



Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

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<http://www.imp.gda.pl/archives-of-thermodynamics/>



Physical modelling in heat and fluid flow

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Received: 29.11.2025; revised: 13.02.2026; accepted: 25.02.2026

Abstract

Experimental research, especially physical modelling, is vital when direct study of real systems is impractical. By applying similarity theory and dimensionless variables, models allow reliable analysis of results. While experiments alone are often qualitative and time-consuming, combining them with theoretical modelling yields stronger quantitative insights. Careful planning, computer-based data collection, and awareness of measurement errors ensure precision and efficiency. The theory of similarity defines the criteria under which the behaviour of a model can be considered representative of the real system. By satisfying these criteria, experimental results obtained from the model can be reliably extrapolated to the actual phenomenon. The present paper aims to present Authors' up-to-date experiences in advanced research using scaled models based on similitude theory, ever since its establishment as a branch of the engineering science to convince the reader about the benefits of physical modelling in comparison to advanced computer based methods governing the contemporary research.

Keywords: Experiment planning; Modelling; Energy; Entropy

Vol. 47(2026), No. 1, 205–212; doi: 10.24425/ather.2026.158670

Cite this manuscript as: Mikielwicz, J. & Mikielwicz, D. (2026). Physical modelling in heat and fluid flow. *Archives of Thermodynamics*, 47(1), 205–212.

1. Introduction

One of the fundamental methods for solving problems in science and engineering is experimental research, particularly through physical modelling of phenomena and processes. Careful planning of the experiment can provide a significant amount of required information at a moderate effort. In many cases, direct investigation of a real device is impractical due to high costs, safety concerns, or the scale of the system. In such situations, studies are conducted using physical models that replicate key aspects of the original system.

Physical models enable comprehensive investigations of technical devices or isolated phenomena under controlled laboratory conditions. The theory of similarity defines the criteria un-

der which the behaviour of a model can be considered representative of the real system. By satisfying these criteria, experimental results obtained from the model can be reliably extrapolated to the actual phenomenon.

Physical modelling serves as a tool for acquiring new experimental data in cases where no reliable mathematical model exists or where available data are insufficient to validate existing theoretical models. This approach relies on inductive reasoning, involving observation, experimentation, data collection and inference. When supported by the theory of similarity, experimental findings can be generalised to other systems governed by the same dimensionless differential equations and boundary conditions.

However, research based solely on experimentation is often

Nomenclature

a – thermal diffusivity, m^2/s
 c_p – specific heat, $\text{J}/(\text{kg}\cdot\text{K})$
 F – force, N
 g – gravitational acceleration, m^2/s
 l – characteristic dimension, m
 p – pressure, Pa
 q – unit heat, J/kg
 T – temperature, K
 w – velocity, m/s
 x, y, z – coordinates, m

Greek symbols

α – heat transfer coefficient, $\text{W}/(\text{m}^2\cdot\text{K})$
 β – coefficient of volumetric expansion, $1/\text{K}$
 λ – thermal conductivity, $\text{W}/(\text{m}\cdot\text{K})$
 ρ – density, kg/m^3
 ν – kinematic viscosity, m^2/s
 τ – time, s

Subscripts

0 – reference
 v – volumetric

Superscripts

$+$ – non-dimensional

time-consuming and typically yields qualitative insights. Combining experimental methods with theoretical modelling – formulating mathematical models based on empirical data – leads to more robust, quantitative conclusions.

Modern experimental research is guided by a well-established methodology that facilitates the design, execution and interpretation of experiments. The preparatory phase includes a literature review, identification of independent and dependent variables, design of the experimental setup, selection of measurement techniques, and acquisition or fabrication of instrumentation. Each step requires a deep understanding of the phenomenon and substantial research experience.

To streamline the research process, variables are commonly expressed in dimensionless form, which reduces their number and simplifies analysis. The principles of similarity, modelling strategies and the formulation of dimensionless variables will be discussed later in detail.

Careful planning of experimental or numerical studies helps minimise effort and cost while maintaining the desired level of precision. Data acquisition and processing are typically performed using computer systems equipped with specialised software and sensor interfaces. All measurements are subject to error, stemming from limitations of both instruments and human perception. Although a detailed statistical analysis of measurement uncertainty is beyond the scope of this paper, it is worth noting that heat and flow measurements are often single-instance observations, which do not require extensive treatment of random error as in other experimental contexts.

The present paper aims to present Authors' up-to-date experiences in advanced research using scaled models based on similitude theory, ever since its establishment as a branch of engineering science, to convince the reader about the benefits of physical modelling in comparison to advanced computer based methods governing the contemporary research. Advances in computational power have also encouraged hybrid strategies that integrate similitude principles with finite element modelling. These methods allow researchers to construct reduced-scale models of highly complex structures with greater fidelity, positioning them as one of the most promising directions for future research. Collectively, the reviewed methodologies highlight both the progress achieved and the challenges that remain in establishing robust similitude frameworks for modern engineering structures. Presented are also the capabilities of physical modelling in application to complex flows.

2. Theory of similarity and principles of physical modelling

Interest in the similarity of physical phenomena dates back centuries. The work presented by Rayleigh [1] is consensually recognised as the first presentation of the use of scientific models, based on dimensional analysis. Research on this topic has been ongoing since the earliest mathematical descriptions of physical processes. Considerations related to similarity were already undertaken by Galileo, Reynolds, Fourier [2], Carnot [3] and others.

Research on reduced-scale models continues to expand as engineers seek reliable methods for predicting the behaviour of complex systems, identifying areas requiring further investigation. Historically, dimensional analysis has served as the dominant methodology, originating from the foundational work of Goodier and Thomson [4]. Although widely presented in academic literature through simplified examples, its successful application in aerospace case studies demonstrates its capability to address highly complex structural configurations. Nevertheless, the method demands meticulous selection of all relevant physical quantities and careful construction of non-unique dimensionless groups, making its practical use both labour-intensive and dependent on expert judgment.

In contrast, Kline's [5] formulation introduced a more intuitive approach based on governing differential equations that explicitly describe system response. Subsequent research has followed two paths: employing the solutions of these equations or using the equations directly to derive similitude conditions. While the latter avoids the need for analytical solutions, it requires complete knowledge of the physics and availability of governing equations – conditions rarely met for structures more complex than plates or shells. A derivative of this approach has been applied in modelling of complex flows presented as examples later.

To address this limitation, a recent work by De Rosa et al. [6] proposes an energy-distribution-based methodology formulated in modal coordinates. Since 2008, this approach has enabled detailed analysis of dynamic and acousto-elastic systems, offering a promising alternative where governing equations are incomplete or unavailable. Despite its mathematical complexity, it represents a significant step toward extending similitude theory to intricate structural assemblies.

Work in this area continues to evolve, with significant contributions such as those by Macagno [7] or Mikielwicz [8].

The principles of similarity are applied in:

- the search for analytical solutions,
- interpretation of experimental research results,
- modelling of phenomena and systems when direct investigation of the real object is impractical.

Similarity is generally an abstract property that may or may not be present in the analysed phenomena or systems. It is most commonly associated with geometric similarity, which is a limited concept that does not encompass the physical similarity of phenomena. Physically similar phenomena do not necessarily exhibit geometric similarity.

In a broader sense, geometric similarity includes the following types of geometric transformations:

- **uniformity**, involving proportional scaling of all geometric dimensions (uniform scale),
- **affinity**, involving transformations with different scaling factors in different directions,
- **affine transformation**, involving arbitrary linear transformations of geometric dimensions (defined by a transformation matrix).

Geometric similarity exists between objects if a unique transformation of geometric dimensions can be established between them. Such a transformation is referred to as a *geometric mapping*. Examples of geometric similarity are illustrated in Fig. 1.

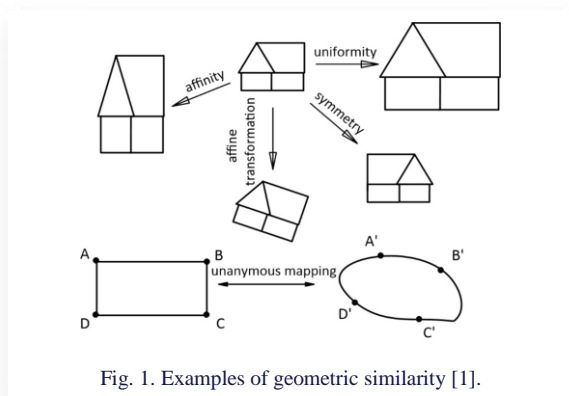


Fig. 1. Examples of geometric similarity [1].

However, geometric similarity alone does not guarantee similarity of physical phenomena – such as fields of physical quantities or the functioning of devices. If it did, a human being geometrically similar to an ant would be able to lift a piano with ease, and a flea scaled to human dimensions would be capable of jumping hundreds of meters. This is not the case, because physical quantities such as force scale with the cross-sectional area of muscles (proportional to the square of a linear dimension), while weight scales with volume (proportional to the cube of a linear dimension). As a result, actions such as lifting are not directly similar.

Phenomena or device behaviours are considered similar if the behaviour of one system allows the prediction of the other. Mathematically, this means that under joint geometric and physical similarity, the vector describing a point in space-time and its physical parameters are uniquely reproduced in the second, similar system. Such a mapping exists if both systems are gov-

erned by the same dimensionless set of equations and boundary conditions. In the most general case, these equations form a system of differential-integral equations with uniqueness conditions.

It follows that the criteria for similarity are most easily identified when a mathematical description of the phenomena is available. However, a complete description is not always possible. Therefore, various methods are used to determine similarity criteria, depending on the available information:

- analysis of differential-integral equations describing the phenomena,
- analysis of global balance equations (e.g. forces or energy) – partial analysis,
- dimensional analysis, based on the dimensions of selected physical and geometric parameters.

The reliability of these methods depends on the amount and quality of information available to the researcher. Dimensional analysis is considered the least reliable among them.

These methods will be discussed in more detail later in this paper. Regardless of the method used, the resulting similarity criteria can be applied in physical modelling of phenomena or devices. Depending on the degree to which the criteria are satisfied, modelling may be complete, approximate or partial.

2.1. Analysis of the mathematical description of similar phenomena

As mentioned in the introduction, similar phenomena are governed by the same system of differential-integral equations expressed in dimensionless form. Determining similarity criteria based on the mathematical description relies on the following principles [9–13]:

- Similar phenomena are described by dimensionless differential-integral equations along with appropriate boundary conditions,
- The solutions to these equations are expressed in terms of dimensionless parameters – known as *similarity numbers* – which appear both in the equations and in boundary conditions,
- These similarity numbers, formed as combinations of physical parameters, must have identical values for similar phenomena. The similarity criteria are constructed from independent similarity numbers.

The core task is therefore to transform the dimensional (physical) equations describing the phenomena into a dimensionless form, and then derive the similarity criteria from the resulting mathematical equations.

Dimensional (physical) equations are homogeneous with respect to physical dimensions, meaning that each term in the equation has the same dimensional unit. The process of converting a physical equation into a mathematical (dimensionless) one involves:

- Defining the relevant dimensions and characteristic values,
- Introducing dimensionless variables into the physical equations.

As a result, each term in the equation can be expressed as a product of two components: one dimensional (constructed from characteristic quantities and units), and one dimensionless

(constructed from non-dimensional variables). This transformation follows from *Bridgman's theorem*. By dividing the entire equation by one of the dimensional components – since all such components share the same dimension – a fully dimensionless equation is obtained.

This method will be applied to a system of equations describing thermo-fluid processes. For the purpose of analysis, we consider a general system of conservation equations, where the equation of motion is represented by the Navier–Stokes equation, and the energy equation by the Fourier–Kirchhoff equation [10].

According to the presented method, let us define the characteristic quantities:

- a_0 – characteristic thermal diffusivity,
- l_0 – characteristic dimension,
- τ_0 – characteristic time,
- T_0 – characteristic temperature,
- ΔT_0 – characteristic temperature drop,
- λ_0 – characteristic thermal conductivity coefficient,
- c_{p0} – characteristic specific heat,
- ρ_0 – characteristic density,
- q_{v0} – characteristic intensity of heat sources,
- F_0 – characteristic mass force,
- Δp_0 – characteristic pressure drop,
- ν_0 – characteristic kinematic viscosity,
- w_0 – characteristic speed.

Using characteristic quantities, let us define dimensionless quantities:

- $x^+ = x/l$, $y^+ = y/l$, $z^+ = z/l$ – dimensionless coordinates,
- $\tau^+ = \tau/\tau_0$ – dimensionless time,
- $T^+ = T/\Delta T_0$ – dimensionless temperature,
- $w^+ = w/w_0$ – dimensionless speed,
- $\lambda^+ = \lambda/\lambda_0$ – dimensionless thermal conductivity coefficient,
- $c_p^+ = c_p/c_{p0}$ – dimensionless specific heat,
- $\rho^+ = \rho/\rho_0$ – dimensionless density,
- $q_v^+ = q_v/q_{v0}$ – dimensionless intensity of internal heat sources,
- $F^+ = F/F_0$ – dimensionless force acting on a unit of mass,
- $p^+ = p/\Delta p_0$ – dimensionless pressure,
- $\nu^+ = \nu/\nu_0$ – dimensionless coefficient of kinematic viscosity.

Moreover, dimensionless spatial differentiation operators should be introduced. Operations ∇ and ∇^2 have a dimension and are related to the adopted scale defined by the characteristic dimension l_0 . Therefore, the operators can be written:

- $\nabla^+ = l_0 \nabla$ – dimensionless Hamiltonian operator,
- $\nabla^{+2} = l_0^2 \nabla^2$ – dimensionless Laplace operator.

By introducing the above relationships into the balance equations for incompressible fluids, we obtain:

- for the mass balance:

$$\frac{w_0}{l_0} \nabla^+ w^+ = 0, \quad (1)$$

- for the Navier-Stokes equation:

$$\frac{w_0}{\tau_0} \frac{\partial w^+}{\partial \tau^+} + \frac{w_0}{l_0^2} \nabla^{+2} w^+ = F_0 F^+ - \frac{\Delta p_0}{l_0 \rho_0} \frac{\nabla^+ p^+}{\rho^+} + \frac{\nu_0 w_0}{l_0^2} \nu^+ \nabla^{+2} w^+, \quad (2)$$

- for the Fourier-Kirchhoff equation:

$$\frac{\Delta T_0}{\tau_0} \frac{\partial T^+}{\partial \tau^+} + \frac{w_0 \Delta T_0}{l_0} w^+ \nabla^+ T^+ = \frac{\lambda_0 \Delta T_0}{l_0^2 c_{p0} \rho_0} \frac{\nabla^+ (\lambda^+ \nabla^+ T^+)}{c_p^+ \rho^+} + \frac{q_{v0} q_v}{c_{p0} \rho_0 c_p^+ \rho^+}. \quad (3)$$

In Eqs. (1)–(3), the relations between scales, called dimensional factors, present in each term of the equations must be equal to each other in order to be used in modelling between the model equation and a modelled equation. Dividing both sides of the equations by the dimensional factor gives:

$$\nabla^+ w^+ = 0, \quad (4)$$

$$\frac{l_0^2}{\tau_0 \nu_0} \frac{\partial w^+}{\partial \tau^+} + \frac{w_0 l_0}{\nu_0} w^+ \nabla^+ w^+ = F_0 \frac{l_0^3}{\nu_0^2} \frac{\nu_0}{w_0 l_0} F^+ + \frac{\Delta p_0}{w_0^2 \rho_0} \frac{w_0 l_0}{\nu_0} \frac{\nabla^+ p^+}{\rho^+} + \nu^+ \nabla^{+2} w^+, \quad (5)$$

$$\frac{\partial T^+}{\partial \tau^+} + \frac{w_0 \tau_0}{l_0} w^+ \nabla^+ T^+ = \frac{\lambda_0 \tau_0}{l_0^2 c_{p0} \rho_0} \frac{\nabla^+ (\lambda^+ \nabla^+ T^+)}{c_p^+ \rho^+} + \frac{q_{v0} q_v \tau_0}{\Delta T_0 c_{p0} \rho_0 c_p^+ \rho^+}. \quad (6)$$

The mass balance equation does not provide dimensionless parameters. The equation of motion, however, has dimensionless complexes:

$$1. \quad \frac{l_0^2}{\tau_0 \nu_0} = \frac{l_0^2}{\tau_0 a_0} \frac{a_0}{\nu_0} = \frac{1}{\text{PrFo}}, \quad (7a)$$

$$2. \quad \frac{w_0 l_0}{\nu_0} = \text{Re}, \quad (7b)$$

$$3. \quad F_0 \frac{l_0^3}{\nu_0^2} \frac{\nu_0}{w_0 l_0} = F_0 \frac{l_0^3}{\nu_0^2} \text{Re}^{-1}, \quad (7c)$$

$$4. \quad \frac{\Delta p_0}{w_0^2 \rho_0} \frac{w_0 l_0}{\nu_0} = \text{Eu Re}, \quad (7d)$$

which are expressed by known dimensionless numbers.

The third dimensionless number $F_0 \frac{l_0^3}{\nu_0^2}$ can appear in different forms depending on the nature of the force F_0 in the Navier-Stokes equation. These mass forces can result from the existence of different fields: gravitational, electric, magnetic and others. Let us define this number in the case of the existence of a gravitational field.

Let a body of volume V and density ρ_0 be in a medium of density that differs by $\Delta \rho_0$ from the body's density. Then, according to Archimedes' principle, the body is subjected to a buoyant force:

$$Vg\Delta\rho_0 = Vg(\rho_0 - \rho). \quad (8)$$

In the case of gravitational field, its intensity is defined by the magnitude of the unit force F_0 , that is the ratio of the buoyancy force $Vg\Delta\rho_0$ and the mass of the body, i.e. $V_0 \rho_0$:

$$F_0 = g \frac{\Delta \rho_0}{\rho_0}. \quad (9)$$

Substituting Eq. (9) into Eq. (7c), the Archimedes number is obtained:

$$\text{Ar} = \frac{g l_0^3}{\nu_0^2} \frac{\Delta \rho_0}{\rho_0} \quad (10)$$

If the density of the medium is negligible in relation to the density of the body, i.e. $\Delta \rho_0 \approx \rho_0$, then Archimedes' number is equal to Galileo's number:

$$Ga = \frac{g l_0^3}{\nu_0^2}. \quad (11)$$

A special case of buoyancy forces are the forces resulting from thermal expansion or compressibility of the medium. The change in density $\Delta\rho_0$ is defined by the relationship:

$$\Delta\rho_0 = \beta\rho_0\Delta T_0. \quad (12)$$

Substituting the relationship (11) into (9), the Grashof number is obtained:

$$Gr = \frac{g l_0^3 \beta \Delta T_0}{\nu_0^2}. \quad (13)$$

Let us return to the last of the dimensionless numbers appearing in the equation of motion – Eq. (5). It is called the Euler number:

$$Eu = \frac{\Delta p_0}{\rho_0 w_0^2}. \quad (14)$$

In turn, the product of Grashof and Prandtl numbers is called the Rayleigh number:

$$Ra = \frac{g l_0^3 \beta \Delta T_0}{\nu_0 a_0} = \frac{g l_0^3 \beta \Delta T_0}{\nu_0^2} \frac{\nu_0}{a_0} = Gr Pr. \quad (15)$$

The solution to the equation of motion is therefore described by the following relationship, which includes all dimensionless numbers occurring in the fluid flow:

$$Eu = Eu \left(Pr, Fo, Re, \frac{F_0 l_0^3}{\nu_0^2} \right), \quad (16)$$

and defining the similarity of the velocity fields.

There are three complexes of dimensionless numbers in the energy equation:

$$\frac{\partial T^+}{\partial \tau^+} + \frac{w_0 \tau_0}{l_0} W^+ \nabla^+ T^+ = \frac{\lambda_0 \tau_0}{l_0^2 c_{p0} \rho_0} \nabla^+ (\lambda^+ \nabla^+ T^+) + \frac{q_{v0} q_v \tau_0}{\Delta T_0 c_{p0} \rho_0 c_p^+ \rho^+},$$

$$1. \quad \frac{w_0 \tau_0}{l_0} = \frac{w_0 l_0}{a_0} \frac{a_0 \tau_0}{l_0^2} = Pe Fo, \quad (17a)$$

$$2. \quad \frac{\lambda_0 \tau_0}{l_0^2 c_{p0} \rho_0} = \frac{a_0 \tau_0}{l_0^2} = Fo, \quad (17b)$$

$$3. \quad \frac{q_{v0} \tau_0}{\Delta T_0 c_{p0} \rho_0} = \frac{q_{v0} l_0^2}{\lambda_0 \Delta T_0} Fo, \quad (17c)$$

and a dimensionless number characterising the internal heat source: $\frac{q_{v0} l_0^2}{\lambda_0 \Delta T_0}$. From the above, it turns out that the temperature field depends on:

$$\frac{T}{\Delta T} = T^+ \left[x_i^+, Fo, Pe, \frac{q_{v0} l_0^2}{\lambda_0 \Delta T_0} \right]. \quad (18)$$

The similarity of temperature fields results in the similarity of temperature gradients at the wall. From the boundary condition – $\lambda \nabla T = \alpha \nabla T$, the Nusselt number can be derived:

$$Nu = \frac{\alpha l_0}{\lambda_0} = \frac{\alpha \Delta T^+}{\lambda_0 \Delta^+ T^+}. \quad (19)$$

From Eq. (19), it follows that the Nusselt number is a function of the temperature field; taking into account Eq. (18), we have:

$$Nu = Nu(T^+) = Nu \left[x_i^+, Fo, Pe, \frac{q_{v0} l_0^2}{\lambda_0 \Delta T_0} \right]. \quad (20)$$

Summarising the above considerations, it can be stated based on Eqs. (16) and (20) that in problems in which, in addition to geometric similarity, there is a similarity of temperature and velocity fields, the dependent Nusselt number can be expressed in the form:

$$Nu = Nu(T^+) = Nu \left[x_i^+, Fo, Re, Pr, \frac{q_{v0} l_0^2}{\lambda_0 \Delta T_0}, \frac{F_0 l_0^3}{\nu_0^2} \right], \quad (21)$$

where $\frac{F_0 l_0^3}{\nu_0^2}$ may be the Archimedes, Galileo or Grashof number.

If the fluid is compressible, the velocity field and temperature field will also be influenced by the Mach number:

$$Ma = \frac{w}{w_s}, \quad (22)$$

where w_s is the speed of sound in a given medium.

2.2. Partial analysis of similar phenomena

If we do not have a full mathematical description of the phenomenon, we can rely on the balances of global forces or energy when establishing the similarity criteria.

The following forces related to the unit mass can be distinguished:

- inertia: $F_b \approx \frac{w_0^2}{l_0}$,
- pressure: $F_p \approx \frac{p_0}{l_0 \rho_0}$,
- viscosity: $F_t \approx \frac{\nu_0 w_0}{l_0^2}$,
- gravity: $F_g \approx g_0$,
- surface tension: $F_\sigma \approx \frac{\sigma_0}{l_0^2 \rho_0}$.

By dividing the balance of forces acting in a phenomenon by one of the forces, we obtain dimensionless force ratios that can be reduced to the following relations:

$$\frac{F_p}{F_b} \approx \frac{p_0}{\rho_0 w_0^2} = Eu, \quad \frac{F_b}{F_t} \approx \frac{w_0 l_0}{\nu_0} = Re, \quad (23)$$

$$\frac{F_b}{F_g} \approx \frac{w_0^2}{g_0 l_0} = Fr, \quad \frac{F_b}{F_\sigma} \approx \frac{\rho_0 w_0^2 l_0}{\sigma_0} = We.$$

The balance of forces does not always take into account all dimensionless numbers of energy relationships. Energy balance provides additional criteria numbers. Energy balances are treated in a similar way to force balances. They are divided by one of the terms, obtaining energy ratios that lead to criteria numbers.

The following types of energy are distinguished in relation to the volume of the system:

- conduction energy: $e_p \approx \frac{\lambda_0 \Delta T_0}{\rho_0 l_0^2}$,
- convection energy: $e_k \approx \frac{w_0 c_{p0} \Delta T_0}{l_0}$,

- friction energy: $e_t \approx \frac{v_0 w_0^2}{\rho_0 l_0^2}$,
- heat source energy: $e_v \approx q_{v0}$,
- energy accumulated in a non-stationary state:

$$e_n \approx \frac{c_{p0} \Delta T_0}{\tau_0}.$$

The ratios of these energies lead to the following criterion numbers:

$$\frac{e_t}{e_k} \approx \frac{w_0 v_0}{c_{p0} \Delta T_0 l_0} = \frac{Ec}{Re}, \quad (24a)$$

$$\frac{e_n}{e_p} \approx \frac{\rho_0 c_{p0} l_0^2}{t_0 \lambda_0} = \frac{l_0^2}{a_0 t_0} = Fo, \quad (24b)$$

$$\frac{e_k}{e_p} \approx \frac{w_0 l_0 \rho_0 c_{p0}}{\lambda_0}, \quad (24c)$$

where $Ec = \frac{w_0^2}{c_{p0} \Delta T_0}$ is the Eckert number.

Having the correct balance of forces and energy, one can find an appropriate set of criteria numbers describing the studied phenomenon.

2.3. Dimensional analysis

Dimensional analysis requires the least information about the phenomenon being studied, but is also the most unreliable. It is used only when the mathematical model of the phenomenon is not known or when we are unable to correctly formulate the balance of forces and energy [14,15].

With the help of dimensional analysis, a smaller number of dimensionless parameters m ($m < n$) is obtained from n physical parameters describing the phenomenon. This leads to the synthesis of information about the phenomenon, and thus to a reduction in the workload of research and then its description.

The basis of dimensional analysis is the so-called Buckingham-Driest theorem. It is based on the so-called Bridgman equation, stating that every physical quantity x can be represented as the product of a dimensionless number x^+ and the nomenclature:

$$x = x^+ [M]^a [L]^b [T]^c [\theta]^d, \quad (25)$$

where x^+ – number, M, L, T – basic dimensions, whose powers a, b, c, d are numbers. In the SI system of units, four basic units are sufficient to express the title of derived mechanical-thermal units (M – mass, L – linear dimension, T – time, θ – temperature).

Using the Bridgman equation, the Buckingham–Driest theorem can be formulated. If there is a functional relationship between n physical quantities:

$$f(x_1, x_2, x_3, \dots, x_n) = 0, \quad (26)$$

then it can be presented in the form of a relationship of m dimensionless numbers π composed of physical quantities:

$$G(\pi_1, \pi_2, \pi_3, \dots, \pi_m) = 0, \quad (27)$$

where $m = n - r$, r denotes the rank of a rectangular matrix created from exponents of powers of basic units representing the denomination of physical quantities describing the phenome-

non. The rank of a matrix created from exponents of powers of denominations of physical quantities is understood as the rank of the highest non-zero determinant of a rectangular matrix. Obtaining the correct set of dimensional numbers using dimensional analysis requires knowledge of all physical parameters important for the phenomenon. Adding an insignificant parameter or omitting a significant one leads to an erroneous result.

The dimensional analysis method will be illustrated with a simple example of cooling steel spheres at a constant ambient temperature [9].

The excess temperature of the ball over the ambient temperature is $T - T_0$ and depends on:

- time elapsed from the moment the ball was inserted into the coolant (environment), $\tau [T]$,
- initial temperature difference, $(T_p - T_0) [\theta]$,
- heat capacity of the ball, $C = \rho c V [ML^2 / T^2 \theta]$,
- thermal resistance of the ball surface, $R = 1/(\alpha A)$ or $(\alpha A) [ML^2 / T^3 \theta]$.

Physical quantities are therefore related to the relationship:

$$f[\tau, (T_p - T_0), C, \alpha A, (T - T_0)] = 0, \quad (28)$$

and the names of these quantities can be represented by independent basic quantities: M, L, T, θ . T_p stands for the initial temperature of the sphere, whereas the temperature T models the current temperature, which varies with time.

It is therefore possible to create a matrix of exponents of the powers of the basic units in which they appear in the names of quantities describing the phenomenon under consideration. This matrix is presented in the form of Table 1 as follows:

Table 1. Matrix of exponents of the powers of the basic units.

	τ	$T_p - T_0$	C	αA	$T - T_0$
$[M]$	0	0	1	1	0
$[L]$	0	0	2	2	0
$[T]$	1	0	-2	-3	0
$[\theta]$	0	1	-1	-1	1

Looking at the rows of the matrix, we find that the second row of the matrix can be obtained by multiplying the first row by two. So the two rows of the matrix are linearly dependent, which means that the matrix is of order three, not four. Hence, $r = 3$, and there are $m = n - r = 5 - 3 = 2$ dimensionless numbers. As a result, we should obtain a relationship of the type:

$$G(\pi_1, \pi_2) = 0. \quad (29)$$

The numbers π_1 and π_2 can be constructed in various ways. Most often, three of the five quantities are selected, the name of which has all the basic dimensions, e.g. $t, T_p - T_0, C$; then, adding one of the remaining physical quantities, we obtain the required two dimensionless numbers in the form:

$$\pi_1 = \tau^{a1} (T_p - T_0)^{b1} C^{c1} (T - T_0), \quad (30a)$$

$$\pi_2 = \tau^{a2} (T_p - T_0)^{b2} C^{c2} (\alpha A). \quad (30b)$$

The unknown exponents of the powers $a1, b1, c1$ and $a2, b2$ and $c2$ in each of the numbers are determined by equating the

exponents at each of the dimensions of the basic names of physical quantities (M, L, T, θ) to zero, which means that the resulting parameter from the physical quantities is dimensionless. In the example, the following two non-dimensional numbers were obtained:

$$\pi_1 = \frac{T-T_0}{T_p-T_0}, \quad \pi_2 = \frac{\alpha A \tau}{c}. \quad (31)$$

So, the dimensional functional relationship f from Eq. (28) transforms into a dimensionless relationship G :

$$G\left(\frac{T-T_0}{T_p-T_0}, \frac{\alpha A \tau}{c}\right) = 0. \quad (32)$$

It is known from theory that the problem under consideration has an exact solution in the form:

$$\frac{T-T_0}{T_p-T_0} = \exp\left(-\frac{\alpha A \tau}{c}\right). \quad (33)$$

Based on the relationship found in the dimensional analysis, one can look for an experimental correlation of the type:

$$\frac{T-T_0}{T_p-T_0} = f\left(\frac{\alpha A \tau}{c}\right), \quad (34)$$

and then compare it with the theoretical relationship and experimental data. A similar result is obtained from the analytical model applied to that issue based on the lumped capacitance method.

2.4. Principles of physical modelling

As previously mentioned, a model is a representation of one phenomenon by another, or of the same phenomenon under laboratory conditions. In many cases, testing a device or process under real-world conditions is not feasible due to high research costs, large dimensions of the system, or safety concerns. In such situations, experiments are conducted in laboratory settings using model devices.

For the model to be valid, both the model and the original system should be governed by the same set of differential-integral equations and boundary conditions expressed in dimensionless form. Experiments performed on the model allow researchers to identify relationships between similarity numbers. If the model enables experimentation under conditions where all independent dimensionless numbers (i.e. similarity criteria) are equal both in the model and in the original system, the modelling is considered *complete*.

Achieving complete similarity often requires the use of materials with physical properties different from those in the original system. This is necessary to satisfy the relationships implied by the equality of dimensionless numbers. However, if the number of required similarity conditions exceeds the number of available free variables, complete modelling becomes impossible. In such cases, *approximate* or *partial* modelling is employed.

One of the modelling approaches extensively practised by Authors are considerations of dissipation energy in application to complex convective flows. The energy has an additive property, and it is this property that has been used in the considered cases. Attention was focused on modelling flows with boiling or condensation under saturated conditions [16–18]. The proposed

approach was also applied to the analysis of subcooled flow [19], mist flow [20], as well as supercritical heat transfer [21]. In these cases too, consistent results were obtained, which confirm the ability of the presented modelling to predict complex flows with heat transfer and non-linear effects. Recently, it proved to return satisfactory results in calculations of pressure drop at pressures close to the thermodynamic critical point [22] as well as during heat transfer under these conditions [23].

It should also be noted that the approach to obtaining a solution in a complex flow based on the summation of dissipation energies has also worked well in other cases. An approach using the concept of a dissipation energy summation model has previously been applied to the analysis of two-phase bubbly flow in a near-wall layer [24]. As a result of modelling, a non-dimensional velocity distribution in the boundary layer was obtained, inclusive of an effect of void fraction. Energy-based methods are an alternative to traditional modelling approaches, providing reliable results. Energy-based methods are appreciated in many engineering applications, see, for example, the Galerkin method or Huber stress folding in solids in mechanics.

Approximate modelling is used when less critical similarity criteria are satisfied to a sufficient degree, or when the research is conducted within a parameter range where one of the criteria has negligible influence on the phenomenon. This range is referred to as the *self-modelling area*. Approximate models are particularly useful in early-stage research or feasibility studies, where full similarity is not essential for drawing meaningful conclusions.

If no self-modelling region exists, **partial modelling** is applied. In this approach, the focus is placed on isolating the unknown effect of a phenomenon, assuming that all other effects are well understood and can be reliably predicted in both the original system and the model. Only the unknown aspect is subject to modelling. For example, when studying the resistance of a ship moving through water, it is known that the total resistance consists of frictional resistance and wave resistance. If the frictional resistance is well characterised, it can be subtracted from the total measured resistance, allowing the wave resistance to be isolated and modelled independently.

In practice, the choice between complete, approximate, and partial modelling depends on the research objectives, available resources, and the complexity of the phenomenon under investigation. It is also influenced by the desired accuracy of the results and the feasibility of reproducing the necessary similarity conditions.

Moreover, the design of a physical model must consider not only the similarity criteria but also practical aspects such as manufacturability, measurement accessibility, and the influence of scaling on material behaviour. For instance, certain materials may exhibit scale-dependent properties (e.g. surface roughness effects or thermal conductivity), which must be accounted for during model construction and interpretation of results.

Ultimately, physical modelling serves as a bridge between theoretical analysis and real-world application. When properly designed and interpreted, it provides valuable insights into complex systems that are otherwise difficult or impossible to study directly.

3. Conclusions

As has been shown, the modelling principles discussed are general in nature and can be applied to a wide variety of physical systems. Regardless of the specific substance or device under consideration, the method of analysis follows a common structure. First, the reference state of the system must be defined, which requires specifying the appropriate constitutive relations, such as for example governing equations, or alternatively other well established relations. Once the state description is established, the appropriate modelling approach can be applied.

In practice, determining the parameters that describe a system often relies on simplified analytical relations that capture the essential behaviour of the material or device. These developed models are not exact descriptions of real cases but idealised constructs that preserve the dominant physical mechanisms while neglecting secondary effects. For example, a real gas may be approximated by the ideal (perfect) gas model when intermolecular interactions are negligible; a simple magnetic material may be represented by the Curie law, which relates magnetisation to temperature and magnetic field; and an elastic solid may be described by Hooke's law, which assumes linear proportionality between stress and strain.

These models are examples of similitude in a broad sense: they allow different physical systems to be analysed using the same mathematical structure by identifying the relevant variables, reducing them through dimensional analysis, and expressing the governing relations in non-dimensional form. This approach underlies the Buckingham π theorem, which states that any physically meaningful relation between variables can be rewritten in terms of dimensionless groups. Such groups – like the Reynolds number in fluid mechanics or the Prandtl number in heat transfer – capture the essential similarity between systems of different scales or compositions. Thus, the use of idealised thermodynamic models is consistent with the general philosophy of similitude: replacing complex reality with a simpler, dimensionally consistent representation that preserves the key physical behaviour. For more information, the interested reader is referred to recent publications [25,26].

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