ARCHIVES OF CIVIL ENGINEERING, LVI, 4, 2010

DOI: 10.2478/v.10169-010-0017-6

PERMEABILITY OF SAND-CLAY MIXTURES

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This study deals with the behavior of composite blends constituted of rigid and impervious grains included in saturated clay paste of kaolin, considered as permeable and deformable. Permeability tests performed during standard oedometr tests (before each load step) highlight the key role of the original and actual state of the clay paste, and show the existence of a threshold of sand grain concentration above which a structuring effect influences its permeability. In the light of these experiments some usual homogenization methods (with simplifying assumptions to make the problem manageable) are considered in order to model the mixture permeability. Qualitative and quantitative comparisons with experimental data point out their respective domain of interest and limitations of such approaches.

Key words: Sand-clay mixtures, Clay paste void ratio, Grain volume ratio, Permeability, Homogenization.

1. Introduction

Requirement of knowledge of hydraulic characteristics of materials made of sand and clay results from the fact that such mixtures are widely encountered in civil engineering either as common natural soils on which structures are settled, or as reconstituted materials used for road embankment, earth dam, and contamination barrier. Their ability to be compacted is of the essential (first) importance in design, either in purpose of predicting or controling the structural settlement due to consolidation, or reaching the required performances in terms of mechanical resistance or impermeability, particularly in the case of clay liners. Practice shows that the mixed materials characteristics highly depend on the nature of the constituents, especially on the mineralogical structure of clay, on their relative proportions, as well as on the overall density and content of water. However, if empirical rules are well established (for instance for the design of

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embankments, LCPC/SETRA [1]), the derivation of the sand-clay mixture properties from that of the constituents, remains an open question. Relating phenomena in the macroscale to the physics in the microscale is typically an upscaling problem. This philosophy, widely developed in material science has also attracted a great attention in soils mechanics. To cite just a few contributions that tend to explain by a systematic approach the macro observations from the microstructure of mixed soils, let us mention the works of Boutin & Auriault [2] on bituminous sand, of Georgianou et al. [3] and Wood & Kumar [4] on clayed sands, of Thevanayagam [5] on silty sands, of Lee et al. [6] on micaceous sands, of Kim & Santamarina [7] on sand-rubber. Within the same philosophy, comparison of theoretic results and experimental data of permeability tests was carried out [8], [9] in order to identify up to which extent different homogenization schemes are able to reasonably describe the mixture hydraulic conductivity.

The paper is divided into four sections. First, empirical formulas of permeability of pure sand, clay, and sand-clay mixtures are reminded in order to get on to fundamental dependencies of permeability. Then the basic principles considered in purpose of developing this study are given. The third section is focused on the experimental results and their physical interpretations. Finally, by using classical homogenization approaches in association with simplifying assumptions, the basic modeling of the mixture permeability are presented and discussed.

2. Empirical formulas of permeability

Formulas of permeability of mixed materials are significantly less developed than those devoted to the behavior of pure sands or clays. In Polish hydrogeological practice, permeability of granular soils are usually appointed by using well known empirical formulas, especially equation of Hazen, Slichter, Krüger, Pałagin, Seelheim, Amer, USBSC and Beyer table which are based on soil granulometry or/and porosity (see Table 1). Such way of proceeding ensued from Białas and Kleczkowski studies (1970), who showed very good conformance between permeability determined by using some of equations mentioned above, and the results of trial pumping. Liszkowska [10] discussed casual character of this coincidence. She paid attention to the fact that permeability of a porous medium depends on a number of factors, mostly related to the pores architecture. She reminded also one of the most popular equations used for calculating permeability of a random porous medium e.g. the Kozeny-Carman relation:

$$k = C_{KC}\,\frac{g}{\eta\rho_w}\frac{e^3}{S^2 D_R^2(1+e)},$$

where e is porosity index, S is specific surface [m2/kg], C_{KC} is a coefficient of pore turtosity, D_R is specific bulk of solid phase ($D_R = \rho$s/ρw), η is dynamic viscosity of water and g is gravity.

Table 1

Empirical formulas for sand permeability coefficient.
Wzory empiryczne opisujące współczynnik wodoprzepuszczalności piasku

Hazen (Biernatowski et al. [13], Kollis [14], Pazdro [15], Pisarczyk & Rymsza [16])	$k = C_H d_{10}^2$,(m/s) with $C_H \in (0,005; 0,014)$, $C_H = 400 + 40(n - 26)$	sand with d_{10}=0,1-3,0 mm and U⩽5
Krüger (Biernatowski et al. [13], Kollis [14], Pazdro [15], Pisarczyk & Rymsza [16], BN-76/8950-30 [17])	$k = 13,5\frac{n}{S^2}$,(m/s,) with S – summarized surface of soil particle in 1 cm^3 of soil sample	sand with d_{10}=0,06-0,28 mm and n=0,32-0,47
Beyer (BN-76/8950-30 [17])	Table of Beyer	sand with d_{10}=0,06-0,6 mm and U=1-20
Slichter (Biernatowski et al. [13], Kollis [14], Pazdro [15], Pisarczyk & Rymsza [16])	$k = 7,8n^{3,26}d_{10}^2$,(cm/s)	sand with d_{10}=0,01-5,0 mm
Pałagin (Pisarczyk & Rymsza [16])	$k = \frac{39,7d_{50}^2 n}{U^{0,523}}$,(m/24h) with d_{50} average grain size in cm	sand and gravel with d_{50}=0,016-14 cm and 1,2<U<19
Amer [18]	$k = 3,5 \cdot 10^{-4}\left(\frac{e^3}{e+1}\right) \cdot U^{0,6}d_{10}^{2,32}$,(cm/s)	non cohesive soil
Seelheim (Biernatowski et al. [13], Kollis [14], Pazdro [15], Pisarczyk & Rymsza [16])	$k = 325d_{50}^2$,(m/24h)	sand with C<5
USBSC	$k = 0,36d_{20}^{0,67}$,(cm/s)	sand with d_{20}=0,01-2 mm

n – porosity of soil

Basing on the latter formula, Chapuis & Albertin [11] presented a new one in which utility for geo-engineering practice was enlarged to majority of soil, also for sand-clay blends:

$$\log k = 0,5 + \log\left[\frac{e^3}{S^2 D_R^2(1+e)}\right]$$

It is also worth previously cited studies of Tkaczukowa [12] who modified Hazen equation to use it for permeability prediction of clayey sand, sandy silt, and sandy clay with content a of particle with diameter d<0,001 mm in an interval a between 2% and 20%:

$$k = \frac{0,0093}{a^2}d_{10}^2, \text{(m/s)}$$

where d_{10} is an effective grain size.

However, the empirical formulas for permeability of sand-clay blends generally base, as well as formulas of clay, on a reference state (where original coefficient of permeability k and porosity index e are known), and show permeability evolution with variation of porosity index in the semi-logarithm or power form, see Table 2 and Table 3.

Table 2

Empirical formulas for clay permeability coefficient.
Wzory empiryczne opisujące współczynnik wodoprzepuszczalności iłu

Tavenas [19]	$\log k = \log k_0 - \frac{e_0 - e}{0,5e_0}$,(m/s)	
Samarasinghe et al. [20]	$k = k_0 \frac{e^b}{1+e}$,with b=4-5, (m/s)	normal consolidated clay
Nagaraj et al. [21]	$\log k = 5.128 \frac{e}{e_0} - 11.087$,(cm/s)	normal consolidated clay
Mesri et Olson [22], Tabaa et Wood [23], Hamidon [24]	$\log k = A \log e + B$,(m/s)	Kaolin

Table 3

Empirical formulas for clay-sand mixtures permeability coefficient.
Wzory empiryczne opisujące współczynnik wodoprzepuszczalności mieszanek ilasto-piaszczystych

Kumar [25]	$k = 4.66 e_c^{3.18} 10^{-10}$,(m/s)	mixture of kaolin and sand with kaolin content > 40%
Kenney [26]	$k = 0.5k_B$,(cm/s) with k_B permeability of bentonite	compacted mixture of bentonite and sand
Chapuis [27]	$\log k = 20\,(n^* - 0,45)$, (cm/s) with $n^* = n \cdot (S_{r100\%}) - 2\frac{V_B}{V_{dry}}$,	compacted mixture of bentonite and sand
Chapuis [27]	$\log k = 20\,(n^* - 0,52)$, (cm/s) with $n^* = n \cdot (S_{r,\%sand}) + \frac{m_B}{m_{dry}} - 2\frac{V_B}{V_{dry}}$,	compacted mixture of bentonite and sand

n – porosity of soil

3. Guidelines of the study

This study is devoted to saturated mixtures made of the three following constituents:

- Hostun's sand grains (G), having a density ρ_g equal to 2650 kg/m^3, and quasi-uniform size distribution with D_{50} close to 0.5 mm. The minimum and maximum void ratio of this sand are respectively of e_{min} = 0.65; e_{max} = 1.04.
- Kaolin (C), having a density ρ_c equal to 2650 kg/m^3 and typical size of platelets equal to 5 μm. The water content at the liquid and plastic limits is equal to w_L = 0.51 and w_P = 0.24, respectively.
- Water (W) having a density ρ_w equal to 1000 kg/m^3. Note that the use of kaolin avoids or greatly minimizes any phenomena of swelling induced by water, whatever is its ionic concentration.

The mixtures are macroscopically homogeneous, and the characteristic size of their Representative Elementary Volume (REV) is much smaller than the size of the samples tested for this study.

3.1. Triangular representation of saturated mixtures

For saturated mixtures, the volume ratio of grains G, clay platelets C, and water W, defined by:

$$C = \frac{V_C}{V_C + V_G + V_W}, \quad G = \frac{V_G}{V_C + V_G + V_W}, \quad W = \frac{V_W}{V_C + V_G + V_W},$$

are related by $C + G + W = 1$, and the water volume ratio W is the usual porosity n.

Conveniently any mixture may be represented by point **M** in an equilateral triangle {**C,G,W**} whose summits **C, G, W** correspond respectively to $C = 1$, $G = 1$, $W = 1$, and the opposite basis, e.g., **GW** corresponds to $C = 0$, (and **CW** to $G = 0$; **CG** to $W = 0$). The three volume ratios of the mixture represented by **M** are provided by the three orthogonal projections of **M** on the three altitudes (Fig. 1 – left). The clay content ratio R_c that determines the grains size distribution of the mixture (either in terms of mass or volume, since the grains and platelets specific gravities are identical) is defined by :

$$R_C = \frac{C}{C + G}$$

Graphically, R_c is obtained by the intersection of the line **WM** with the base **CG**. (Fig. 1 right). Note that all mixtures located on the line **WM** have the same clay content ratio, i.e. the same constitution of solid particles (and therefore same grain size distribution). For this reason, during its oedometric consolidation, a mixture necessarily evolves on the line **WM**. The void ratio e of the saturated mixture is given by:

$$e = \frac{W}{C + G} = \frac{W}{1 - W}$$

It is obtained in the triangular graph – with a non-linear scale – by the projection of **M** on the altitude issued from the vertex **W**, perpendicular to **CG**. Thus, all the mixtures located on a line parallel to **CG** have identical void ratio e, or also the same porosity $n = W$.

3.1.1 The two control parameters e_c and e_g

Because of the great difference of size between sand grains and kaolin platelets, such mixtures can be considered as bi-composite materials made of sand grains embedded in a matrix of saturated clay paste (i.e. kaolin and water). These two components present highly contrasted properties, since the grains are rigid and impervious while the clay paste is deformable and permeable.

The mechanical properties of the clay paste strongly depend of its water content w_c. The behavior varies from that of a quasi-liquid suspension for $w_c > w_L$ (undrained shear strength c_u typically less than 2 kPa, Wood [28]) to that of a plastic paste for

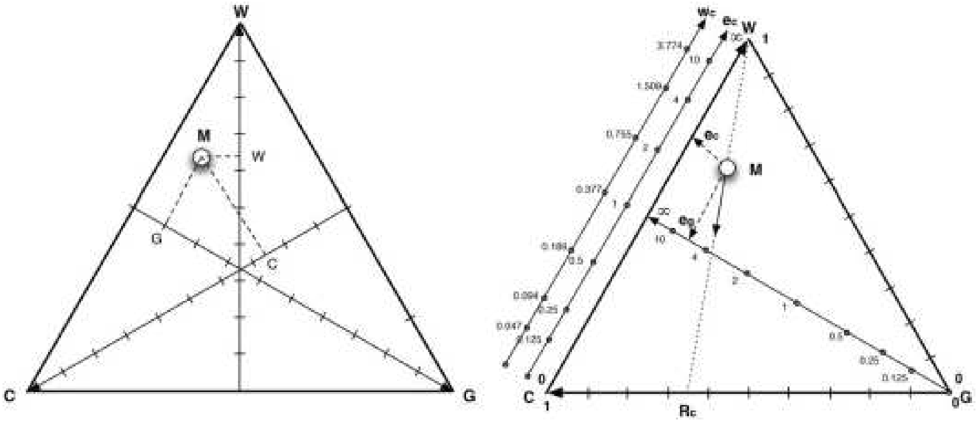

Fig. 1. Triangular representation of saturated sand clay mixtures. On the left, determination of the mixture composition by volume ratio parameters C, W, G. On the right, identification of the two control parameters e_c and e_g. The extended line **WM** corresponds to mixtures characterised by identical R_c and presents consolidation path in oedometer test.

Rys. 1. Przedstawienie mieszanek piasku i iłu na planie trójkąta (stan pełnego nasycenia). Po lewej, określenie budowy mieszanki poprzez parametry proporcji objętościowej C, W, G. Po prawej, określenie parametrów kontroli e_c i e_g. Punkty leżące na przedłużeniu linii **WM** odpowiadają mieszankom o identycznej wartości R_c i pokazują ścieżkę konsolidacji w badaniu edometrycznym

$w_L > w_c > w_P$ (undrained shear strength typically comprised between 2 and 200 kPa), and reaches a solid behavior for $w_c < w_P$ (c_u typically higher than 200 kPa). When the shrinking limit $w_s < w_P$ is reached, the arrangement of the clay platelets cannot adapt any more to the weak amount of water and the saturation is no more possible. In this state, the behavior of the material is progressively affected by other mechanism such as the suction by desaturation, and cracks start to develop.

Moreover, the density of the mixture meets the restrictions imposed by the sand skeleton. Sand grain arrangement in the mixture cannot be denser than the densest arrangement of pure sand alone (R_c=0), but oppositely, can be looser than the loosest arrangement of pure sand. Then, considering the paste to grain volume ratio, $e_g = (C+W)/G$, Mitchell [29]:

– necessarily $e_g > e_{\min}$
– when $e_{\max} > e_g > e_{\min}$, one may infer that the grains are in contact and constitute a skeleton
– the larger the grain volume ratio $e_g (> e_{\max})$, the lower the contacts between grains. Consequently, grains do not built a skeleton and the mixture turns to a clay paste matrix embedding dispersed grains.

These observations lead to use the two volumetric control parameters e_c and e_g for specifying the mechanical behavior of the mixtures:

• the clay paste void ratio defined by :

$$e_c = \frac{W}{C},$$

is related to the water content of the paste (defined in terms of mass ratio) by $e_c = (\rho_c/\rho_w)w_c = 2{,}65w_c$. The triangular graph e_c is obtained – with a non linear scale – by the intersection of the line **GM** with the base **CW** (Fig. 1 – right). Note that all mixtures located on the line **GW** have the same clay void ratio, i.e. the same clay behavior at the initial state.

- the granular "void" ratio related to the concentration in grains. By analogy with pure sand, the granular void ratio is the relation of volume between sand grains to the volume of grains in a unit volume of mixture. Its expression is:

$$e_g = \frac{C+W}{G} = \frac{1-G}{G},$$

As e_g is a function of G, its representation in the triangular graph lies on the same axis than G (i.e. the projection of **M** on the line perpendicular to the base **CW**, Fig. 1 right), but with a non linear scale (Fig. 1 left). Consequently, all the mixtures located on a line parallel to**CW** have identical granular void ratio.

3.2. Mixture composition and mechanical behavior

By crossing the characteristics of sand ($e_{\min}, e_{\max}$) and of clay paste (w_L, w_P, w_S) one may define different morphologies of the mixtures (Fig. 2). Three regions are defined by two lines parallel to **CW** corresponding to $e_g = e_{\min}$ and $e_g = e_{\max}$. Mixtures of $e_g < e_{\min}$ are impossible to realize, between the lines $e_g = e_{\min}$ and $e_g = e_{\max}$ they present a granular skeleton, and between $e_g = e_{\max}$ and **CW**, their morphology turns progressively to dispersed grains into a clayed matrix.

The lines **GL**, **GP**, **GS** corresponding to $w_c = w_L, w_P, w_S$ define four sectors. In between **GW** and **GL** the clay paste behaves as a quasi-liquid suspension, in between **GL** and **GP** the clay paste behaves as a plastic paste, in between **GP** and **GS** the clay paste is solid. Finally, between **GS** and **GC** it is impossible to saturate the clay, therefore the mixture. The shaded regions in Fig. 2 below e_{min} and below w_S indicate the impossible states of saturated sand-clay mixtures.

From this representation it is clear that, by modifying the amount of water, mixtures made of the same sandy clay material may vary from a quasi-liquid suspension of clay with dispersed sand grains (that may even be unstable because of the sand grains sedimentation) to a sand skeleton surrounded by a plastic paste. Similarly, at the same porosity (or void ratio), by changing the sand content, the mixture may vary from a clayed sand made of a solid clay matrix embedding sand grains, to a sandy clay having a grain skeleton surrounded by a plastic paste.

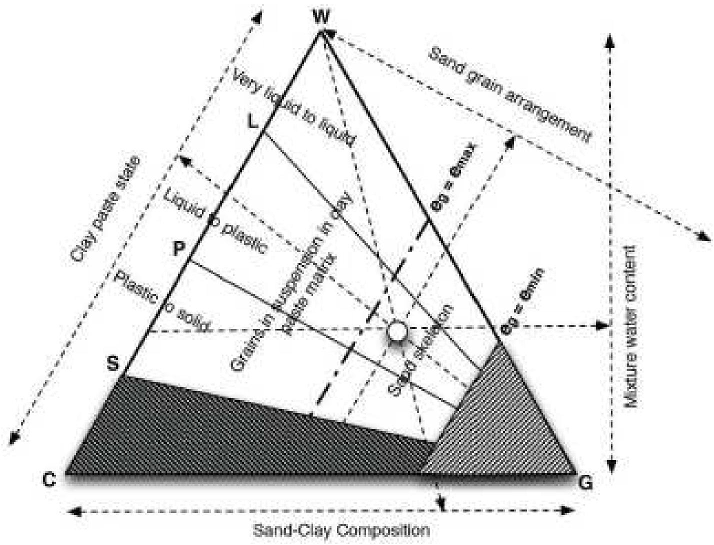

Fig. 2. Possible to achieve morphologies of saturated sand-clay mixtures.
Rys. 2. Możliwe do osiągnięcia morfologie mieszanek piasku i iłu w stanie całkowitego nasycenia

These two examples show that when varying independently the usual soil mechanics parameters e (or W) and R_c, both clay water content and sand content are modified. Thus, the links between the behaviors of the constituents and of the mixture would be difficult to identify. For this reason, in this study, the mixtures have been created in view of controlling the paste behavior – by w_c or e_c – and the sand content, by e_g.

3.3. Previous studies

While the content of fines is recognized as of significant importance in engineering practice, there is relatively few systematic studies on the response of clay-sand mixtures for different mixture density, water content and clay-sand ratio. Łuczak-Wilamowska [30] performed some laboratory tests on blends constituted of dune sand and miopliocen brown clay from Budy Mszczonowskie mine (Poland). An analysis of behavior of manufactured blends was done in function of different sand content (0, 20, 40, 60, 80 and 100%) with optimal water content different for each material. Fig. 3 shows that prepared materials characterize different clay water content and also different sand grains arrangement. Georgianou et al. [3] studied the response of Ham river sands merged in a very liquid speswhite kaolin slurry ($5w_L$), and found that the mixture behaves almost as a sand saturated by pure water. This result indicates that, for such a high level of clay paste liquidity, the suspensions of sand grains are unstable, so that mixtures necessarily present a skeleton of sand grains. Wood & Kumar [4] investigated mixtures made of a liquid kaolin clay ($1.5w_L$) and coarse sand. They evidenced two kinds of behavior : (i) behavior of stable suspensions of grains, mainly controlled by the clay paste, and (ii) behavior of skeletons of grains in contact filled by the clay paste, mainly controlled by the grains. However, the conclusions drawn in this two latter studies are limited to the case of clay paste with water content higher than the liquid limit, that is with a very low shear strength (see Fig. 3). As a consequence, the clay paste cannot carry shear forces and cannot consequently act as a bridge between the sand particles. The study mentioned before does not allow to link the behavior of the blend with that one of the components. So, the objective of the present study is to provide some insights into the behavior of clay-sand mixtures where the stiffness and the strength of the clay paste range from low values to values close to that of the sand skeleton.

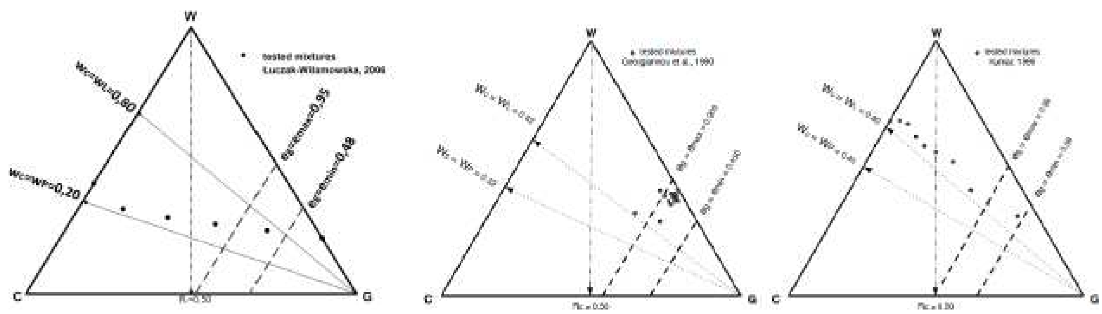

Fig. 3. Triangular representation of the mixtures tested by Łuczak-Wilamowska, Georgianou et al., Wood and Kumar to be compared with those of present study presented on Fig. 4.
Rys. 3. Mieszanki użyte w badaniach przez Łuczak-Wilamowska, Georgianou et al., Wood and Kumar, pokazane w celu porównania z mieszankami poddanymi badaniom przez autora (patrz Fig. 4)

3.4. Tested mixtures and experiments

More than twenty mixtures were realized [8]. Three initial water contents (w_{ci}) were selected to study mixtures made of paste at the liquid limit ($w_{ci} = w_L = 0.51$; $c_u = 2$kPa), at an intermediate water content ($w_{ci} = 0.35$; $c_u = 31$kPa), and close to the plastic limit ($w_{ci} = 0.30 > w_P = 0.24$; $c_u = 73$kPa), see Fig. 4. Values of e_g close to $e_g = 1 = e_{max}$, 1.5, 2, 3, 4, 6 and ∞ (no sand) were chosen to investigate mixtures where grains might be in contact or were dispersed in the paste in a dense or more dilute concentration. Detailed fabrication procedure of the saturated sand-clay mixtures and the testing program of permeability measurement are described in [8].

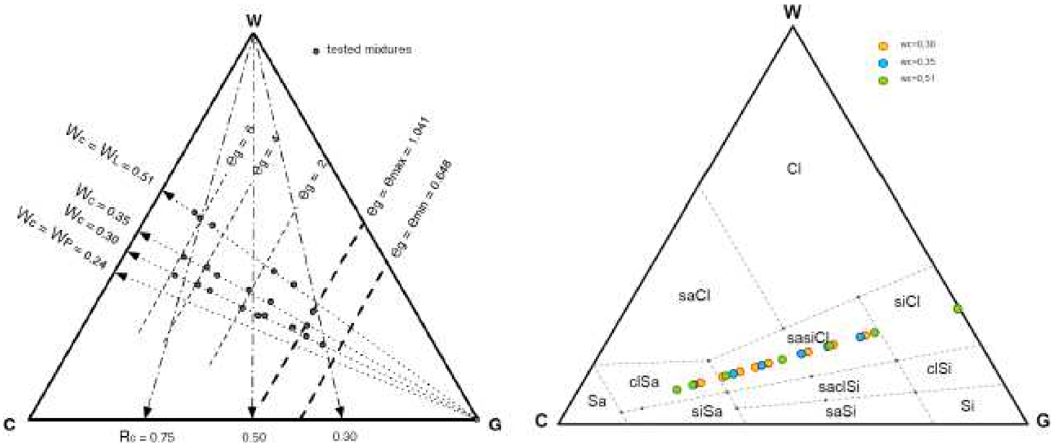

Fig. 4. Triangular representation of the tested sand clay mixtures.
Rys. 4. Mieszanki piasku i iłu poddane badaniom przez autora

4. Analysis and treatment of experimental data

4.1. Mean clay paste void ratio $\hat{e}_c$

At each loading stage, the effective volume variation of the saturated sand clay mixture is calculated by subtracting the instantaneous settlement (solely due to air bubbles compression) from the total settlement of the soil. This procedure gives the void ratio e of the saturated mixture:

$$e = \frac{W}{C+G},$$

hence the mixture compression curve e versus $Log(\sigma')$.

Now, the mean void ratio of the clay paste, denoted $\hat{e}_c$ to emphasize the fact that the clay paste may become non homogeneous during the test, can be obtained by dividing e by the clay content R_c, constant during each oedometric tests:

$$\hat{e}_c = \frac{W}{C+G}\frac{C+G}{C} = \frac{W}{C} = \frac{e}{R_c}.$$

Then, the apparent compression curve ($Log(\sigma'); \hat{e}_c$ of the clay paste within the mixture, can be deducted. The term "apparent" underlines that, in most cases, the effective stress σ' applied to the mixture is not the stress supported by the clay itself. The presented analysis of the behavior of the different mixtures bases only on the apparent compression curve of the clay paste. The discussion is organized by distinguishing the effect of the initial water content of the clay paste (from the liquid to the plastic limit) and the granular void ratio (from dilute to dense).

4.2. Analysis of the results concerning permeability

An analysis was performed in order to relate the permeability $K(e)$ (m/s) of mixture at a void ratio e, to the permeability$K_c(\hat{e}_c)$ of the pure clay paste at a void ratio $\hat{e}_c$ and the initial granular void ratio e_{g0}. Because of the small amount of air (around 10% as previously discussed), its influence on the water flow is considered negligible and the mixture permeability is assimilated to the soil permeability. Fig. 5 shows the variation of *Log(K)* with $\hat{e}_c$ for different e_{g0}. A linear trend is observed with a slope that decreases as e_{g0} increases, conforming to the physical intuition. For mixtures with dispersed grains ($e_g > 2e_{\max}$) or made of paste at the liquid state, *Log(K)* can be reasonably deducted from *Log(K_c)* by a translation factor depending on e_g only, following the expression:

$$K(e) = H(e_{g0}, e_{c0}).K_c(\hat{e}_c) \tag{4.1}$$

The mixture permeability can be decoupled into the product of the clay paste permeability by a pure geometrical factor related to grain concentration. For instance, considering mixtures prepared with paste at the liquid state:

$$K_c(\hat{e}_c) = 10^{-9}10^{2(1-\hat{e}_c)}1 = H(\infty) < H(e_{g0}, e_{cL}) < H(2e_{\max}) = 3$$

Mixtures with closer grains ($e_g < 2e_{\max}$) and less liquid paste are much less permeable, and the influence of $\hat{e}_c$ and e_g cannot be decoupled. In these cases, the experimental results may be summarized by the following empirical relationships:

(4.2)
$$Log(K(e)) = \phi'(e_{g0}, e_{c0}) + \beta'(e_{g0}, e_{c0})Log(K_c(\hat{e}_c)) or K(e) = \phi'(e_{g0}, e_{c0})[K_c(\hat{e}_c)]^{\beta'(e_{g0}, e_{c0})}$$

with $1 < \beta' < 3$. The strong reduction of permeability and its interdependence with the grain concentration, argue in favor of a structuring process previously mentioned. Indeed, the significant decrease in permeability can be interpreted as being caused by the presence of a network of denser (and thus less permeable) zones within the clay paste.

5. Basic modeling in the framework of homogenization

Phenomena in heterogeneous media can be upscaled and formulated in terms of macroscopic behavior, provided the condition of a scale separation is fulfilled. This latter requires that the medium morphology is sufficiently regular as described by a Representative Elementary Volume (REV), and that the characteristic size of the phenomena is much larger than the REV size [31]. These two conditions are satisfied in the present work. For linear phenomena, the homogenization of periodic media [32] – based on a

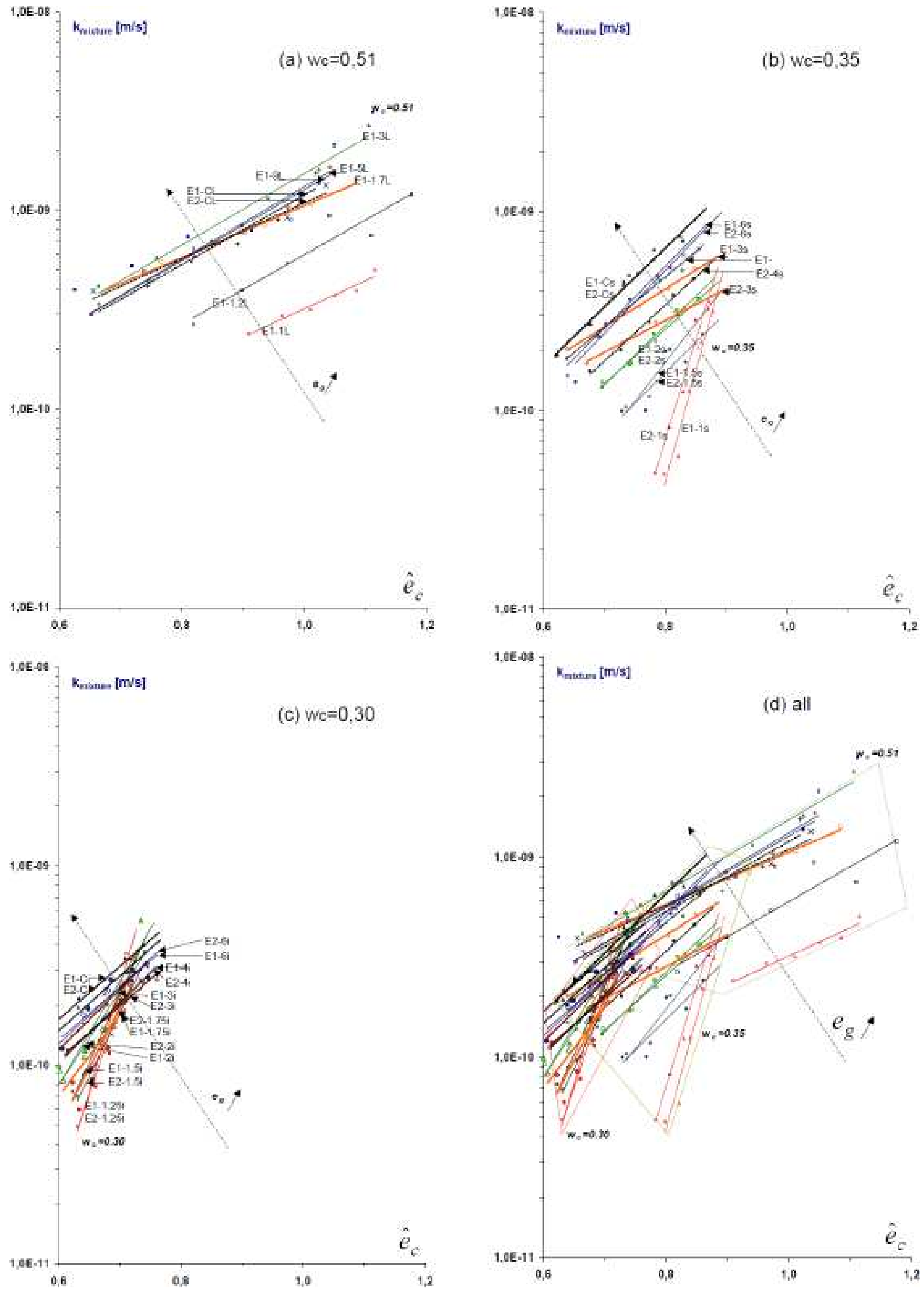

Fig. 5. Permeability coefficient of the mixtures versus $\hat{e}_c$ parametrized by e_g. Mixtures initially made of clay paste at the liquid state (a), at intermediate state (b), at quasi-plastic state (c). The data of all samples are plotted together on (d).

Rys. 5. Współczynnik wodoprzepuszczalności mieszanek w funkcji $\hat{e}_c$ sparametryzowany przez e_g. Mieszanki z pastą ilastą na etapie produkcji w stanie płynnym (a), w stanie przejściowym (b), w stanie quasi-plastycznym (c). Wszystkie wyniki zestawiono na jednym planie (d) two scale asymptotic expansions of both physical variables and differential operators – provides a rigorous theoretical framework to justify the existence of the macroscopic behavior and to determine its features and limits of validity.

Non-linear behavior significantly increases the theoretical difficulties. The heterogeneities generally imply an inhomogeneous stress distribution in the REV that in turn modifies the local properties of the constituents within the REV because of their non linear behavior. Thus, the derivation of the macroscopic description requires an incremental procedure. A new step of calculation requires the construction of a new REV with properties determined by the previous steps and accounting for the heterogeneous state of the clay paste (for example in poroplastic soil mechanics, see [33]). Note also that, conversely to the linear case, there is at the moment no proof of the uniqueness of the local solution in the REV.

In order to reduce difficulties of the rigorous procedure, different simplifying assumptions are introduced in the homogenization procedure, for instance, to cite just a few contributions, by [34], [35], [36], [37], [38]. However, it appears that the resulting macroscopic descriptions, mostly established in the context of plasticity without volume variations, are quite sensitive to the adopted simplifying assumption [39].

This section aims at identifying up to which extent different homogenization schemes are able to reasonably describe the permeability of the mixtures. A corollary aim is also to point out the domain of interest and the limitations of such approaches by using the experiments described in this work for qualitative and quantitative comparisons. Two ways of reducing the difficulty, without missing the conceptual tools of the homogenization methods, will be successively considered.

The first way consists in reducing the real 3D geometry to a simplified 1D geometry, i.e. in comparing the real mixture to a stratified mixture.

The second way is to linearize the problem at each state and to investigate the tangent properties. At the expense of assumption of local homogeneous state, the usual homogenization process can then be applied to determine the incremental tangent macroscopic properties.

In this context, no theoretical derivation of the uniqueness of the local solution will be looked for but one will rely on the fairly good experimental repeatability to assume its existence.

Notice that this assumption is necessary to justify the search of a macroscopic description.

5.1. Permeability of stratified mixtures

The simplest approach consists in idealizing the mixture by a 1D stratified media made of layers of compressible and permeable clay paste, alternated with rigid and impervious layers corresponding to the sand in a proportion identical to the grain concentration G, see Fig. 6-left. This morphology imposes the homogeneity of the paste, i.e. $\hat{e}_c = e_c$, and enables a rigorous derivation of the macro-behavior, including non-linearity and large deformation, without any additional assumptions.

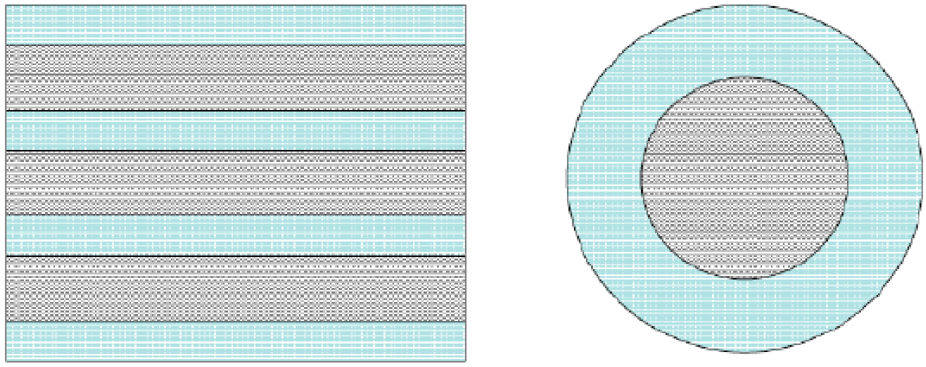

Fig. 6. Simplified REV morphology. Left: idealized stratified mixtures studied in section (5.1). Right: Bi-composite sand coated clay sphere considered for the self consistent estimates of the tangent homogenized parameters (Section 5.2.3). In both cases, the granular content G is the same to that of the real mixture.

Rys. 6. Uproszczona morfologia Reprezentatywnej Objętości Elementarnej (REV). Po lewej: wyidealizowane mieszanki o budowie warstwowej analizowane w sekcji (5.1). Po prawej: kompozyt dwuelementarny piasku otoczonego sferą iłu użyty w procedurze homogenizacji stycznej (sekcja 5.2.3). W obu przypadkach, zawartość frakcji piaszczystej G została przyjęta na poziomie odpowiadającym wartościom rzeczywistym z przygotowanych mieszanek

Within the stratified mixture, the "sand layer" being impervious, the local flux takes place in the clay paste and in the direction of the layers only. The mean flux is then reduced by a factor in proportion to the clay paste content, and the permeability in the direction of the layers reads :

$$K_m(e) = K_c(e_c).(1 - G) = K_c(e_c).\frac{e_g}{1 + e_g} \tag{5.1}$$

Compared to the clay paste permeability, the effective permeability is reduced by a factor in proportion to the grain content.

5.2. Modeling of permeability by tangent homogenization and self-consistent estimates

5.2.1 Tangent homogenization and simplifying assumptions

In order to reduce the complexity of the up-scaling problem, instead of determining the whole behavior, it is to focus on the tangent behavior at any state. To proceed with the tangent homogenization, one considers a fictitious REV made of elastic linear materials whose properties match the tangent properties of the real material in the

REV. The homogenization applied on the fictitious REV enables the derivation of the tangent macroscopic properties of the real media. Despite the linearization, the implementation of this method remains complex for two reasons.

A "technical" difficulty results from the general anisotropy of the tangent behavior on any point of the REV. As a matter of fact, if the non linear constitutive behavior is expressed by:

$$\sigma = \mathbf{L}(\mathbf{E}) : \mathbf{E}$$

where $\mathbf{E}$ is the strain tensor, the tangent behavior reads :

$$d\sigma = \mathbf{L_t}(\mathbf{E}) : d\mathbf{E}$$

and, from the tensor representation theorem, [40], as $\mathbf{E}$ is generally anisotropic, the tangent tensor $\mathbf{L}_t(\mathbf{E})$ is also anisotropic. Then, the calculation of the macroscopic tangent properties has to be performed on a REV with inhomogeneous distribution of anisotropic elastic tensors of the more general form.

The fundamental difficulty lies in the required knowledge of the state of the non linear material within the REV to determine the tangent properties at a given loading step. Since the current state results from the previous loadings, it is still necessary to solve and integrate the incremental homogenization procedure from the initial state up to the considered loading. These remarks lead to introduce the two following simplifying assumptions.

Firstly, it is assumed that the non-linear behavior depends only on the isotropic invariant of $\mathbf{E}$, i.e. on the volume variation. Consequently, the tangent behavior can be taken as isotropic. In this case, the tangent macroscopic properties could be derived from calculations on the REV with heterogeneous isotropic elastic media. However, the problem remains quite complex because of the necessity to specify the local tangent elastic tensor within the REV, which needs the resolution of the previous loadings steps.

Secondly, in order to overcome the question of the determination of the current state, a homogeneous state within the clay paste is considered, i.e. $\hat{e}_c = e_c$. Then, without solving the previous loading steps, the macroscopic tangent behavior can be estimated from the usual homogenization of a simple bi-composite.

As the permeability of the clay paste depends on its void ratio, the knowledge of the current state is a key point for determining the effective permeability.

5.2.2 General form of the effective tangent permeability

The homogenization method applied to bi-composites made of rigid impervious inclusions and to a homogeneous isotropic elastic and permeable matrix shows that the effective properties can be expressed in function of the parameters of the matrix and form factors that depends on the geometry of the constituents within the REV.

Consequently, the effective permeability of mixture K_{eff} is related to the permeability of clay paste K_c by a dimensionless form factor B function of the REV morphology:

$$K_{eff} = K_c B(REV)$$

In first approximation, it is reasonable to consider that the dependence on the REV morphology can be reduced to the dependence on the most basic geometrical information on the REV that is the volume ratio between the two constituents defined by G or e_g. Then, we are left with :

$$K_{eff} = K_c(\hat{e}_c)B(e_g) \tag{5.2}$$

The results of the tangent approach Eq. (5.2) and those established for stratified mixtures Eq. (5.1) are formally identical and may differ only by the form factor. They are also qualitatively valid, in accordance with the empirical equations Eq. (4.1) for mixtures with sufficiently dispersed grains ($e_g > 2e_{\max}$). These similarities indicate that the assumption of constant e_c within the REV is acceptable for mixtures such as that $e_g > 2e_{\max}$.

Conversely, the discrepancy between Eq. (5.1) and the power law observed experimentally Eq. (4.2) indicates that the homogeneity of the clay paste within the REV is missed for $e_g < 2e_{\max}$, and both tangent homogenization and stratified modeling fail to describe the actual behavior.

When a qualitative agreement is observed (i.e. mixtures with $e_g > 2e_{\max}$), quantitative comparisons could be performed by determining numerically factor B on a chosen REV's morphology, with the periodic homogenization method. In absence of detailed knowledge about grain arrangement in the REV, an alternative is to assess the effective properties by a self consistent approach.

5.2.3. Self consistent estimates of tangent properties

The observation that the microstructure is essentially made of grains coated by the clay paste suggests to describe the 3D geometry of the REV by spherical perfectly rigid grains embedded in the (homogeneous) clay paste concentric shell of shear rigidity μ_c, the Poisson ratio ν_c , and permeability K_c, respecting the same volume proportion than the mixture, see Fig. 6-right. For such configuration, and additionally assuming the mixture isotropy, the form factor B can be assessed analytically from a self consistent approach as established by [41] for the effective conductivity.

The principle consists in (1) applying to the bi-composite sand-clay paste sphere the macroscopic homogeneous solicitation, (2) determining the local field that satisfies the continuity conditions and the equilibrium of the system, (3) expressing the energy consistency, i.e, the identity of the energies contained in the bi-composite sphere, and the same volume of the effective homogeneous media submitted to the same homogeneous solicitation. The reader will find the main steps of the resolution in the Appendix.

As for the effective permeability, the self-consistent estimate of $B(e_g)$ is also comparable to the form factor (1 - G) determined for stratified mixtures and reads:

$$B(e_g) = \frac{e_g}{\frac{3}{2} + e_g} = \frac{1 - G}{1 + G/2}$$

Depending on initial arrangement of sand grains (in contact or dispersed in the paste), value of e_{g0} is known, and changes form 1 to ∞ (no sand). Using the value of permeability of the pure clay paste (with initial water contents w_{ci}) K_c obtained experimentally, one can calculate the effective permeability of mixture K_{eff} (Eq. 5.2) and the permeability of 1D model K_m (Eq. 5.1). Then, experimental values of the permeability of the mixtures in the same state of clay paste ($\hat{e}_c$=const.) can be compared with theoretical estimations.

5.3. QUANTITATIVE COMPARISON WITH EXPERIMENTS

A comparison of the experimental results and the results derived by the stratified (K_m), and tangent homogenization (K_{eff}) models for the permeability are drawn in Fig. 7. From a quantitative point of view, a reasonable agreement is observed for granular void ratio above the threshold $2e_{\max}$. This leads to a conclusion that for such mixtures the assumption of inner homogeneity is acceptable, and that the influence of the geometrical distribution of grains exists but is not a dominating factor. However, this homogenization scheme does not give qualitative nor quantitative results in agreement with the experimental results obtained on mixture with $e_g < 2e_{\max}$. The progressive departure of the curves as e_g decreases confirms this remark and suggests the occurrence of structuring phenomena for $e_g < 2e_{\max}$ that influences permeability.

From a theoretical point of view, the structuring effect can be modeled provided the mixture is represented by a composite with inhomogeneous matrix (instead of a bi-composite). However, the necessity to know the local values of the permeability within the REV makes the problem much more complex practically.

6. CONCLUSION

The response of mixtures of sand and saturated kaolin paste was studied in the oedometer for different states of the clay paste (function of the initial water content) and different sand concentrations. Results were drawn as a function of two volumetric state parameters, the clay void ratio e_c and the granular void ratio e_g, instead of the overall void ratio parameter classically used in soil mechanics. The results were interpreted by considering the material as a composite made of a non-linear and permeable matrix (the kaolin paste) and rigid and impervious inclusions (the sand grains). The final

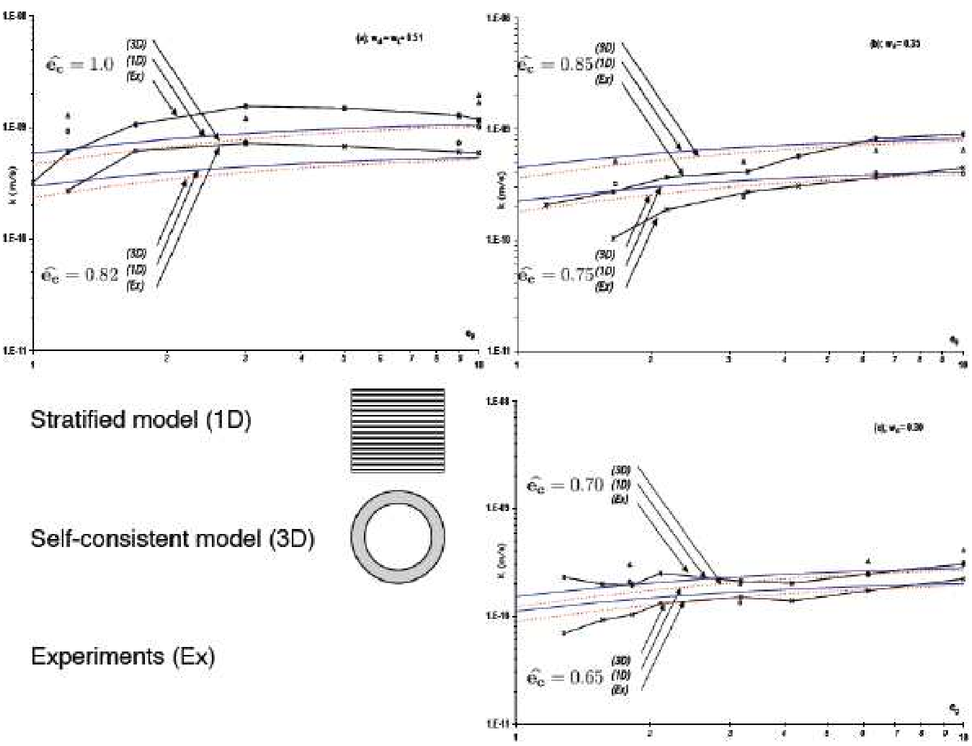

Fig. 7. Variation of the permeability coefficient with the granular void ratio: comparison between stratified and tangent homogenization expressions K_m, K_{eff} and experimental values, K.
Rys. 7. Zmienność współczynnika wodoprzepuszczalności w funkcji współczynnika porowatości frakcji piaskowej: porównanie wyników teoretycznych modelu warstwowego K_m i homogenizacji stycznej K_{eff} z wynikami doświadczalnymi K

objective of this study was to investigate the possibility to build by homogenization a macroscopic description related to strongly non-linear local behavior where volume variation is an essential feature.

Permeability test results highlight the key role of the deviatoric strength of the clay paste in the perturbations introduced by the grains. They also point out a threshold of granular void ratio of about $2e_{\max}$, below which both structuring and skeleton effects influence the permeability. Finally, the good repeatability of the results provide in some way an experimental proof of the uniqueness of the solution at the local scale (at least statistically), which justifies the search of a macroscopic description. Incidentally, the structuring effect explains the possibly higher inefficiency of the static uniaxial compaction with respect to the dynamic one. Indeed, the rotation of the principal stresses associated to the dynamic or rolling compaction imposes the stress paths to move, and thus avoid, the construction of stable hardening force chains that channel the stress flux. Location and orientation of the denser zones are constantly redistributed in the material and there is consequently no creation of stiffer material response in the preferential direction of the load. In the light of these experiments, the usual and tangent homogenization process (with strongly simplifying assumptions to make the problem manageable) were applied to estimate the mixture permeability. Qualitative and quantitative comparisons with experiments show that this approach reasonably succeeds to capture the macroscopic properties over the whole range of loading when the granular void ratio is larger than the threshold of $2e_{\max}$ but fails in other cases. For these latter, the structuring and/or skeleton effect plays a significant role and must be taken into account at the local scale.

7. APPENDIX

SELF CONSISTENT ESTIMATE OF THE PERMEABILITY

Similarly, when the bi-composite sphere is submitted to an imposed macroscopic pressure gradient **G**, the argument of spherical symmetry of the bi-composite pattern and isotropy of space imposes the pressure to be an isotropic function of a position vector originated at the center of the sphere **r** and **G,** and therefore :

$$p = \mathrm{P}(\mathbf{r}.\mathbf{r}, \mathbf{G}.\mathbf{G}, \mathbf{r}.\mathbf{G})$$

In the present case, the pressure depends linearly on **G,** and consequently:

$$p = \mathbf{r}.\mathbf{G}P(r)$$

Reporting this latter expression in the Laplace equation satisfied by p, one derives that in the clay shell $P(r) = \alpha.r + \beta.r^{-2}$. The coefficients α and β are obtained by expressing the vanishing flux at the sand-paste interface and the mean gradient condition : $\mathbf{G} = \int_{REV} \mathbf{grad}(p)dv$. Finally, the energy consistency provides the effective permeability [41]:

$$K_{eff}(e) = K_c(\hat{e}_c)\frac{1-G}{1+G/2} = K_c(\hat{e}_c)\frac{e_g}{3/2+e_g}$$

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AN EXTENDED ABSTRACT

Soil materials made of sand and clay are widely encountered in civil engineering/geoengineering either as common natural soils on which structures are settled, or as reconstituted materials used for road embankment, earth dam or contamination barrier. Practice shows that such mixture characteristics highly depend on the nature of the constituents, especially on the mineralogical structure of the clay, on their relative proportions, as well as on the overall density and on the water content. However, if empirical rules are well established for design of geotechnical construction, the derivation of the sand-clay blends properties from that of the constituents remains an open question. Hence, authors try to better understand behaviour of sand-clay mixtures at the macroscale by relating them to the physics at the microscale what is typically an upscaling problem of homogenization.

Presented study deals with the permeability of composite blends constituted of rigid and impervious grains included in saturated clay paste of kaolin, considered as permeable and deformable. Permeability tests performed during standard oedometr tests (before each load step) highlight the key role of the original and actual state of the clay paste, and show the existence of a threshold of sand grain concentration above which a structuring effect influences its permeability. At the light of these experiments some usual homogenization methods (with simplifying assumptions to make the problem manageable) are considered to model the mixture permeability. Qualitative and quantitative comparisons with experimental data point out their respective domain of interest and limitations of such approaches.

Polish translation of an extended abstract

W inżynierii lądowej/geoinżynierii bardzo często spotykamy się z materiałami gruntowymi zbudowanymi z frakcji piaskowej i ilastej. Materiały te stanowią naturalne podłoże gruntowe dla posadawianych na nim budynków lub jako mieszanki rekonstytuowane używane są do budowy nasypów drogowych, tam, zapór jak również barier uszczelniających. Doświadczenie pokazuje, że cechy tych materiałów zależą od właściwości komponentów, szczególnie od mineralogii iłu, od wzajemnych proporcji składników, od stopnia zagęszczenia, jak również od wilgotności mieszanki. Mimo, że do projektowania konstrukcji geotechnicznych stosuje się proste i powszechnie znane zasady (równania empiryczne) bazujące na właściwościach składników, wielokrotnie stawia się pytanie czy jest to podejście właściwe. W związku z tym autorzy podjęli próbę lepszego zrozumienia właściwości mieszanek piasek-ił w skali makro poprzez powiązanie ich ze zjawiskami fizycznymi jakie mają miejsce w materiale w skali mikro. Takie podejście „skalowania" jest typowe dla homogenizacji.

Prezentowane badania dotyczą filtracji kompozytów/mieszanek zbudowanych ze sztywnych i szczelnych ziaren piasku zanurzonych w nasyconej wodą paście kaolinitu, rozważanej jako materiał podlegający filtracji i deformacji. Badania wodoprzepuszczalności wykonane w czasie typowych badań edometrycznych uwidaczniają ważną rolę początkowego i aktualnego (uzależnionego od stanu naprężenia) stanu matrycy ilastej, jak również pokazują istnienie wartości progowej zagęszczenia ziaren piasku, powyżej której zjawisko „strukturyzacji" matrycy wpływa istotnie na wodoprzepuszczalność mieszanki. W świetle przeprowadzonych badań, wyniki doświadczalne porównano w sposób jakościowy i ilościowy z oszacowaniami empirycznymi typowych modeli homogenizacji, co pozwoliło na wskazanie zakresu poprawnego działania użytych modeli matematycznych.

Remarks on the paper should be sent to the Editorial Office no later than March 30, 2011

Received February 15, 2010
revised version
December 17, 2010