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A STUDY ON THE FOUR-SECTION PITCH MEASUREMENT METHOD ON THE DEVIATION OF CHARACTERISTIC PARAMETERS OF HERRINGBONE GEARS

Yadong Zhang, Jianhua Wang, Yunbo Shen, Peili Yin

School of Mechatronic Engineering, Xi'an Technological University, Xi'an, Shaanxi 710021, China
(✉ *jdgcxy_wjh@163.com*)

Abstract

To enhance measurement efficiency for herringbone gear deviations, this study leverages the fundamental principle of tooth surface generation through helical and involute line families to systematically establish a mathematical model of tooth surface incorporating Herringbone Gear Characteristic Vertex-P (HGCV-P) parametric deviations. Based on this foundation, an innovative four-section tooth pitch method was proposed for HGCV-P deviation measurement, and a corresponding measurement system was developed. First, two cross-sections (within the evaluation range) on the upper gear for tooth pitch measurement should be selected, then the same method applied to measure two cross-sections on the lower gear; finally, the measured points are transformed into a two-dimensional coordinate system through coordinate transformation to calculate the HGCV-P deviation. The experimental results demonstrate the total HGCV-P centring deviation $F_p = 0.0288$ and the mean HGCV-P deviation $F_{pS} = 0.0243$, showing fundamental consistency with the results [19]. The research findings elucidate the correlation between HGCV-P deviations and the transmission stability of herringbone gears, providing critical information for gear optimisation design. Furthermore, the proposed HGCV-P measurement protocol significantly enhances the inspection efficiency of herringbone gears, establishing a novel inspection methodology to advance manufacturing precision.

Keywords: measurement efficiency, HGCV-P, four-section tooth pitch method, measurement system, centring deviation.

1. Introduction

Herringbone gears, compared to helical gears, exhibit significant performance advantages in systems due to their unique symmetrical meshing characteristics: their axial force self-balancing mechanism enables high load-bearing capacity, along with vibration suppression and improved transmission stability, making them indispensable in high-speed and heavy-duty applications such as turbofan engines of large commercial aircraft, main reducers in helicopters, and propulsion systems of large container ships and cruise liners [1–5]. However, due to inherent errors in the manufacturing process and dynamic deformation in service, a herringbone gear pair is prone to

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abnormal vibration and broadband noise problems, which directly threaten the operational safety and service reliability of the transmission system. With the development of high-end equipment in terms of high power density and low vibration noise, more stringent requirements have been put forward for tooth precision control and dynamic meshing performance of herringbone gears. In this context, realising accurate prediction and control of gear sub manufacturing quality has become a key technical path to suppress transmission chain vibration noise and improve equipment reliability.

In recent years, international scholars have conducted systematic research on mapping the relationship between manufacturing errors and transmission performance of herringbone gears. Xu [6] presents an innovative method for predicting the wear of herringbone gears in the presence of alignment errors. It is shown that misalignment significantly affects the wear characteristics of the gears: on the overrunning contact side, the gear teeth exhibit higher contact temperature, greater wear depth, and higher contact pressure. It is interesting to note that due to the alignment error, the wear of the gear teeth on both sides shows a significant asymmetric distribution. Wear depth is also found to show a decreasing trend with increasing gear pair speed. More importantly, the change in wear depth dynamically affects the value of alignment error, which in turn changes the contact characteristics and load distribution pattern of the gear pair. Zhou [7] constructed a life model of herringbone gears considering the dual factors of centring and angular errors and systematically investigated the coupled effects of the two types of errors on the life of "crack-budding" herringbone gears. The study revealed that these errors significantly shorten the crack initiation life of herringbone gears. Additionally, it was found that appropriate angular adjustments can effectively balance the load distribution between the left and right teeth of herringbone gears, thereby extending the gear's service life. Chen [8] used theoretical analysis to study the nonlinear vibration response characteristics of herringbone gears under the influence of pitch errors and cumulative pitch errors. During the research, a dynamic model of herringbone gears was developed that incorporates axial deflection and gyroscopic effects. Solving the model demonstrated that under high-speed operating conditions, pitch errors significantly excite higher-order natural frequencies. Zhang [9] developed a comprehensive *transmission error* (TE) prediction model for herringbone gears by integrating multi-source error factors that include tooth flank errors, pitch errors, and alignment errors. This model systematically reveals the mechanisms of coupled interaction between these three types of errors and TE. Through comparative simulations under both no-load and loaded operating conditions, the study provides an in-depth analysis of the mechanisms through which these errors influence transmission error. Based on the *loaded tooth contact analysis* (LTCA) model, Liu reveals the multi-source error coupling mechanism such as integrated meshing stiffness and integrated meshing error, and then constructs a herringbone gear dynamics model containing asymmetric tooth pitch characteristics by considering the multi-factor error excitation conditions, and finally reveals the mechanism of influence of rotational speed, asymmetric meshing impact force, and other multi-conditions on the three-dimensional vibration response [10]. Wang [11] innovatively proposed a quasi-static analytical model for herringbone planetary gear trains based on the slice theory. Through iterative solving of nonlinear differential equations, the study systematically reveals the displacement/angular dynamic response characteristics of planetary transmissions under coupled load-error conditions and developed a semi-analytical transmission accuracy evaluation method. The research specifically clarifies the synergistic mechanism between pitch deviations and alignment errors in influencing transmission accuracy under loaded operating conditions. Li [12] established an improved dynamic model of the herringbone gear incorporating misalignment angle parameters. Based on the ROMAX software platform, the study revealed the nonlinear coupling mechanism between misalignment angles and axial forces/dynamic performance in gear pairs. The research found that the influence of angle misalignment on axial forces exhibits periodic variations with pitch deviations. Furthermore, although misalignment angles induce axial forces, their rational selection can reduce fluctuations

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in the comprehensive meshing stiffness of herringbone gears, thus stabilising the dynamic meshing forces in star-type herringbone gear systems. Dong [13] considered the asymmetric error of the herringbone gear and proposed a 12-degree-of-freedom static analysis model to analyse the impact of the asymmetric error on transmission error. Li [14] analysed the effects of different misalignment errors on the contact and axial vibration of herringbone gears and proposed a compensation method that uses the coupled effects of axial assembly errors and misalignment errors. The series of studies mentioned above reveals that manufacturing errors in herringbone gears exert a profound influence on the system's vibration characteristics and load distribution patterns.

In the specialised field of herringbone gear measurement, many scholars have devoted themselves to research work. Through many experiments, in-depth analysis, and the use of various types of advanced technology and theory, they have achieved a series of results with reference value. Kawasaki *et al.* [15] proposed an innovative manufacturing process for herringbone gears based on a universal *computerised numerical control* (CNC) machining platform: first, a geometrical model of the tooth surface was constructed using a 3D *computer-aided design* (CAD) system, and then a *computer-aided manufacturing* (CAM) system-driven CNC machining centre was used to realise precision forming. The finished herringbone gears were quantitatively inspected for key parameters such as tooth profile deviation, helix deviation, and surface roughness. Guo *et al.* [16] carried out a precision inspection of the tooth-face helix of the herringbone gear using a coordinate measuring machine, quantitatively analysed the tooth-face symmetry deviation by means of the collected geometrical parameter data, and constructed an evaluation system for the transmission accuracy of the gear pair based on the obtained measurement values. Liang's [17, 18] research team developed a multi-degree-of-freedom symmetry detection device and proposed a method to measure symmetry errors in herringbone gears and internal keyways of deep holes using this apparatus. They also established control strategies to optimise their effectiveness. The test results demonstrate that the proposed method can precisely control symmetry errors within 0.02 mm. The bilaterally independent measurement paradigm commonly adopted in existing studies has systematic limitations: by discretising the herringbone gears into independent helical gear pairs for discrete detection, the spatial constraints formed by the cross-coupling of the tooth surfaces are not adequately considered. This simplification leads to absence of characteristic parameters associated with the meshing phases of gear teeth, highlighting the need for the overall linkage detection of herringbone gears. For this purpose, the authors determined the characteristic parameters of the herringbone gears and their deviations using the mapping principle of a tooth-faced helix from the literature [19], studied the evaluation algorithm of its circumferential deviation, and developed a modular measurement system based on the VC++ platform using this algorithm. Subsequently, from the perspective of gear transmission performance, the authors' team used the mapping principle of the tooth contact line to characterise its characteristic parameters and their deviations, studied the axial deviation evaluation algorithm of its characteristic parameters, and designed measurement software to carry out the real test for theoretical verification [20]. However, although the two measurement methods above can meet the requirements of laboratory environment and high-precision herringbone gear measurement, they have low measurement efficiency in practical applications, making it difficult to meet the requirements of large-scale or time-sensitive herringbone gear inspection. For measuring herringbone gears that do not require high precision, traditional methods such as template inspection, manual measuring instruments, and even ordinary coordinate measuring machines [16, 21] are used, but these methods have limitations in terms of efficiency and accuracy. The measurement of herringbone gears with high precision requirements basically relies on various types of high-precision gear measuring centres imported to China from abroad, such as *KLINGELNBERG* (P100) and GLEASON (GMS 800). However, there are very few foreign publications on this topic, and they do not cover the core technical content. Domestic research generally focuses on summarising and sharing information on the use of instruments. Currently, research frameworks for herringbone gear measurement still exhibit significant deficiencies. Critical aspects such as measurement methods, precision control, measurement efficiency, and expansion of applicable scenarios require further exploration and refinement. Additionally, studies on the influence of deviations of herringbone gear characteristic parameters on transmission systems remain insufficiently comprehensive and systematic, urgently necessitating further in-depth investigation.

This paper is based on the principle of the tooth surface formed by the helix and involute families, and thus a mathematical model of the tooth surface with their characteristic parameters is established. Furthermore, an innovative measurement method, the four-section pitch method, is proposed to enhance the efficiency of herringbone gear inspection. Next, supporting measurement software is developed and experimental tests are conducted to validate the approach.

2. Tooth face model with HGCV-P deviation

2.1. The concept of HGCV

As specified in the AGMA 940-A09 technical specifications [22], the unique geometric configuration of herringbone gears is characterised by the symmetric V-shaped distribution of bilateral tooth profiles, where the theoretical extensions of the left and right tooth flanks intersect in a spatial coordinate system to form a characteristic geometric vertex, as depicted in Fig. 1. This vertex, designated as *P*, is defined as the *Herringbone Gear Characteristic Vertex* (HGCV), a core geometric parameter intrinsic to herringbone gears.

From a geometric metrology perspective, this characteristic parameter can be quantitatively characterised by the intersection properties of the helical lines (or contact lines) of the left and right tooth flanks within a two-dimensional coordinate system.

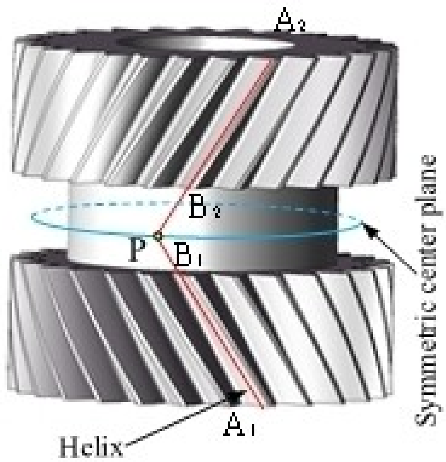

Fig. 1. Characterization of HGCV-P.

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2.2. Tooth Flank Model Incorporating HGCV Deviations

When reconstructing the actual machined herringbone tooth face, random angle error in the form of a sinusoidal is introduced by considering the standard angle of the circumferential array of point coordinates on the standard tooth face. The mathematical model of the actual tooth face thus constructed needs to satisfy the following constraints:

$$\Delta\mu_j = \left[(j-1)\cdot\frac{2\pi}{Z_0} + \xi_j\right] \; (j = 1, 2, 3 \ldots Z_0)\,, \tag{1}$$

where Z_0 is the number of teeth. ξ_j is the random angle error in the sinusoidal form.

According to (1), the constraints are introduced into the standard equations according to the modelling method of the standard herringbone gear tooth model formed by the helix family and the involute family. In the gear coordinate system as shown in Fig. 2, its right tooth mathematical model after considering the HGCV deviation can be expressed as follows:

$$S_R(\delta,\gamma) = \begin{bmatrix} x(\delta,\gamma) \\ y(\delta,\gamma) \\ z(\delta,\gamma) \end{bmatrix} = \begin{bmatrix} r_b(\cos\delta + \delta\sin\delta)\cos\left(\gamma + \Delta\mu_\mathrm{j}\right) - r_b(\sin\delta - \delta\cos\delta)\sin\left(\gamma + \Delta\mu_\mathrm{j}\right) \\ r_b(\cos\delta + \delta\sin\delta)\sin\left(\gamma + \Delta\mu_\mathrm{j}\right) - r_b(\sin\delta - \delta\cos\delta)\cos\left(\gamma + \Delta\mu_\mathrm{j}\right) \\ P_H\gamma \end{bmatrix}, \tag{2}$$

where δ and γ are the tooth parameters, P_H is the helix parameter, and r_b is the base radius.

Based on the cogging transformation matrix H [17], the left cogging equation can be derived as follows:

$$S_L(\delta,\gamma) = H \cdot S_R(\delta,\gamma)\,. \tag{3}$$

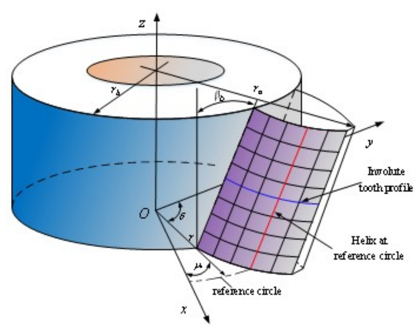


Fig. 2. Tooth flank formed by the family of helical lines and involutes.

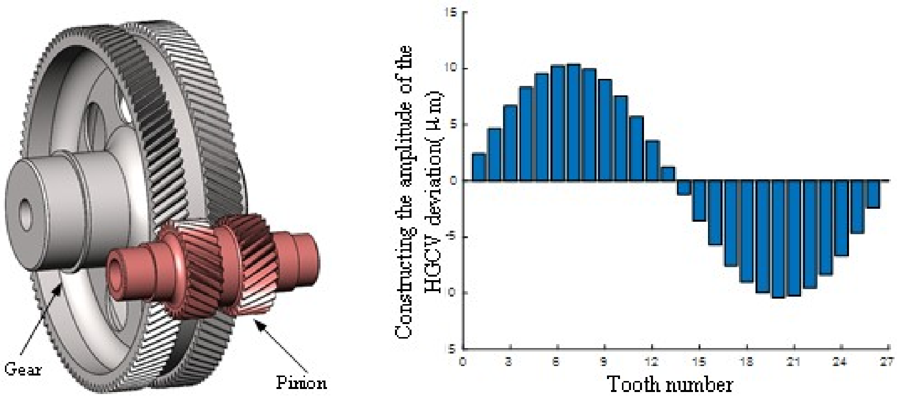
2.3. Effect of HGCV-P deviation on its meshing characteristics

The specific parameters of the pinion are detailed in Table 1. The number of teeth in the large gear that fits the pinion is 106, and the other parameters are the same as those of the pinion. Next, a three-dimensional mathematical model is established. In this study, a sinusoidal excitation function is employed to simulate the stochastic deviation characteristics of HGCV, as shown in Fig. 3.

The analytical model of transient dynamics is established using the finite element method, and after the calculations, the characteristic evolution of the axial dynamic movement of the gear pair can be obtained, as shown in Fig. 4.

Table 1. Parameters of the herringbone gear investigated.

Parameters	Numerical value
normal modulus m_n $[mm]$	2.5
tooth number Z_0	27
normal pressure angle α_n [°]	22.5
helix angle β [°]	30
tooth width B $[mm]$	30
over-travel slot width W $[mm]$	20
coefficient of variation	0.2



(a) (b)

Fig. 3. 3D model of a herringbone gear pair: a) 3D model, b) constructed HGCV deviations.

From Fig. 4, it can be seen that the axial movement of the herringbone gear pair exhibits a significant quasi-periodic fluctuation pattern during the meshing phase turnover. The simulation results show that the presence of different P-point deviations in the herringbone gear will directly affect the axial movement of the gear train after loading. Therefore, it is necessary to improve the machining accuracy of the herringbone gear to reduce this error.

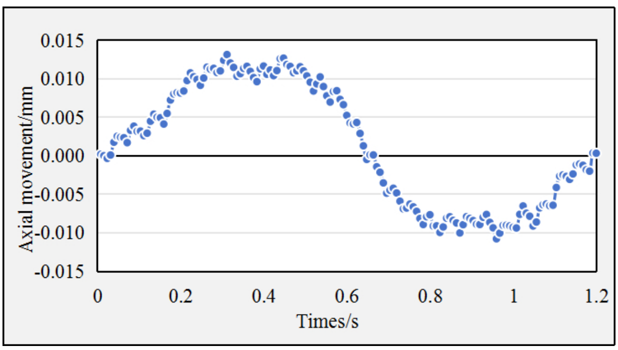


Fig. 4. Axial movement at P-point deviation for finite element analysis simulation.

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3. Principles for evaluating HGCV deviations

3.1. Coordinate conversion

The herringbone gear tooth flank characteristic line is defined by extremely complex three-dimensional parametric equations, a characteristic that reveals significant limitations when performing evaluation work for herringbone gears. In view of this, this paper proposes a strategy to transform the tooth surface in the three-dimensional space to a two-dimensional plane by applying the basic principle of involute helix formation. The specific implementation process is to unfold the cylindrical surface of the tooth in the three-dimensional coordinate system, so that it is presented in the two-dimensional plane, and then through the precise coordinate transformation operation, the characteristics of the tooth surface are realised in the two-dimensional coordinate plane of intuitive expression. The transformation formula is as follows:

$$\begin{bmatrix} X_p \\ Y_p \end{bmatrix} = \begin{bmatrix} r\gamma \\ z_0 \end{bmatrix}. \tag{4}$$

3.2. Calculation of HGCV-P coordinates with the four-section tooth pitch method

As shown in Fig. 5, four sections were selected in the axial direction of the herringbone gear to measure the pitch, among which Sections I and II were on one gear, while Sections III and IV were on the other gear.

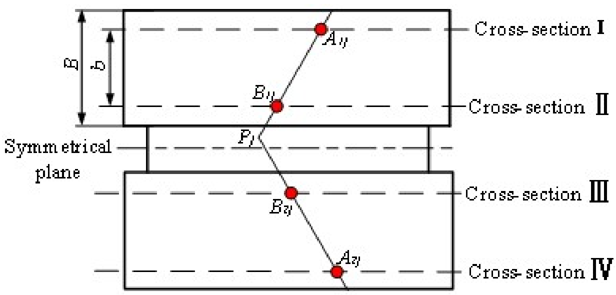

Fig. 5. Four-section position.

To minimize the impact of tooth profile modification on the measurement deviation of HGCV-P, the distance b between two cross-sections on one side of the herringbone gear teeth, as shown in Fig. 3, is determined by the tooth profile modification amount and is typically selected as $b = 0.2B - 0.8B$. To ensure measurement stability and reduce the influence of tooth profile modification, this paper first selects $b = 0.6$B. Four sets of data could be measured on the corresponding flanks $A_{1j}(x_{A1j}y_{A1j})$, $B_{1j}(x_{B1j}y_{B1j})$, $A_{2j}(x_{A2j}y_{A2j})$, $B_{2j}(x_{B2j}y_{B2j})$, j represented the tooth number, x was the arc length coordinate at the reference circle, and y was the axial coordinate of the corresponding section. Equation (5) can be obtained from point $A_{1j}(x_{A1j}y_{A1j})$ and point $B_{1j}(x_{B1j}y_{B1j})$. Equation (6) can be obtained from the same point of reason$A_{2j}(x_{A2j}y_{A2j})$ and point $B_{2j}(x_{B2j}y_{B2j})$:

$$\left(y_{B1j} - y_{A1j}\right) X_j - \left(x_{B1j} - x_{A1j}\right) Y_j - \left(x_{A1j}y_{B1j} - x_{B1j}y_{A1j}\right) = 0, \tag{5}$$

$$(y_{B2j} - y_{A2j})X_j - (x_{B2j} - x_{A2j})Y_j - (x_{A2j}y_{B2j} - x_{B2j}y_{A2j}) = 0. \tag{6}$$

Similarly, by solving (5) and (6), the HGCV-P $P_j(X_j, Y_j)$ of tooth j was obtained, as in (7).

$$\begin{bmatrix} X_j \\ Y_j \end{bmatrix} = \begin{bmatrix} \dfrac{(x_{B1j}-x_{A1j})(x_{A2j}y_{B2j}-x_{B2j}y_{A2j})-(x_{B2j}-x_{A2j})(x_{A1j}y_{B1j}-x_{B1j}y_{A1j})}{-(y_{B1j}-y_{A1j})(x_{B2j}-x_{A2j})+(y_{B2j}-y_{A2j})(x_{B1j}-x_{A1j})} \\ \dfrac{-(y_{B2j}-y_{A2j})(x_{A1j}y_{B1j}-x_{B1j}y_{A1j})+(y_{B1j}-y_{A1j})(x_{A2j}y_{B2j}-x_{B2j}y_{A2j})}{-(y_{B1j}-y_{A1j})(x_{B2j}-x_{A2j})+(y_{B2j}-y_{A2j})(x_{B1j}-x_{A1j})} \end{bmatrix}. \tag{7}$$

3.3. HGCV-P deviation assessment

(1) Individual HGCV-P centring deviation $\boldsymbol{f_{Pj}}$ – denotes the perpendicular distance of the p-point with respect to the plane of symmetry, and the sign is specified as positive when the p-point is close to the left-handed tooth, and negative when the opposite is true, as in Fig. 6.

(2) Difference between neighbouring P points:

$$f_{Pu} = \max\left|f_{Pj+1} - f_{Pj}\right|. \tag{8}$$

(3) HGCV-P total centring deviation F_p – maximum minus the minimum of the single HGCV-P centring deviation:

$$F_p = \max f_{Pj} - \min f_{Pj}. \tag{9}$$

(4) Mean deviation of HGCV-P F_{pS}:

$$F_{pS} = \frac{f_{Pj} + f_{Pj+1} + f_{Pj+2} + \ldots + f_{PZ_0}}{Z_0}. \tag{10}$$

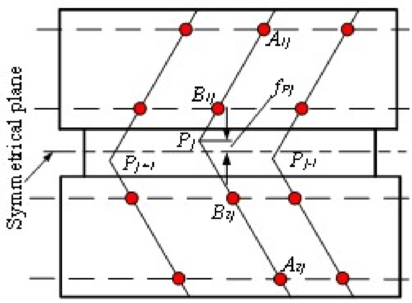

Fig. 6. Characterization of P-point deviation

4. Measurement of HGCV-P deviations

4.1. Measuring instruments and environment

The analysis in Section 1 reveals that there is a significant time-varying correlation between the morphological characteristics of the HGCV-P deviation and the service performance of the system, according to which it is necessary to establish the mapping control criterion between the manufacturing accuracy and the service performance. For the multi-dimensional evaluation model of HGCV deviation constructed in Section 3, this section adopts the GMC650 gear measurement centre platform, as shown in Fig. 7, to carry out experimental validation on the herringbone gear in Table 1.

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Fig. 7. GMC650 and the pinion gear under test.

The GMC650 gear measurement centre is located in a constant temperature ultra-clean experimental environment and its precision measurement system consists of five core modules: 1) the measurement system integrating a high-precision rotary table and an aerostatic guideway with the use of a high-precision grating encoder to form a closed-loop feedback system in order to achieve the four-axis (X/Y/Z and the spindle) linkage of coordinate measurements; 2) a Renishaw SP600Q three-dimensional scanning probe for system configuration; 3) a control system equipped with a self-developed SoC motion controller and an integrated all-digital servo drive unit; 4) a computer system to build a distributed data acquisition architecture; 5) a software system with the algorithm described in Section 3.2 based on the VC++ development environment for the independent development of software systems. The overall design structure of the software encompasses the following stages: a human-computer interaction interface (input of basic parameters of herringbone gears), determination of measurement benchmarks, and measurement methods (the four-section tooth pitch method: including measurement parameters, measurement motion control, sampling and data processing, *etc.*). The experimental environment implements precise environmental control: the fluctuation of temperature is controlled at 20 ± 1°, and the vibration suppression system meets the vibration isolation requirements of the isolation frequency ≤ 10 Hz and the amplitude ≤ 2 µm. The equipment geometric accuracy guarantee system includes: the coaxiality error of the top centre $\delta \leq 3$ µm, and the radial runout tolerance of the bottom centre ±2 µm. The module of the gear to be tested is: 0.5 ~25, the maximum outer diameter of the gear being tested is Φ 650 mm, the vertical measurement range of the probe is 0–800 mm, the resolution of the probe is 0.1 um, and the maximum weight of the workpiece that can be measured is 500 kg.

4.2. Measurement procedure

The measurement flow chart for herringbone gears is shown in Fig. 8. The detailed steps for measuring herringbone gears are as follows:

(1) Zero calibration. This part of the selection of the ISO standard [23] mandrel or standard ball for the whole zero calibration.
(2) Installation of the herringbone gear. The measured herringbone gear is precisely mounted between the upper/lower data centres of the GMC650 rotary table. In order to eliminate the

influence of the spindle rotary error, the measurement of the use of the drive shown in Fig. 7 is applied to compensate for it.

(3) Configuration of measurement parameters. The basic geometric parameters of the herringbone gear (as shown in Table 1) and the measurement engineering parameters (effective diameter of the probe ϕ1.5 mm, positional range of the measurement point of the four-section method and other key indexes) are set in the human-machine interface of the software.

(4) Next, the zero point is coordinated. Based on the centre of symmetry of the herringbone gear to establish the zero point of the Z-axis, the probe scans the involute tooth profile on both sides of the tooth groove, determines the angle value corresponding to the position of the middle of the tooth groove, and establishes this value as the zero point of the main axis.

(5) Then, the measurement is performed using the four-section tooth pitch method. According to the installation position of the herringbone gear and based on the measurement principle of HGCV-P deviation as described in Section 3.2, the tooth pitch in Section 1, Section 2, Section 3 and Section 4 is subsequently measured and the measured data are saved after completing the data acquisition for all sections.

(6) The three-dimensional measured data points are then expanded to the two-dimensional plane using the coordinate transformation given in Section 3.1, the coordinates of the *P*-point position are calculated according to (6), and the method discussed in Section 3.2 is used for the calculation of the various deviations of the HGCV-P.

(7) Finally, the measured herringbone gear is dismantled in according to the standard and placed in a dry and ventilated environment for encapsulation and storage after cleaning and lubrication maintenance.

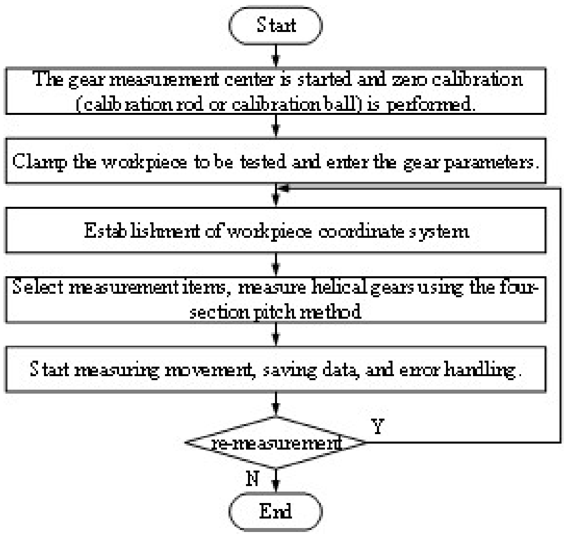

Fig. 8. Measurement flow chart for herringbone gears.

4.3. Four-section pitch method measurement results and analysis

The herringbone gear is measured on the GMC650 using the four-section pitch method described in this paper, and the results are shown in Fig. 9.

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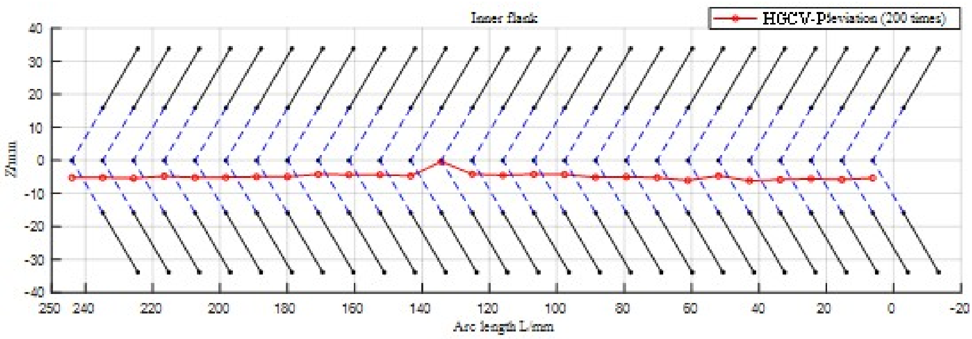

Fig. 9. Measurement results for the four-section tooth pitch method.

Using the four-section pitch method described in Section 3.2 and taking the axial distance b between the two sections to be 18 mm, the P-point deviation curve for the Herringbone gear was as shown in Fig. 10.

From Fig. 10, the range of the individual HGCV-P centring deviation f_{Pj} was –0.0309 to –0.0020 mm, HGCV-P total centring deviation $F_p = 0.0288$ mm and the difference between the neighbouring P points $f_{Pu} = 0.0215$ mm. Thus, the mean deviation of HGCV-P : $F_{pS} = -0.0243$ mm. For the full helix measurement method [19], the f_{Pj}, F_p and f_{Pu} obtained are greater than those for the full helix method. The mean deviations of HGCV-P obtained by both methods are essentially the same.

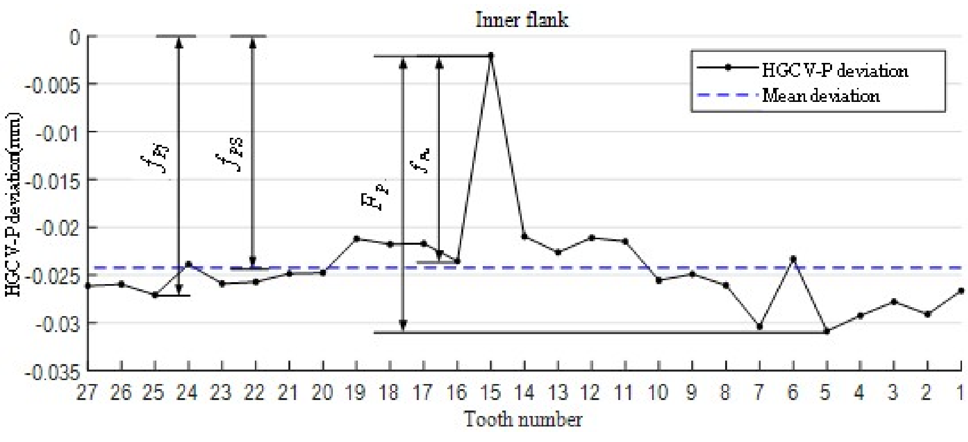

Fig. 10. HGCP-P deviation.

The reason for different data stability of the two measurement methods was that the straight lines obtained with the full helix method included the least square midlines of the helices, which had the average effect of multi-point data, while the straight lines obtained with the four-section pitch method were connected by two points, for which the measurement error affected the uncertainty of the straight-line equation and then affected the coordinate data of the intersection point. In addition, in the four-section pitch method, the axial distance b between the two sections also affected the data stability of the centring deviation; therefore, a larger value b had to be used. Although the full helix method is in accordance with the definition of centring deviation, it is inefficient to measure all helices tooth by tooth. However, the four-section pitch method is a highly efficient centring deviation measurement scheme suitable for day-to-day production.

5. Conclusions

1. Based on the principle of constructing tooth flanks by helical and involute families, a mathematical model of tooth flanks containing HGCV-P deviation is established. The model provides a theoretical basis for an in-depth study of the measurement and system dynamic characteristics of herringbone gears.
2. An innovative method for the evaluation of HGCV-P deviation, the four-section tooth pitch method, is proposed. This method measures the tooth pitch of the four cross sections of the herringbone gear, maps the spatial three-dimensional measurement points to the two-dimensional parameter plane using the principle of coordinate transformation, and then deduces the calculation method of HGCV-P deviation.
3. Based on the VC++ platform combined with the evaluation principle of the four-section tooth pitch method, the HGCV deviation measurement system was developed, integrated in the GMC650 measuring machine, and carried out measurement experiments. The measured results show that the total centring deviation of HGCV-P $F_p = 0.0288$ and the mean deviation of HGCV-P $F_{pS} = -0.0243$.

This method greatly shortens the single-piece measurement cycle of herringbone gears and provides a new method for batch inspection of high-precision gears.

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Yadong Zhang received his M.Sc. degree in mechanical engineering from Henan University of Science and Technology in 2018 and is currently pursuing a Ph.D. degree at the School of Mechatronic Engineering, Xi'an Technological University, Xi'an, China. His current research interests include precision measurement techniques for herringbone gears.

Yunbo Shen received his Ph.D. degree in engineering from Northwestern Polytechnical University. He is now a professor and a doctoral supervisor. His scientific interests focus on the high-performance gear transmission theory, gear design, processing and preparation and drive system dynamics. He has completed more than 10 national and provincial and ministerial-level projects, won the Office of the Bureau-level Science and Technology Achievement Award 4, 4 authorized invention patents, published 45 papers, of which 18 were SCI or Ei indexed.

Jianhua Wang received his Ph.D. degree in engineering from Xi'an Jiaotong University, Xi'an, China. He is now a professor and a Ph.D. supervisor, He has been awarded 3 second prizes for scientific and technological progress at provincial and ministerial levels, 6 third prizes awards, 10 authorised invention patents, 1 translated book, more than 100 papers, 1 first prize for excellent teaching achievements in Shaanxi Province, China, and participated in the preparation of two textbooks. His main research interests: precision measurement theory and application, precision machinery design and manufacturing.

Peili Yin received her Ph.D. degree in 2018 from Xi'an Technical University, majoring in mechanical and electronic engineering. She is now an associate professor, masters' tutor. She has presided over one subproject of the provincial scientific and technological major project and one special project of the Department of Education; she has participated in five scientific research projects at the national, provincial, and ministerial levels. She has published 8 academic papers as the first author, including 2 SCI II, 1 SCI IV, 2 EI; 2 authorised invention patents; and participated in a translation. Her main research interests are precision measurement theory and application, digital twin theory and technology.