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Setpoint weighted PID controller tuning for unstable system using heuristic algorithm

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Most of the real time chemical process loops are unstable in nature and designing a suitable controller for such systems are difficult than open loop stable processes. In this work, an attempt is made with a two degree of freedom setpoint weighted PID controller tuning procedure for a class of unstable systems using the recent heuristic algorithms such as Particle Swarm Optimization and Bacterial Foraging Optimization. The problem considered in this study is to aptly tune the controller in order to enhance the overall closed loop performance. A novel objective function proposed in this study is used to monitor the heuristic algorithms in order to get the optimal controller parameters like K_p, K_i, K_d, and α with minimized iteration number. The proposed method is validated with a simulation study and this helps to accomplish enhanced system performance such as smooth reference tracking, satisfactory disturbance rejection, and error minimization for a class of unstable systems.

Key words: setpoint weighted PID, unstable system, particle swarm optimization, bacterial foraging optimization, objective function

1. Introduction

Controller tuning is an essential preliminary procedure in almost all the industrial process control systems. Despite the significant developments in advanced process control schemes such as predictive control, internal model control, sliding mode control, etc., Proportional + Integral + Derivative (PID) controllers are still widely used in industrial control application because of their structural simplicity, reputation and easy implementation. The merits of PID controller are as follows: (i) obtainable in variety of structures such as, academic PID, series PID, parallel PID and IMC-PID [25], (ii) provides optimal and robust performance for stable, unstable and nonlinear processes, (iii) supports online/offline tuning and retuning based on the process performance requirement, (iv) advanced arrangement such as 2DOF and 3DOF is possible. Many researchers proposed PID tuning rules to control various stable and unstable systems by different

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schemes to enhance closed loop performance [18,23,35]. For stable systems, PID controller offers a viable result for both the reference tracking and disturbance rejection. However, for unstable systems, it can effectively work either for reference tracking or disturbance rejection. The proportional and derivative kick in the controller also results in large overshoot and large settling time.

In chemical process industries, processing units such as jacketed continuous stirred tank reactor (CSTR), biochemical reactor, polymerization reactor, etc., are to be operated at unstable operating region for economic and safety reasons. In the control literature, a plethora of PID and modified configuration PID controller tuning methods are elaborately examined for unstable systems.

The conventional controller tuning methods proposed by most of the researchers are model dependent. The tuning rule proposed for a particular process model (first order or second order unstable process model) will not offer a fitting response for other process models (higher order models, model with a positive or negative zero, model with a large delay time to process time constant ratio, etc.). Most of the classical PID tuning methods require numerical computations in order to get the best possible controller parameters. Due to these reasons, in recent years, heuristic algorithm based PID controller tuning is greatly attracted the researchers. From the recent literature, it is observed that the heuristic algorithm based optimization procedures have emerged as a powerful tool for finding the solutions for variety of control engineering problems. Most recent heuristic methods such as Particle Swarm Optimization (PSO) [11] and Bacterial Foraging Optimization (BFO) [2,12,13] are extensively addressed by the researchers to tune controllers for a class of process models. In case of unstable systems, the PID parameter tuning seems to be a difficult task and is limited by delay time to process time constant ratio ($theta/\tau$ ratio) [26]. Therefore heuristic approach requires a modified PID structure or a modified objective function.

The Setpoint Weighted PID (SWPID) is discussed by most of the researchers [3,4,24,26]. Recently Nelendran and Poobalan [20] proposed a PSO based SWPID tuning for a class of unstable process models. Based on the setpoint weighting parameter α, it is possible to obtain a variety of modified PID controller structures [3]. In the present work, we propose an optimally assigned PSO and BFO algorithm to discover the best possible PID controller parameters such as K_p, K_i, K_d and setpoint weighting parameter α for a class of unstable process models by maintaining the guaranteed accuracy. We also proposed a time domain related novel Objective Function (OF) in order to improve the correctness of the optimized controller parameters. The purpose of this paper is to present tuning formulae based on the PSO and BFO algorithm to tune a classical and modified structured PID controller for unstable processes to meet setpoint tracking and disturbance rejection specifications.

The remaining part of the paper is organized as follows: Section 2 presents the outline of the SWPID controller. A brief description of PSO, BFO, problem formulation and the OF based controller tuning is provided in section 3. Setpoint weighted PID tuning and the proposed objective function is discussed in Section 4. Section 5 discusses the simulated results on different process models. Section 6 provides conclusion of the present research work.

2. Setpoint weighted PID controller

In industries, PID controller is used to shape the steady state as well as the transient response of the process control system. In a closed loop control system, the controller $C(s)$ continuously corrects the value of $U(s)$ until the difference among reference input $R(s)$ and the process output $Y(s)$ is zero irrespective of the external disturbance signal $D(s)$. Figure 1 shows the structure of One Degree Of Freedom (1DOF) PID controller. For stable systems, this structure provides an excellent result for both the reference tracking and disturbance rejection operations. For unstable systems, it fails to provide a smooth reference tracking performance due to the occurrence of proportional and derivative kick [18]. In order to reduce these effects and also to improve the time response characteristics, it is essential to consider a 2DOF PID structure.

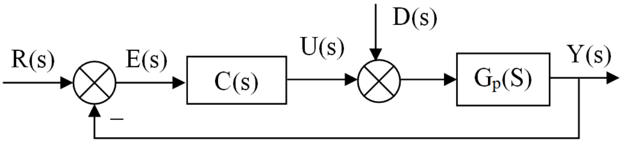


Figure 1. Structure of 1DOF PID control system.

In the proposed work, an attempt has been made with the setpoint weighted PID controller. Figure 2 depicts the SWPID controller widely considered by the researchers [19,20,23]. It has a 2DOF structure and the number of parameters to be tuned are K_p, T_i, T_d, α, and β.

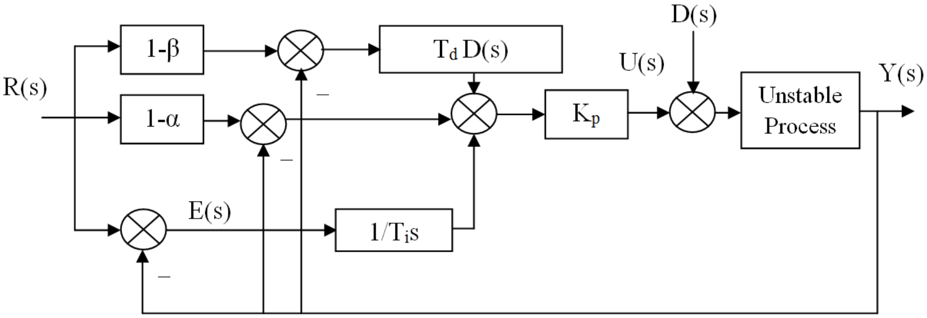


Figure 2. Structure of setpoint weighted PID control system.

The above 2DOF PID structure can be mathematically represented as follows [19]:

$$U(s) = U_1(s) - U_2(s) \tag{1}$$

$$U_1(s) = K_p\left\{(1-\alpha) + \frac{1}{T_i s} + (1-\beta)T_d D(s)\right\} \tag{2}$$

$$U_2(s) = K_p\{\alpha + \beta T_d D(s)\} \tag{3}$$

where: K_p is proportional gain, T_i is integral time constant, T_d is derivative time constant, α is setpoint weighting parameter for proportional controller, β is setpoint weighting parameter for derivative controller, $D(s) = s/(1+\tau_f s)$ is first order derivative filter, $K_p/T_i = K_i$, and $K_p \cdot T_d = K_d$.

In the proposed work, initially we considered the basic PID structure for heuristic algorithm based tuning. The modified PID structures such as PI-D, ID-P, I-PD, and PI-PD are formed by assigning appropriate values for parameters α and β as given in Tab. 1.

Table 10. Setpoint weighted PID and modified PID structures.

Setpoint weighting parameters		**Controller structure**
α	β	
0	0	PID
0	1	PI-D
1	0	ID-P
1	1	I-PD
$0<\alpha<1$	1	PI-PD

When $\alpha = \beta = 0$, the SWPID provides a PID structure (i.e. (2) will be acting and (3) will be eliminated). In this structure, the PID controller works based on the error signal $e(t)$. When $t = 0$, the $e(t)$ will be maximum, since the PID structure results in large overshoot because of proportional and derivative kick.

When $\alpha = 0$ and $\beta = 1$, the controller will be a PI-D structure. In this, PI part responds for $e(t)$ and D works on $y(t)$. In this structure proportional kick by the P is maximum and the kick by D is minimum (since, when $t = 0$, $e(t) = \max$, and $y(t) = 0$). The response of PI-D structure is similar to PID.

When $\alpha = 1$ and $\beta = 1$, SWPID forms an ID-P structure, which is free from proportional kick. The effect derivative kick by this structure is considerably small and it can provide a smooth reference tracking response compared to PID, PI-D.

When $\alpha = 1$ and $\beta = 1$, we can get an I-PD structure, which is free from proportional and derivative kick.

When $\alpha = 0$ and $\beta = 1$, we can construct the PI-PD structure. Where the PI part works based on (2) and PD part represents (3). From (2), it is observed that, the value of proportional gain in $U_1(t)$ is $K_p(1-\alpha)$. Since, in PI-PD controller, the effect of proportional kick is $1-\alpha$ times lesser than the PID controller. In this the PD part is available in feedback loop and $U_2(t)$ is free from proportional and derivative kick effect. Since the overshoot by PI-PD will be always lesser than PID, PI-D structures.

3. Brief overview of heuristic algorithm

3.1. Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) technique, developed by Kennedy and Eberhart [11], is a population based heuristic optimization technique developed due to the inspiration of the social activities in flock of birds and school of fish, and is widely applied in various engineering problems due to its high computational efficiency [8,14,17,18,21,28,32]. In PSO algorithm, the number of parameters to be assigned is very few compared to other nature inspired algorithms. In this, a group of artificial bird is initialized with arbitrary positions S_i and velocities V_i. At early searching stage, each bird in the swarm is scattered randomly throughout the D dimensional search space. With the supervision of the Objective Function (OF), own flying experience and their companions flying experience, each particle in the swarm dynamically adjust their flying position and velocity. During the optimization search, each particle remembers its best position attained so far (i.e. pbest – $(P^t_{i,D})$), and also obtains the global best position information achieved by any particle in the population (ie. gbest – $(G^t_{i,D})$). The search operation is mathematically described by the following equations

$$V^{t+1}_{i,D} = WV^t_{i,D} + C_1R_1(P^t_{i,D} - S^t_{i,D}) + C_2R_2(G^t_{i,D} - S^t_{i,D}) \tag{4}$$

$$W = W_{\max} - \frac{W_{\max} - W_{\min}}{iter_{\max}} iter \tag{5}$$

$$V^{t+1}_{i,D} = \Psi\left[V^t_{i,D} + C_1R_1(P^t_{i,D} - S^t_{i,D}) + C_2R_2(G^t_{i,D} - S^t_{i,D})\right] \tag{6}$$

$$\Psi = \frac{2}{\left|2-\phi-\sqrt{\phi^2-4\phi}\right|}, \quad \text{where} \quad \phi = C_1 + C_2, \ \ \phi > 4 \tag{7}$$

$$S^{t+1}_{i,D} = S^t_{i,D} + V^{t+1}_{i,D} \tag{8}$$

where: W – inertia weight, $V^t_{i,D}$ – current velocity of the particle, $S^t_{i,D}$ – current position of the particle, R_1, R_2 are the random numbers in the range $0-1$, C_1, C_2 are the cognitive and global learning rate respectively, $V^{t+1}_{i,D}$ – updated velocity, $S^{t+1}_{i,D}$ – updated position, $W_{\max}$ – maximum iteration number, $W_{\min}$ – minimum iteration number, $iter$ – current iteration, $iter_{\max}$ – maximum iteration, Ψ – constriction factor, and $i = 1,2,\ldots,N$, – particles.

Equation (4) represents the velocity update equation for the PSO algorithm. In this equation, the updated velocity depends on the inertia weight W. From (5), it is noted that, the inertia weight W requires additional parameters such as $W_{\min}$, $W_{\max}$, $iter$, and $iter_{\max}$. In the literature, there is no clear guide line to assign the value for these parameters. Due to the above reason, in this study, we considered (6) for velocity update [14]. The updated velocity depends mainly on the constriction factor 'Ψ', and its value can be easily assigned as in (7). Equation (8) shows the position update for the PSO algorithm, and it depends on the current position of the ith particle and the updated velocity of the ith particle in the D dimensional search space.

3.2. Bacterial Foraging Optimization (BFO)

Bacteria Foraging Optimization (BFO) algorithm is a new division of biologically inspired stochastic search technique based on mimicking the foraging (methods for locating, handling and ingesting food) behavior of Escherichia coli (E.coli) bacteria. This algorithm is developed by Kevin M. Passino [12, 13]. Due to the merits such as high computational efficiency, easy implementation and stable convergence, it is widely applied to solve a range of complex engineering optimization problems [1,27,29,34]. The basic operations of BFO algorithm have the following key steps.

Chemo-taxis: The initial stage of BFO search, which directs the bacteria towards the food source with the action of swimming and tumbling. Through swimming, it can move in a specified path and during tumbling action, the bacteria can modify the direction of search. These two operations are continuously executed by a bacteria its whole lifetime.

Swarming: After reaching the best food source, the bacteria which have the knowledge about the optimum path will shares the information with the other bacteria by using an attraction signal. The signal communication between cells in E.coli bacteria is represented mathematically as

$$\begin{aligned} &Jcc(\theta, P(j,k,l)) \\ &= \sum_{i=1}^{N}\left[-d_{att}\exp(-W_{att}\sum_{m=1}^{P}(\theta_m-\theta_m^i)^2)\right]+\sum_{i=1}^{N}\left[h_{rep}\exp(-W_{rep}\sum_{m=1}^{P}(\theta_m-\theta_m^i)^2)\right] \end{aligned} \tag{9}$$

here $Jcc(\theta, P(j,k,l))$ represents objective function value, N is the total number of bacterium, P the total parameters to be optimized. The other parameters such as d_{att} are the depth of attractant signal released by a bacteria and W_{att} is the width of attractant signal. The signals h_{rep} and W_{rep} are the height and width of repellent signals between bacterium.

Reproduction: In swarming process, the bacteria accumulated as groups in the positive nutrient gradient and which may increase the bacterial density. Later, the bacteria are sorted in descending order based on its health values. The bacteria which have the least health will die and the bacteria with the most health value will split into two and breed to maintain the predefined population.

Elimination-dispersal: This is the final stage in the bacterial search. The population of the bacterium may decrease either gradually or suddenly based on the environmental conditions such as change in temperature, noxious surroundings, and availability of food, etc. In this stage, a group of the bacteria gathered in a restricted region (local optima) will be eliminated or a group may be scattered (dispersed) into a new food location in the search space. The dispersal possibly flattens the chemo-taxis advancement. After dispersal, some bacteria may be located near the superior nutrient. The above procedures are repeated until the ’D’ dimensional search converges to optimal solutions or total number of iterations is reached.

4. Setpoint weighted PID controller tuning

4.1. Objective function

A generalized closed loop response of a system is shown in Fig. 3. The main objective of the controller is to make the rise time, peak overshoot, undershoot, settling time and final steady state error, as small as possible. In this work, we proposed a time domain based objective function (OF), which is to be minimized during the optimization search. This OF can be expressed as follows:

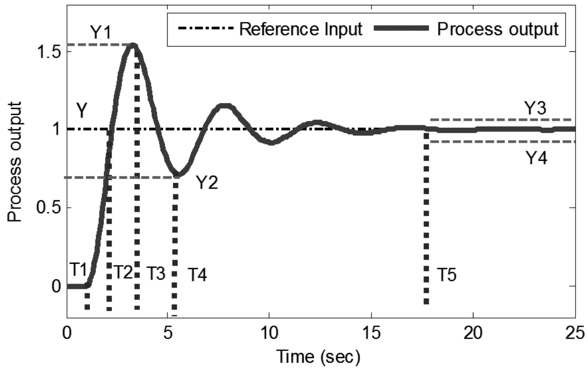


Figure 3. Closed loop response with controller.

$$OF_{\min} = P_1 \int_{T_1}^{T_2} Y + P_2 \int_{T_2}^{T_3} Y1 + P_3 \int_{T_3}^{T4} Y2 + P_4 \int_{T_4}^{T_5} Y^e + P_5 \int_{T5}^{T\max} Y_3 \tag{10}$$

where T_1 – process delay time, T_2 – rise time, T_3 – peak time, T_4 – time for undershoot, T_5 – settling time, Y – desired process output (reference signal), Y_1 – peak overshoot, Y_2 – peak undershoot, Y_e – tolerable error (ś5% of Y), Y_3 – maximum limit of tolerable Y, Y_4 – minimum limit of tolerable Y, P_1–P_5 – weighting factors for performance measures.

The maximum simulation time is fixed based on the delay time present in the process (i.e. T_1).

- If the delay time in the process model is less than 5 sec, the total simulation time $(T_{\max})$ for the heuristic algorithm based search is fixed as 100 sec.
- If the delay time in the process model is greater than 5 sec, the total simulation time $(T_{\max})$ for the search can be chosen as 500 sec.

Based on $T_{\max}$, other time domain values are assigned as follows: $T_2 = T_{\max}/10$, $T_3 = T_{\max}/6$, $T_4 = T_{\max}/4$, $T_5 = T_{\max}/2$, $Y_3 = +5\%$ of Y, and $Y_4 = -5\%$ of Y.

4.2. Algorithm parameters for optimization search

PSO parameters

In the literature, PSO tuned PID controller is widely addressed by the researchers. Gaing proposed a PSO based PID controller for a higher order stable system with a particle size of 50 [36]. Majid et al. discussed about the PID and H_∞ PID controller tuning for a class of stable process models with $N = 30$ [16,17]. Nelendran and Poobalan proposed a SWPID controller tuning fora class of unstable process models using the basic PSO algorithm with $N = 20$ [20]. A large swarm size (N) in the PSO based optimization search sometimes may provide a better solution. But increase in the agent size in heuristic algorithm will increase the iteration number. Eventhough the size of agent may be high, due to the local minima; it may provide a worst solution. In order to reduce the iteration number, always it is necessary to use optimal values for the heuristic algorithm parameters.

In the proposed work, the PSO parameters are assigned as follows: dimension of the search $(D) = 3$ (i.e. K_p, K_i, K_d), total number of swarm $(N) = 20$, number of swarm steps is equal 20, the cognitive learning rate (C_1) is equal to the global learning rate $(C_2) = 2.1$ (i.e. $C_1 + C_2 = \varphi > 4$), total number of iterations during the search is equal N multiplied by the number of swarm steps.

BFO parameters

The BFO based PID tuning for a class of stable system is widely discussed by Ali and Majhi [1], and Korani et al. [34]. Kim and Cho proposed a PID controller tuning for stable AVR system with $N = 10$ [6]. Kim proposed a hybrid algorithm to tune the PID controller for a stable system with a bacteria size of ten [7]. Recently Kanth and Latha proposed a PID controller tuning procedure for a class of process models with BFO algorithm [30]. They provided an empirical relation to assign the BFO parameters.

In this work, the BFO parameters are assigned as follows: Dimension of the search $(D) = 3$, total number of E.Coli bacteria $(N) = 20$, and the other algorithm parameters are assigned as follows [30]:

- The total number of E.Coli bacteria is equal $10 < N < 30$ (even numbers).
- The total number of chemotactic steps $(Nc) = N/2$.
- Swim length during the search (N_s) is equal to total number of reproduction steps $(N_{re}) \approx N/3$.
- The number of elimination - dispersal events $(N_{ed}) =\approx N/4$.
- The total number of bacterial reproduction $(N_r) = N/2$.
- The probability of the bacterial elimination/dispersal $(P_{ed}) = \left(\frac{N_{ed}}{N+N_r}\right)$.
- Total number of iterations during the search is equal N^2.
- Swarming parameters can be assigned as follows:

$$d_{attractant} = W_{attractant} = \frac{N_S}{N} \quad and \quad h_{repellant} = W_{repellent} = \frac{N_C}{N}.$$

4.3. Controller tuning

The controller tuning process is employed to find the best possible values for K_p, K_i and K_d. In order to achieve the superior accuracy during the optimization search, it is necessary to assign appropriate values for OF (refer equation (10)), which guides the optimization search. Prior to the optimization search, it is necessary to assign the parameters for PSO / BFO based search. In this study, the following values are assigned:

- Boundaries for the three dimensional search space is assigned as follows:
  Value 1 = 0% < $K_p < +50\%$ (ie. $0 < K_p < 5$)
  Value 2 = 0% < $K_i < +5\%$ (ie. $0 < K_i < 0.5$)
  Value 3 = 0% < $K_d < +100\%$ (ie. $0 < K_d < 10$).
- The weighting function values are assigned as $P_1 = P_2 = P_3 = P_4 = P_5 = 10$
- Maximum simulation time $(T_{\max})$ is selected based on T_1.
- The reference signal is considered as unity (i.e. $R(s) = 1$).
- For each process example, ten trials are carried out and the finest set of values among the trials is selected as the best optimized controller value.

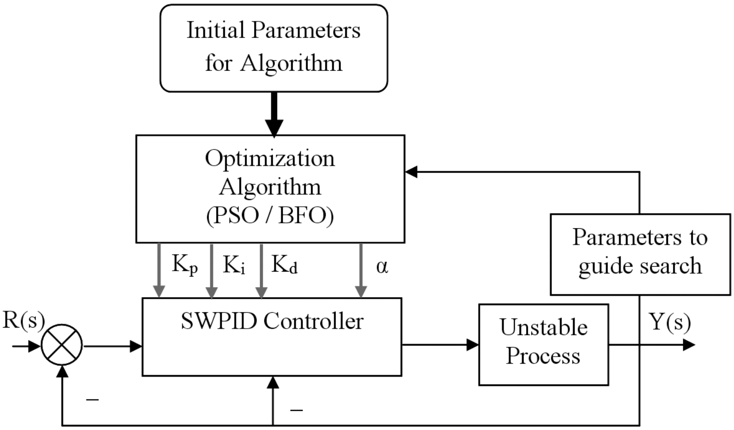


Figure 4. Heuristic algorithm based SWPID controller tuning.

4.4. Setpoint weighting parameter α retuning

Initially the PSO / BFO algorithm is considered to find the best possible values for the parameters K_p, K_i, and K_d. Then the simulation study is executed with various values for setpoint weighting parameters α, and β as given in Tab. 1, to form controller structures such as PI-D, ID-P, and I-PD. Later, a retuning operation for α is then performed by considering the rise time, overshoot and the settling time of the closed loop process as the reference. During the retuning operation, the controller parameters such as K_p, K_i, and K_d are replaced by its optimized numerical values and the setpoint weighting parameter β is assigned as 1 (i.e. $\beta = 1$). In the retuning operation, the number of parameter to be optimized is one (i.e. the dimension of the search $(D) = 1$). During the retuning, the value of α is adjusted until the considered performance criterion is minimized. Retuning of α helps to realize the PI-PD structure, which can improve the closed loop performance of the process. In retuning procedure, the following values are considered.

- Search boundary for the setpoint weighting parameter α is assigned as

$$0\% < \alpha < +10\% \quad (\text{i.e. } 0 < a'lpha < 1)).$$

- Calculate

$$OF_{\min} = P_1 \int_{T_1}^{T_2} Y + P_2 \int_{T_2}^{T_3} Y1 + P_4 \int_{T_4}^{T_5} Y^e \tag{11}$$

where

$\int_{T_1}^{T_2} Y = tr$ – rise time (time required for $y(t)$ to reach 100% of its reference input),

$\int_{T_2}^{T_3} Y1$ – overshoot,

$\int_{T_4}^{T_5} Y^e = ts$ – settling time, time required for $y(t)$ to reach an stay at $r(t)$ [ie. $y(t) = r(t)$],

- The t_r is preferred to be less then 15% of the maximum simulation time ($T_{\max}$).
- The overshoot (Y_1) range is selected as less then 10% of the reference signal.
- The t_s is preferred less then 50% of the maximum simulation time.

4.5. Performance measure

The performance of a closed loop system can be measured with the time domain parameters (rise time, overshoot, undershoot, settling time, and the final steady state error) or with the error values (such as ISE, IAE, ITSE, and ITAE). In this paper we measured the closed loop performance with rise time, overshoot, settling time, ISE, and IAE.

4.6. Steps in setpoint weighted PID controller tuning using PSO / BFO algorithm

Step 1. Initialize the algorithm parameters with appropriate values.

Step 2. Assign the search boundary and the number of iteration for the algorithm convergence.

Step 3. Consider a suitable objective function to guide the optimization search.

Step 4. Check for the best possible values for K_p, K_i, K_d.

Step 5. If best possible values are available, use it. Else, continue step 2 to 4.

Step 6. Change the setpoint weighting parameter values α and β as given in Tab. 1 and record the corresponding performance measures (error values).

Step 7. Without changing the values of K_p, K_i, K_d , retune α value with a new objective function and construct the PI-PD controller. Compare the performance of various controller structures.

5. Simulation results and discussions

To study the closed loop performance of the unstable process with PSO / BFO tuned controller, six examples are considered from the literature. The following simulation study demonstrates the competence of the proposed method.

Example 1 The first order unstable process with the following transfer function model is considered

$$G_p(s) = \frac{4e^{-2s}}{4s-1} \tag{12}$$

This process has θ/τ ratio of 0.5. Many studies have proposed different controller settings for the above process model. The PID controller parameters suggested by Huang and Chen (HC) [9]; Sree, Srinivas, and Chidambaram (SSC) [31]; Jung, Song, and Hyun (JSH) [10]; and the PD controller values by Visioli [33] are presented in Tab. 1. The PSO based SWPID controller tuning is initially proposed for the process (12) as depicted in Fig. 4 with $\alpha = \beta = 0$. The delay time in the process is 2 sec; hence the total simulation time ($T_{\max}$) for the heuristic algorithm search is taken as 100 sec. Other time domain values are selected based on the procedure discussed in section 4.1. The PSO based search converges at 68th iteration and the converged controller parameter (K_p, K_i, K_d) is shown in Fig. 5.

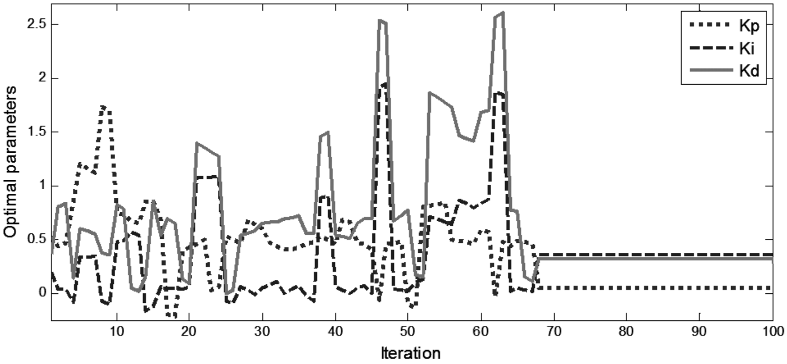
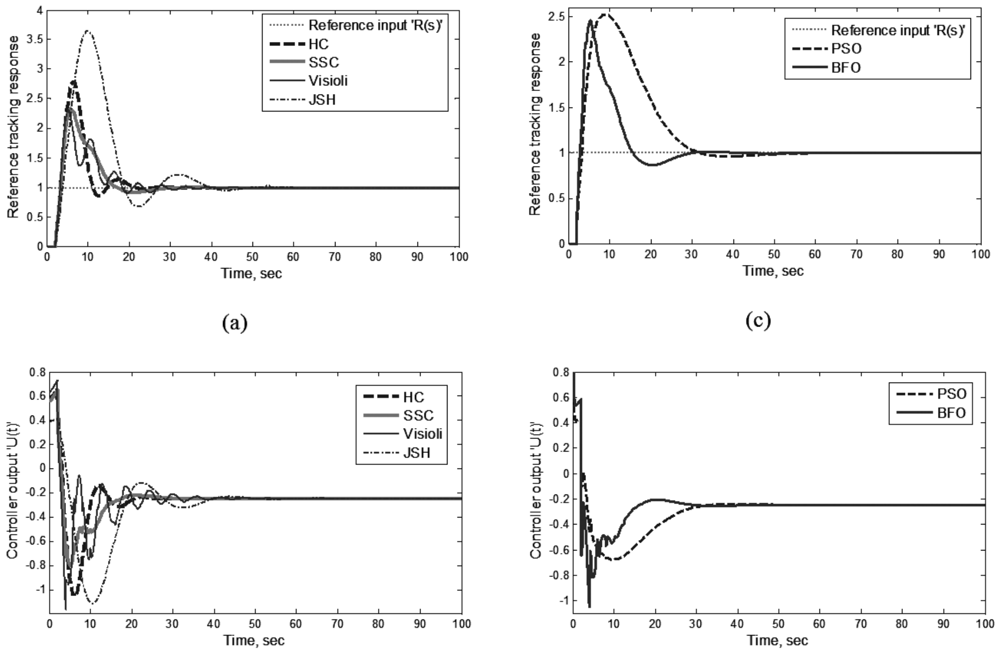


Figure 5. Convergence of control parameters.

The above procedure is repeated with the BFO algorithm and the search converges at 104th iteration. The heuristic algorithm tuned control parameters are presented in Tab. 2.

The closed loop response of the system with PID controller is shown in Fig. 6(a) and (c) for a reference input magnitude of 1 applied at 0sec. The PID controller by SSC and Visioli gives better time response characteristic compared to other methods considered in this study. From Fig. 6(b) it is observed that the controller output $U(t)$ is more oscillatory due to the effect of proportional and derivative kick. From Fig. 6(d) the observation made is, heuristic algorithm tuned PID provides a smooth controller output compared to other methods. The performance of the controller is assessed using t_s, M_p, t_r, ISE, and IAE and the values are given in Tab. 2. Fig 7 graphically represents the relative analysis of various tuning methods considered in this study (X axis represents 1- HC, 2-SSC, 3-Visioli, 4- JSH, 5-PSO, and 6-BFO). HC and SSC provide improved response compared



(a) (c)

(b) (d)

Figure 6. (a),(c) reference tracking response. (b),(d) controller output.

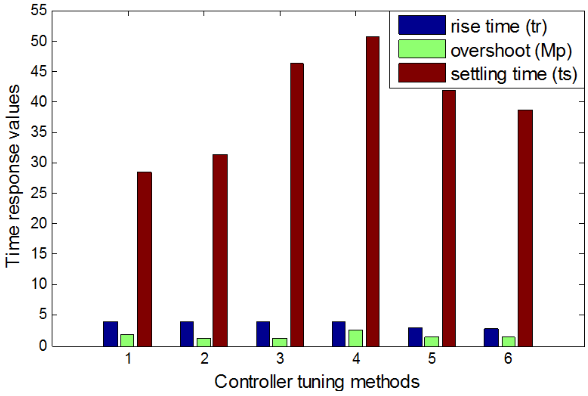
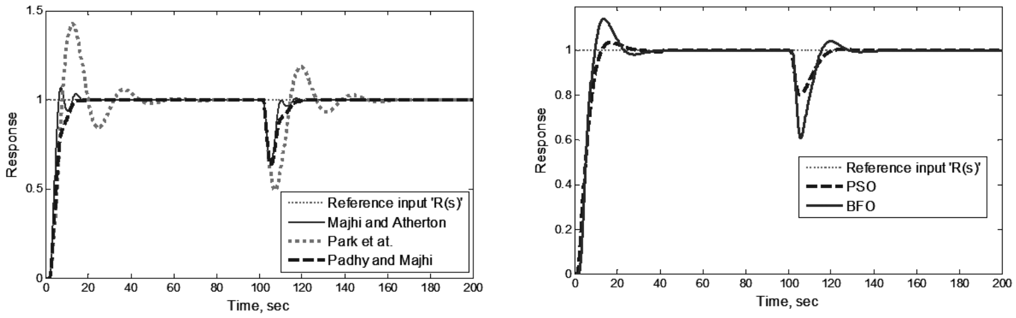


Figure 7. Performance measure for Example 1.

to other methods. The time response value by the PSO and BFO is better than Visioli and JSH. For this process, Park et al. designed a PID-P controller with the parameters

Table 11. Quantitative analysis for conventional and heuristic algorithm tuned PID controller.

Process	Method	Controller parameters			Time response			Error	
		K_p	K_i	K_d	t_r	M_p	t_s	ISE	IAE
Ex 1	HC	0.5650	0.0460	0.3435	3.913	1.782	28.42	29.54	5.435
	SSC	0.5480	0.0493	0.5611	3.889	1.328	31.30	25.71	5.071
	Visioli	0.6240	0.0540	0.7245	3.926	1.309	46.25	21.43	4.630
	JSH	0.3840	0.0127	0.0000	3.975	2.650	50.72	387.5	19.69
	PSO	0.4054	0.0136	0.3081	3.016	1.519	41.85	337.9	18.38
	BFO	0.4988	0.0428	0.4846	2.729	1.415	38.74	34.12	5.841
Ex 2	HC	1.7920	0.1442	0.8602	2.415	2.776	33.95	48.09	6.935
	PC	1.5860	0.1322	0.7597	2.403	2.567	36.77	57.22	7.654
	LLP	1.9490	0.1616	1.6099	2.164	1.675	21.52	38.29	6.188
	PSO	1.7397	0.1982	1.1557	1.821	2.395	16.24	25.46	5.045
	BFO	1.6872	0.2136	1.2206	1.824	2.325	23.50	21.92	4.682
Ex 3	HC	6.1860	0.8628	9.1058	2.931	0.855	24.51	1.343	1.159
	LLP	7.1440	1.0688	11.823	2.853	0.710	25.25	0.8754	0.9356
	PSO	4.2196	0.5419	9.9043	1.992	0.838	19.43	3.411	1.847
	BFO	3.7277	0.4139	7.5711	2.215	0.823	18.26	5.840	2.417



(a) (b)

Figure 8. Reference tracking and load disturbance response for Example 1.

$K_p = 0.068$, $T_d = 4.296$, $T_i = 1.885$, and $K_{p1} = 0.35$ [22]. The four parameters of the PI-PD controller suggested by Majhi and Atherton are $K_p = 0.131$, $T_d = 1$, $T_i = 2$, and

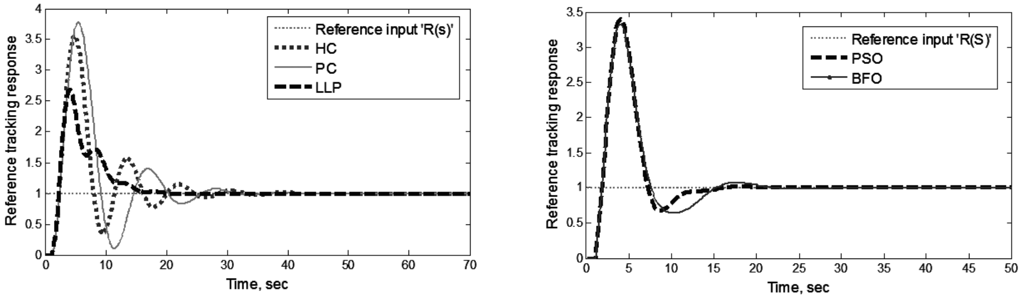
$K_{p1} = 0.5$ [22], and by Pathy and Majhi are $K_p = 0.4548$, $T_d = 0.1999$, $T_i = 0.4151$, and $K_{p1} = 2.229$ [22].

The closed loop performance of the controllers is shown in Fig. 8(a) for a reference input magnitude of 1 and a step load disturbance (d) of 0.1 applied at 0sec and 100sec respectively. The similar response by the heuristic algorithm tuned PI-PD controller with retuned setpoint weighting parameter α (PSO tuned $\alpha = 0.9025$, and BFO tuned $\alpha = 0.8719$) is depicted in Fig. 8(b). The PI-PD controller by Padhy and Majhi gives better time response characteristic compared to other methods considered. The overshoot by PSO and BFO tuned PI-PD controller is considerably small compared to Park et al. method.

Example 2 The second order delayed unstable process with the following transfer function is considered. It has one unstable pole and a stable pole.

$$G_p(s) = \frac{\exp^{-1s}}{(2s-1)(0.5s+1)}. \tag{13}$$

The classical PID parameters suggested by Huang and Chen (HC) [9]; Prashanti and Chidambaram (PC) [26]; Lee, Lee, and Park (LLP) [15] are considered in this study. The delay time in the process is 1 sec; hence the simulation time and the time domain values are selected as discussed in Example 1. PSO and BFO based PID tuning is proposed for the process model (13) with $\alpha = \beta = 0$ and the optimization search converges at 148th iteration with PSO and 163rd iteration with BFO algorithm. The controller values and its reference tracking performance for a unit step input are given in Tab. 2. Fig. 9(a) and (b) depicts the closed loop response of classical and heuristic algorithm tuned PID controller. The reference tracking response by PSO and BFO tuned controller is smooth compared to HC and PC. Fig. 10 represents the relative analysis of various tuning methods considered in this study (X axis represents 1- HC, 2-PC, 3-LLP, 4-PSO,



(a) (b)

Figure 9. Reference tracking response for Example 2.

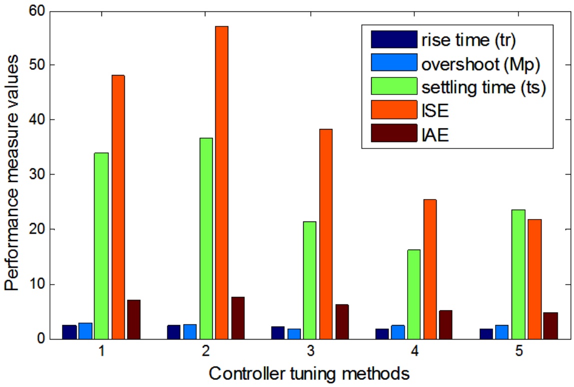


Figure 10. Performance measure for Example 2.

and 5-BFO). The overshoot by LLP is considerably low compared to other methods. The overall performance by heuristic algorithm tuned PID is better than the methods considered in Example 2.

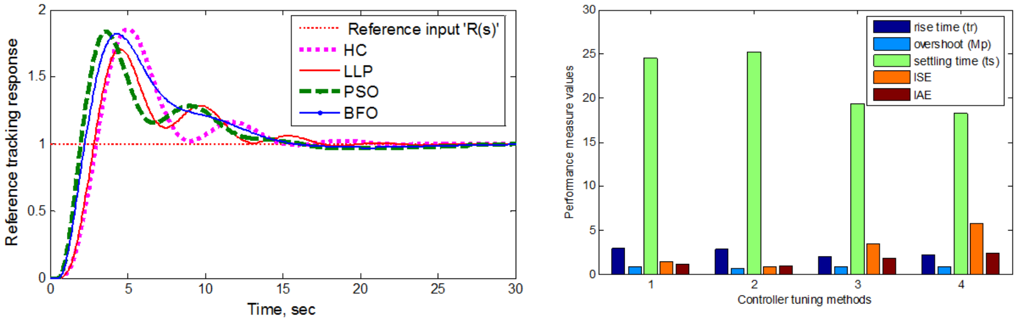
Example 3 The third order delayed unstable process with the following transfer function is considered. It has one unstable pole and two stable poles.

$$G_p(s) = \frac{\exp^{-0.5s}}{(5s-1)(0.5s+1)(2s+1)} \tag{14}$$

The classical PID parameters given by Huang and Chen (HC) [9]; and Lee, Lee, and Park (LLP) [15] are used in this study. Heuristic algorithm based PID tuning is proposed for the process with $\alpha = \beta = 0$ and the search converges at 88th iteration with PSO and 106th iteration with BFO algorithm. The controller values and its reference tracking performance for a unit step input are given in Tab. 2. Fig 11(a) depicts the reference tracking response of classical and heuristic algorithm tuned PID controller. The LLP method offers a reduced overshoot compared to HC, PSO, and BFO. Fig 11(b) shows the relative analysis between the tuning methods considered in this study (X axis represents 1- HC, 2-LLP, 3-PSO, and 4-BFO). Eventhough the iteration number is large, BFO offers improved overall performance compared to PSO and the classical methods.

Example 4 An isothermal continuous stirred tank reactor (CSTR) studied by Liou and Yu-Shu [5] is considered. The final steady state model of this process is represented by the following mathematical model [23]

$$\frac{dc}{dt} = \frac{\mathrm{nQ}}{\mathrm{mV}}(C_f - C) - \left[\frac{K_1 C}{(K_2 C + 1)^2}\right] \tag{15}$$



(a) (b)

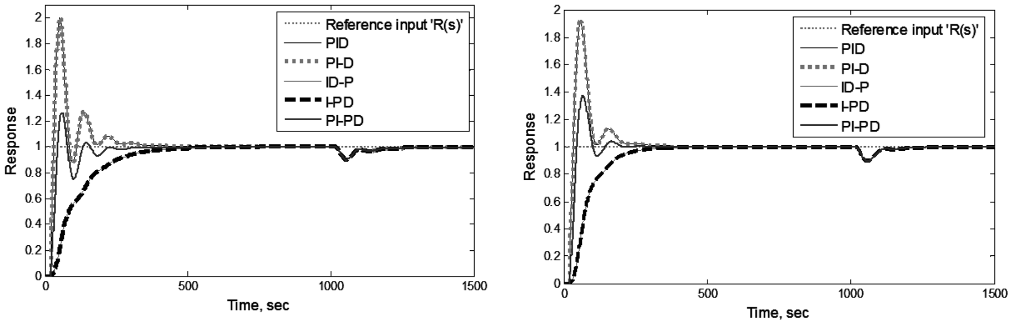
Figure 11. (a) Reference tracking response for Example 3; (b) Performance measure for Example 3.

The modeling parameters for the isothermal CSTR are: flow rate $(Q) = 0.03333$l/sec, volume $(V) = 1$l, $K_1 = 10$l/s, and $K_2 = 10$l/mol, and $n = m = 0.75$. Linearizing the nonlinear model equation around the operating region with, concentration $(C_f) = 3.288$mol/l, gives two stable steady states at $C = 1.7673$mol/l and $C = 0.01424$mol/l. When $C = 1.3065$mol/l, the CSTR provides an unstable steady state and the relation between the reactor concentration to feed concentration can be mathematically represented by the following unstable transfer function model with a measurement delay of 20 sec.

$$G_p(s) = \frac{\Delta C(s)}{\Delta C_f(s)} = \frac{3.3226\exp^{-20s}}{(99.69s-1)} \tag{16}$$

Equation (16) presents the first order unstable model and the controller setting for this model is proposed using heuristic algorithm as discussed earlier. The PSO based SWPID controller tuning is initially proposed for the process with $\alpha = \beta = 0$. The delay time in the process is 20 sec; hence the total simulation time $(T_{\max})$ for the heuristic algorithm search is taken as 500 sec. Other time domain values are assigned based on the procedure discussed in section 4.1. After finding the optimal value for K_p, K_i, and K_d, tuning of parameter α is attempted as discussed in section 4.4. The above procedure is repeated with the BFO algorithm and the optimized controller parameters are presented in Tab. 3.

The closed loop performance of the SWPID controllers is shown in Fig. 12 (a-PSO; b-BFO) for a reference input magnitude of 1 and a step load disturbance (d) of 0.1 applied at 0sec and 1000sec respectively. The response of the isothermal CSTR with PSO/ BFO tuned PID controller results in large overshoot due to the effect of proportional and derivative kick. The ID-P and I-PD structure gives a smooth response compared to other controller structures. However, the rise time, settling time, ISE, and IAE values are larger than other structures (Tab. 3 and Tab. 4). The PI-PD gives a lesser overshoot compared to PID, PI-D and improved ISE and IAE values than ID-P, I-PD structures. The overall performance by the BFO tuned SWPID is better than the



(a) (b)

Figure 12. Reference tracking and load disturbance response for isothermal CSTR.

PSO tuned controller. From Fig. 12 the observation is that, load disturbance rejection response for the classical and the modified structured PID controllers such as PI-D, ID-P, I-PD, and PI-PD are similar.

Example 5 Continuous stirred tank reactor (CSTR) with nonideal mixing discussed by Liou and Yu-Shu [5] has the following transfer function model [23]

$$G_p(s) = \frac{\Delta C_e(s)}{\Delta C_f(s)} = \frac{2.21(1+11.133s)\exp^{-20s}}{(98.3s-1)} \tag{17}$$

The process model has one unstable pole and a zero. The unstable system with a zero may produce a large overshoot or inverse response and such process is very difficult to control with a classical PID controller [23]. Recently, Kanth and Latha proposed a BFO tuned I-PD controller for the above process model [29].

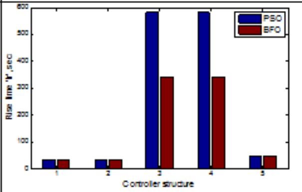
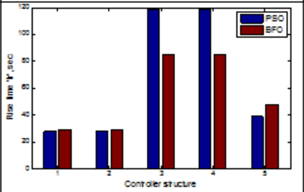
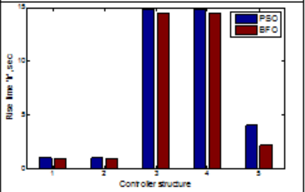
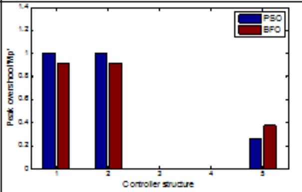
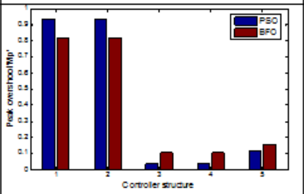
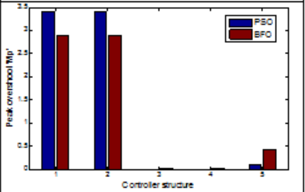
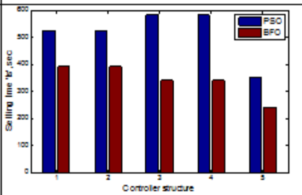
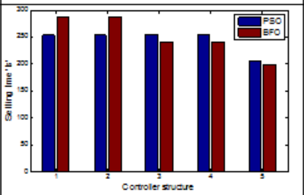
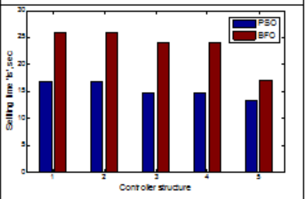
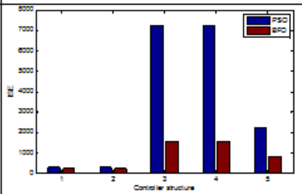
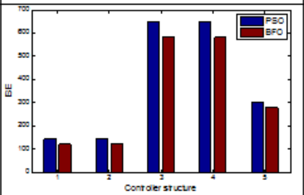
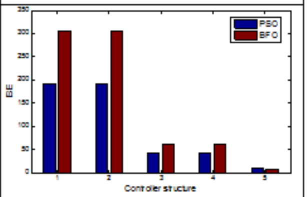
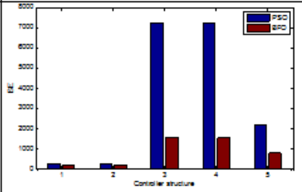
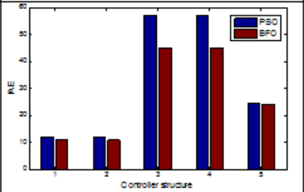
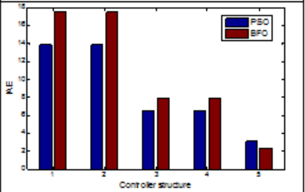
In the proposed work, we initially attempted the classical PID tuning with the PSO algorithm. The total simulation time $(T_{\max})$ for the heuristic algorithm search is taken as 500 sec. The optimization search with the proposed OF provides the result at 59th iteration. The BFO based search provided the optimal values at 65th iteration. From this result, it is observed that, the proposed OF can be used to find the optimized values for the PID controller with minimal number of iteration compared to the previous study in the literature [29]. The controller parameters and the retuned α values are presented in Tab. 3.

Figure 13(a) depicts the closed loop response of PSO tuned SWPID controller for reference input of 1 and a step load disturbance (d) of 0.1 applied at 0sec and 1000sec respectively. Fig 13(b) shows the closed loop response by BFO tuned controller. The overshoot in BFO based PI-PD controller is large compared to PSO tuned PI-PD. From Tab. 3 and Tab. 4, the observation is that, other parameters such as t_r, t_s, ISE, and IAE by BFO tuned SWPID is better than PSO tuned controller. The load disturbance rejection by the entire controller structures are similar and are satisfactory.

Table 12. Quantitative analysis for heuristic algorithm tuned SWPID controller.

Process	Controller parameters			Iteration	α	β	Time response			Error	
	K_p	K_i	K_d				t_r	M_p	t_s	ISE	IAE
Ex 4: PSO	1.7629	0.0119	8.0371	72	0	0	30.91	1.003	524.7	287.6	16.96
					0	1	30.91	1.003	524.7	287.6	16.96
					1	0	581.9	0.000	581.9	7248	131.2
					1	1	581.9	0.000	581.9	7248	131.2
					0.5722	1	44.32	0.267	351.4	2223	47.15
Ex 4: BFO	1.5177	0.0131	6.6163	87	0	0	33.07	0.920	392.3	235.5	15.35
					0	1	33.07	0.920	392.3	235.5	15.35
					1	0	341.6	0.000	341.6	1574	100.5
					1	1	341.6	0.000	341.6	1574	100.5
					0.3755	1	45.28	0.375	239.6	792.8	28.16
Ex 5: PSO	2.0224	0.0293	0.1006	59	0	0	27.81	0.937	253.2	144.7	12.03
					0	1	27.81	0.937	253.2	144.7	12.03
					1	0	119.1	0.037	254.9	648.3	56.99
					1	1	119.1	0.037	254.9	648.3	56.99
					0.5296	1	38.74	0.119	206.2	301.5	24.52
Ex 5: BFO	1.8416	0.0322	0.0793	65	0	0	28.90	0.820	288.1	119.8	10.95
					0	1	28.90	0.820	288.1	119.8	10.95
					1	0	85.42	0.105	241.4	582.8	45.07
					1	1	85.42	0.105	241.4	582.8	45.07
					0.6118	1	47.71	0.154	198.9	278.1	24.04
Ex 6: PSO	-1.1884	-0.0583	-0.3917	42	0	0	1.000	3.416	16.80	192.1	13.86
					0	1	1.000	3.416	16.80	192.1	13.86
					1	0	14.83	0.000	14.83	42.57	6.527
					1	1	14.83	0.000	14.83	42.57	6.527
					0.8322	1	4.000	0.106	13.41	9.636	3.104
Ex 6: BFO	-1.1724	-0.0462	-0.4936	55	0	0	0.904	2.893	25.91	305.9	17.49
					0	1	0.904	2.893	25.91	305.9	17.49
					1	0	14.48	0.033	24.16	62.21	7.887
					1	1	14.48	0.033	24.16	62.21	7.887
					0.7031	1	2.190	0.437	17.08	8.125	2.359

Table 4. Graphical representation of performance measure values

Performance measure values	**Example 4**	**Example 5**	**Example 6**
tr			
Mp			
ts			
ISE			
IAE			

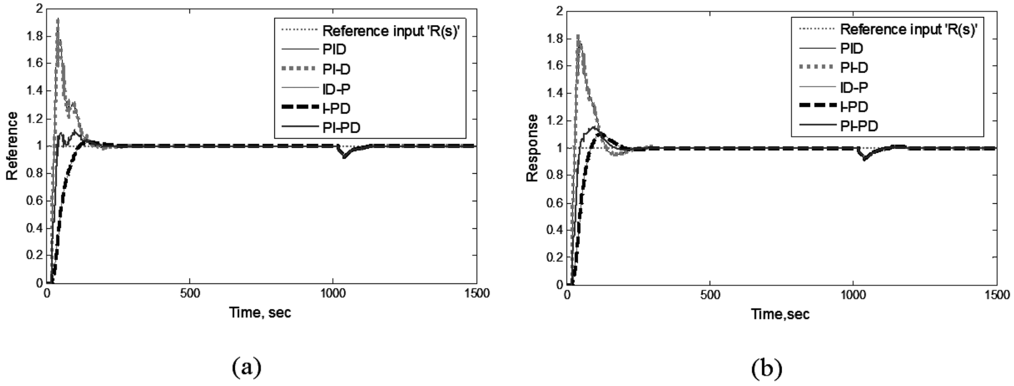


Figure 13. Reference tracking and load disturbance response for Example 5.

Example 6 Jacketed CSTR discussed in the book by Bequette [35] is considered for the study. The process model relating the jacket flow rate to the reactor temperature can be represented as

$$G_p(s) = \frac{-4.4747s - 37.94}{s^3 + 9.332s^2 + 16.89s - 34.45} e^{-\theta s}. \tag{18}$$

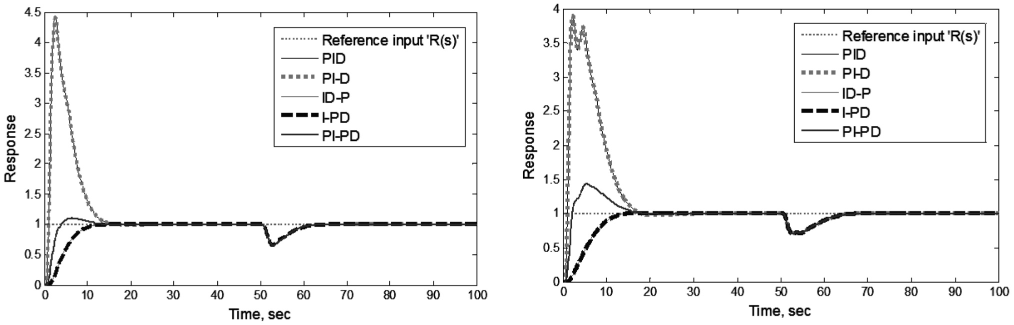
In this study, the value of delay time (θ) is considered as 0.5 sec. The above transfer function has two stable poles and one unstable pole. This process model also has a positive zero. The controller boundaries assigned in section 4.3 cannot provide the viable result during the optimization search (since the numerator has the -ve sign).

The search boundaries for the controller parameters in the three dimensional search space is assigned as follows:

Value 1 = $-50\% < K_p < 0\%$ (ie. $-5 < K_p < 0$)
Value 2 = $-10\% < K_i < 0\%$ (ie. $-1 < K_i < 0$)
Value 3 = $-50\% < K_d < 0\%$ (ie. $-5 < K_d < 0$)

In this study the lower boundary is set with a negative value and the upper boundary is assigned as zero. Initially the PSO based optimization search is executed with the PID controller ($\alpha = \beta = 0$). In this process the delay time is 0.5sec, hence $T_{\max}$ is assigned as 100sec. The optimization search with the PSO provides the result at 42th iteration. Similar procedure is followed in the BFO based search, and it provides the optimal values at 55th iteration. Later tuning of α is attempted with PSO and BFO as discussed in section 4.4. The optimized values are presented in Tab. 3.

Fig 14(a) depicts the closed loop response of PSO tuned SWPID controller for reference input of 1 and a step load disturbance (d) of 0.1 applied at 0sec and 50sec respectively. Fig 14(b) shows the response by BFO tuned controller. The overshoot in BFO



(a)

(b)

Figure 14. Reference tracking and load disturbance response for jacketed CSTR.

based PI-PD controller is large compared to PSO tuned PI-PD. The general performance of PI-PD structure is better than other controller structures considered in this study. From Tab. 3 and Tab. 4, the observation is that, the overall performance by the PSO tuned controller is better than BFO tuned controller. The load disturbance rejection by the heuristic algorithm controller is similar for all the structures are similar and are satisfactory.

6. Conclusions

In this paper, we have discussed the PSO and the BFO tuned setpoint weighted PID controller for a class of unstable process models. A procedure to select the PSO and BFO search parameters are discussed in the proposed work to minimize the complexity during the algorithm parameter assignment. We proposed a novel objective function using the closed loop time response data in order to minimize the convergence time during the heuristic algorithm based optimization search with the PID controller. The proposed objective function monitors the optimization search until the PID parameters converge to a best possible value. The simulation result shows that the reference tracking performance of the PI-PD controller is better than classical and modified PID controller structures considered in this study. The proposed heuristic algorithm based SWPID controller helps to provide better result for the setpoint tracking and error minimization on a class of unstable process models. The load disturbance rejection performance by the classical and modified structured PID controller is similar and its performance measure is found to be satisfactory.

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