) and noise standard deviations $s_{n,1,j}$ (15), $s_{n,2,j}$ (17) estimated by both methods;
5. estimators of the standard uncertainties $u_A(V)_{j,\text{Un},1}$, $u_A(V)_{j,\text{Tr},}$ and $u_A(V)_{j,\text{Un},2}$, $u_A(V)_{j,\text{Tr},2}$ by (10) using estimators of interference amplitude ($V_{m,1,j}$, $V_{m,2,j}$) and noise standard deviations ($s_{n,1,j}$, $s_{n,2,j}$) for uniform and triangle averaging.

The mean values of the uncertainties $u_A(V)_{theor,\ GUM}$ and also those determined for the uniform and triangle averaging theoretical values $u_A(V)_{theor,\ Un}$, $u_A(V)_{theor,\ Tr}$ (for a known V_m and σ_n) normalized to the ratio $\sigma_n/\sqrt{n} = 1.118$ are shown in Fig. 3. From this figure it can be seen that the standard uncertainty determined by Monte Carlo simulation according to the GUM procedure [1] is consistent with the results of the theoretical analysis, which is shown in Fig. 2a.

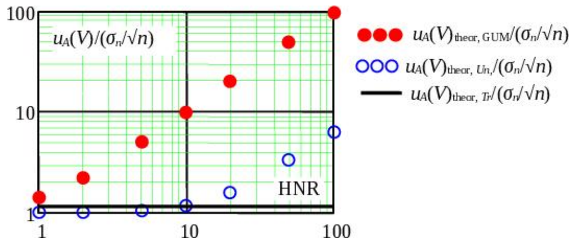


Fig. 3. Dependence on HNR of the normalized-to-$\sigma_n/\sqrt{n}$ values of average standard uncertainties $u_A(V)_{theor,GUM}$ calculated according to GUM and theoretical uncertainties $u_A(V)_{\text{theor,Un}}$, $u_A(V)_{\text{theor,Tr}}$ using uniform and triangular averaging.

The normalized-to-$V_m$mean values of the interference amplitude $V_{m,MC,1}$, $V_{m,MC,2}$ estimated with the two methods are shown in Fig. 4a. As can be seen, both methods ensured estimation of the interference amplitude with imprecision about a few percent, which is acceptable from the point of view of uncertainty evaluation. Fig. 4 b shows the normalized-to-σ_n mean values of noise standard deviations $s_{n,MC,1}/\sigma_n$, $s_{n,MC,2}/\sigma_n$. From this figure it can be seen that, in general, Method 1, based on previous interference amplitude estimation, provides better results in comparison with results obtained with Method 2 based on direct estimation of noise standard deviation. That is, when HNR >10, the normalized mean values increase to about: 1.05 (HNR = 20), 1.36 (HNR = 50) and 2.07 (HNR = 100) using estimation Method 1 and 1.14 (HNR = 20), 1.60 (HNR = 50) and 2.58 (HNR = 100) respectively using Method 2. Besides, at HNR > 20, the instability in determining the noise standard deviation by both methods increases significantly.

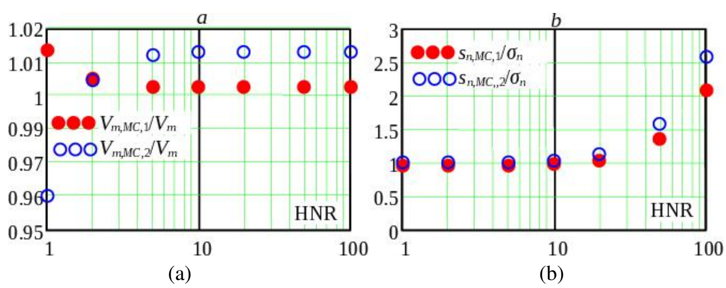
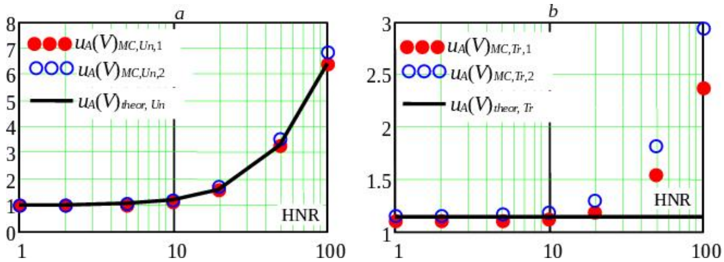


Fig. 4. Dependences on the HNR of the normalized-to-V_m mean values $V_{m,MC,\ 1}$, $V_{m,MC,\ 2}$ of the interference amplitude (a) and normalized-to-σ_n mean values $s_{n,MC,\ 1}$, $s_{n,MC,\ 2}$ of the noise standard deviation (b).

In Fig. 5, the normalized-to-$\sigma_n/\sqrt{n}$ mean values of standard uncertainties values $u_A(V)_{\text{MC,Un,1}}$, $u_A(V)_{\text{MC,Un,2}}$, $u_A(V)_{\text{MC,Tr,1}}$, $u_A(V)_{\text{MC,Tr,2}}$ determined using uniform and triangle averaging and also theoretical uncertainties $u_A(V)_{\text{theor,Un}}$, $u_A(V)_{\text{theor,Tr}}$ are shown. From Fig. 5, it can be seen that at HNR $\approx\leq$ 10 both uniform and triangle averaging practically ensures quite acceptable uncertainty values. However, at HNR >10 when using uniform averaging, the standard uncertainty increases more than 6 times at HNR = 100. This increase in uncertainty is consistent with the theoretical relationship (6) and is caused by insufficient interference suppression in uniform averaging. Using triangular averaging at HNR $\approx\geq$ 20 also results in an increase in standard uncertainty, but this increase is about 2.5 times smaller. The main reason for the increase in uncertainty is the lack of adequate accuracy in estimating the standard deviation of noise at high values of interference amplitude, *i.e.*, at HNR $\approx\geq$ 20, especially when using Method 2.



(a) (b)

Fig. 5. Dependence on the HNR of the normalized-to-$\sigma_n/\sqrt{n}$ mean values of standard uncertainties $u_A(V)_{\text{MC,Un,1}}$ and $u_A(V)_{\text{MC,Un,2}}$ determined using uniform averaging (a) and mean values of standard uncertainties $u_A(V)_{\text{MC,Tr,1}}$ and $u_A(V)_{\text{MC,Tr,2}}$ determined using triangle averaging (b) and also theoretical uncertainties $u_A(V)_{\text{theor,Un}}$, $u_A(V)_{\text{theor,Tr}}$.

One of the main reasons for the insufficient accuracy of the noise standard deviation estimation is the lack of knowledge of the actual value of the interference frequency. In both methods of estimating of the interference and random components it is assumed that the interference frequency is nominal: f_n = 50 Hz, period T_n = 20 ms. While, as noted above, the actual value of the interference frequency may differ from the nominal value by ±1%. Additional processing of noisy signal samples, such as those described in [19,20], can be used to determine the actual interference frequency. For this purpose, the frequency of the power line voltage could also be measured during the signal acquisition. The resulting measurement of the actual frequency (period) could be used in (13)–(18) to estimate V_m and σ_n. However, this requires additional research, which is beyond the scope of this article.

4.2. Experimental results

Test stand. Experimental verification of the theoretical results and obtained by simulations was carried out using a test stand, the block diagram of which is shown in Fig. 6.

The source of a regular part of the input test signal is a RIGOL DG1022 programmable function generator. This generator, programmed according to relation (2), generates the sum of the component V_x, the harmonic interference of the amplitude V_m, and the frequency f. The random component was formed at the output of an NC 6102A noise generator (white noise in the band 10 Hz ... 100 kHz). The test signal $v_{\text{in}}(t)$ is the sum of the outputs of both generators. The voltage is measured by a DMM KEYSIGHT 34465A of AC voltage range: $V_{AC,R}$ = 100 mV. A RIGOL DS1052E oscilloscope is used for observation of the input signal. For the acquisition of the input observations a NI 9222 voltage input module with a cDAQ-9171 measurement board [23] is used,

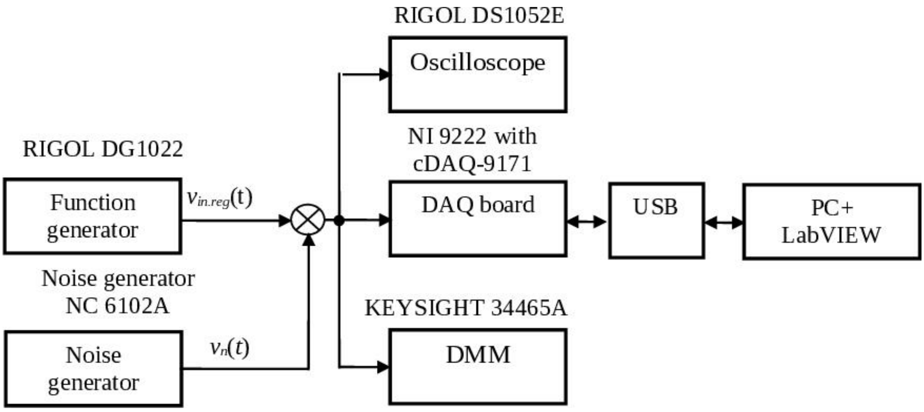


Fig. 6. Block diagram of the test measuring circuit.

main parameters: 4 differential channels, input ranges: ±10 V, 16 bits, 500 kS/s/ch. To control the measurement card and processing registered signal observations a program in LabVIEW environment was used.

Experimental results. In the program controlling the measurement DAQ board, the sampling frequency $f_s = 1$ kHz and the option to record of $N = kn = 1000$ ($n = 40$, $k = 25$) of the input observations ($v_{\text{in},i}$) were established. The standard deviation value in the noise generator output was set to about $\sigma_n = 7.07$ mV by RMS indication of DMM. At the output of the function generator the interference frequency was set in the range from $f_1 = 49.5$ Hz to $f_{11} = 50.5$ Hz ($k_f = 11$ values with a step of 0.1 Hz (maximal deviation is ±1.0%). The $k_u = 7$ values of interference amplitude were as follows: $V_{m1} = 10$ mV; $V_{m2} = 20$ mV; $V_{m3} = 50$ mV, $V_{m4} = 100$ mV, $V_{m5} = 200$ mV; $V_{m6} = 500$ mV, $V_{m7} = 1000$ mV. The values of HNR were follows: 1, 2, 5, 10, 20, 50 and 100.

This section provides only those experimental results that directly relate to the estimating measurement uncertainty. First, the results are given for the estimation with both methods of the interference amplitude and the noise standard deviation, and then the standard uncertainty values when using uniform and triangular averaging are given. At the same time, a comparison of the obtained experimental results with theoretical and simulation results is also given.

For each interference amplitude ($k_u = 7$) and for each harmonic frequency ($k_f = 11$) the $M_s = 1000$ signal observations (total $M_r = k_u k_f = 77$ realizations of 1000 observations) were registered and processed. At the sampling period $T_s = 1$ ms and the averaging duration $T_{\text{av}} = 40$ ms the number of the averaging observations is $n = 40$, therefore theoretically it was possible to process $k_f k_u M_s / n = 1925$ series of input observations. However, to ensure the randomness condition of the initial phase of interference, each series with $n = 40$ averaged observations should start after the end of the previous at a random moment of time in the range of one interference period. Since the sampling period $T_s = 1$ ms, that is, there were 20 observations in the period, the next averaged series started with a delay by a random number of signal observations in the range of 1 to 20. Consequently, the number of observation series averaged for each frequency and each interference amplitude was $k = 17$, that is the total number of series tested was $M_s = 1309$. It is obvious that in this test the interference frequency did not change randomly, it took preset values. For a given $k = 17$ series and $k_f = 11$ different values of interference frequency there are $N_u = 17 \cdot 11 = 187$ estimates of the interference amplitude and also estimates of the standard deviation of the random component and related to them standard uncertainties.

The experimental uncertainty can be evaluated when the amplitude $V_{m,\exp}$ of harmonic interference and the standard deviation $s_{n,\exp}$ of the random component are determined by the two methods described above. Fig. 7 shows normalized-to-given-V_m values of the experimentally determined interference amplitudes ($V_{m,\exp,1}/V_m$, $V_{m,\exp,2}/V_m$) and normalized-to-σ_n values of the experimentally determined noise standard deviations ($s_{n,\exp,1}/\sigma_n$, $s_{n,\exp,2}/\sigma_n$). Comparing the graphs in Fig. 4a with the graphs in Fig. 7a, which present the results of the Monte Carlo simulation, one can see their very good convergence. Similarly, one can notice a very good convergence of the dependencies of the estimated values of the noise standard deviation determined from experimental studies (Fig. 7b) and those determined from Monte Carlo simulations (Fig. 4b).

Experimentally determined and normalized-to-$\sigma_n/\sqrt{n}$ standard uncertainties $u_A(V)_{\exp,\mathrm{Un},1}$, $u_A(V)_{\exp,\mathrm{Un},2}$, $u_A(V)_{\exp,\mathrm{Tr},1}$, $u_A(V)_{\exp,\mathrm{Tr},2}$ obtained when using uniform and triangle averaging are shown in Fig. 8. Comparing the standard uncertainties of measurement determined by the experimental results (Fig. 8) with the results of Monte Carlo simulation (Fig. 5) and with the theoretical values, given in these figures, one can see a good convergence between them. It should be noted that some differences between these results are caused by a large difference in the number of averages: the number of averaged experimental realizations is only 187, and in the Monte Carlo simulations the number of realizations was much higher, namely 10^5.

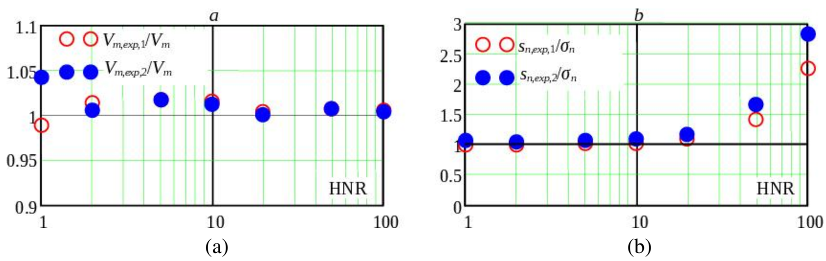


Fig. 7. Experimentally-determined interference amplitude $V_{m,\exp}$ normalized to V_m (a) and the noise standard deviation $s_{n,\exp}$ normalized to σ_n (b).

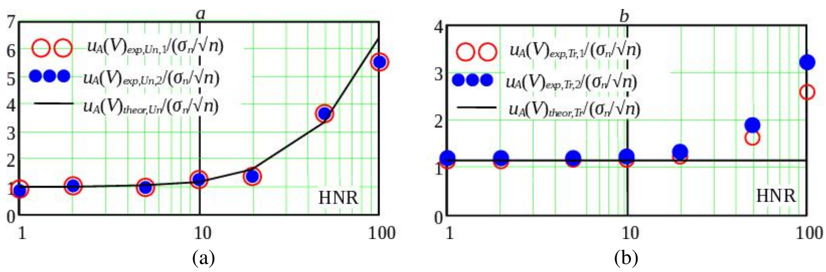


Fig. 8. Experimentally-determined and normalized-to-$\sigma_n/\sqrt{n}$ standard uncertainties when uniform (a) and triangle (b) averaging are used.

The experimental results also confirmed the effect of reducing the accuracy of uncertainty evaluation due to the low accuracy of the estimation of the standard deviation of noise, even though the interference component is sufficiently suppressed by, for example, triangular averaging. The main reason for this is the lack of knowledge of the actual deviation of the interference frequency from the nominal value.

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5. Conclusions

This paper presents the problems of the type A evaluation of the measurement uncertainty with the multiple observations, when the measured signal, in addition to random uncorrelated noise, is distorted by power line interference. The standard uncertainty of such measurement was investigated when uniform and triangular weighted averaging of the input signal were used. The research was carried out theoretically, using the Monte Carlo simulation method, and experimentally.

It was shown that the classical type A method given in GUM [1] does not provide a correct evaluation of the uncertainty of such measurement. Namely, calculated according to the GUM, the standard uncertainty is overestimated in comparison with the theoretical value by a few to several tens of times or even more at high levels of interference (when HNR $\approx$ 100). This problem is caused by the different suppression of random noise and power line interference during averaging the input observations. The influence of the uncorrelated noise decreases in the first approach proportionally to the root of the number of averaged observations. But the influence of the power line interference practically does not depend on the number of observations and only depends on the used averaging weight function and the instability of interference frequency.

It was stated that in order to correctly evaluate uncertainty in such measurement both the amplitude of the power line component and the standard deviation of the random component should be estimated separately. Two simple methods of the separate estimation of these components were investigated.

Assuming that the maximum frequency deviation of the power line interference is $|\delta_f| = 1\%$, it was found that uniform averaging can provide the correct evaluation of measurement uncertainty if HNR $\approx\leq$ 10. This limitation is mainly due to the relatively small interference suppression (about only $1/|\delta_f| = 100$ times) and also the low accuracy of the estimation of the noise standard deviation caused by the instability of interference frequency.

However, using triangular averaging, due to the large value of interference suppression (not less than 10^4 at $|\delta_f| = 1\%$), the standard uncertainty can be calculated correctly for the level of HNR $\approx\leq$ 20. Although the accuracy of the uncertainty estimation using triangular averaging is approximately 2.5 times higher than in the case of uniform averaging, the inaccuracy of the uncertainty estimation is significant at HNR $>$ 20. The main factor in this case also is the lack of knowledge of the exact interference frequency.

To improve the evaluation of uncertainty, the actual value of the power line interference should be determined. This requires additional research which will be performed in the next stage.

The results obtained and presented from the simulations and experimental tests have shown good accordance with the results of the theoretical analysis. Therefore, they confirmed the effectiveness of the proposed solutions of the Type A uncertainty evaluation in the measurements in which the measured signal is distorted by both random noise and high-level power line interference.

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