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Bayesian SVLEDEJ Model for Detecting Jumps in Logarithmic Growth Rates of One Month Forward Gas Contract Prices

Maciej Kostrzewski*

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Abstract

A Bayesian stochastic volatility model with a leverage effect, normal errors and jump component with the double exponential distribution of a jump value is proposed. The ready to use Gibbs sampler is presented, which enables one to conduct statistical inference. In the empirical study, the SVLEDEJ model is applied to model logarithmic growth rates of one month forward gas prices. The results reveal an important role of both jump and stochastic volatility components.

Keywords: jump-diffusion model, stochastic volatility, Bayesian approach, MCMC methods, gas forward prices

JEL Classification: C580, C51

*Cracow University of Economics; e-mail: kostrzem@uek.krakow.pl

1 Introduction

Financial time series of logarithmic rates of return are often modelled by the SV processes (e.g. Jacquier, Polson and Rossi 1994, Pajor 2003, Jacquier, Polson and Rossi 2004, Yu 2005, Omori, Chib, Shephard and Nakajima 2007). Extending basic SV structures to the ones featuring a jump component may often appear empirically superfluous, because of a small number of jumps (if at all) to be identified in a given dataset, which is believed to be a consequence of the SV component's capability of modelling sharp movements in the data, especially in the fat-tailed SV models, see Jacquier, Polson and Rossi (2004).

Energy is an important economy factor having effects on the politics and development economy. The problem of modelling time series from energy markets is important from practical point of view (e.g. forecasting; see e.g. Weron 2014). Such data sets often feature periods of high and low volatility and jumps. Asymmetry of returns justify one's expectation of an asymmetry of jump values. This paper addresses the aforementioned problem by proposing a new model structure, which is believed to be a useful tool in modelling commodity time series.

Bayesian SV models with jumps are not a new concept (Chib, Nardari and Shephard 2002, Johannes and Polson 2010). Some models with Levy jumps were considered e.g. by Li, Wells and Yu (2008), Szerszen (2009). The contribution of the paper is the proposition of the Bayesian stochastic volatility model with a leverage effect, normal errors and jump component with the double exponential distribution of a jump value (the SVLEDEJ model, in short). The assumed distribution of jumps corresponds with one's expectation of asymmetrical (around zero) jump sizes. The main goal is to define the model and present the numerical algorithm, namely the Gibbs sampler (Gamerman and Lopes 2006) which facilitates statistical inference under the SVLEDEJ model. Moreover, an application of the methodology to a problem of estimating parameters, detecting jumps and identifying periods with high volatility is presented.

The double exponential jump-diffusion model is a jump-diffusion structure with the double exponential jump size distribution. The process defines the dynamics of an asset under the Kou model framework which is applied in pricing derivative securities (Kou 2002, Kou and Wang 2004). The specification is a particular case of the Pareto-Beta jump-diffusion specification proposed by Ramezani and Zeng (1998), where two Poisson processes govern the arrival rate of "bad" and "good" news. The discrete version of the double exponential jump-diffusion model (DEJD, in short) is considered by Kostrzewski (2015), who proposed its Bayesian version. The Bayesian DEJD model was applied to detect jumps and investigate the asymmetry of jump values. The main drawback of the DEJD structure is the assumption of independence of the returns. The SVLEDEJ model is an extension of the DEJD model. Bacause of the SV component, the assumption of the independence of returns is no longer in force, under SVLEDEJ framework.

The very term of a jump is in common use. Unfortunatelly, there is no a unique and

generally accepted definition. While analysing a time series which is believed to be a trajectory of some stochastic process, one seems to know which values are jumps "for sure", but does not know how to classify the rest of observations, that is where to put the border between a small movement (diffusion) and a jump. In the paper, it is assumed that a time series is generated by a process with jumps. The problem how to classify a given data point as being (co-)generated by the jump component is solved by introducing latent variables.
The paper is structured as follows. In the first part, the theoretical one, the definition of the model is given. Then the Bayesian version of it is proposed. Finally, the Gibbs sampler is given. The second part, the empirical one, is devoted to application of the new structure. First, the time series is presented. Second, results of estimation and detection of jumps are shown. Finally, the article ends up with conclusions and further plans of research.

2 The SVLEDEJ model

2.1 The model structure

The DEJD model assumes that the logarithm of a risky asset S is governed by a jump-diffusion process. The process is built of two components: a (pure) diffusion part, representing continuous variations in the series, and a (pure) jump component, reflecting abnormal (extreme) movements in the series:

$$d\left(\ln\left(S_t\right)\right) = \underbrace{\left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t}_{\substack{\text{a pure diffusion} \\ \text{component}}} + \underbrace{d\left(\sum_{i=1}^{N_t} \xi_i\right)}_{\substack{\text{a pure jump} \\ \text{component}}},$$

where $\{S_t\}$ is the stochastic process of prices and $\{W_t\}$ stands for a Wiener process. The continuous price behavior between jumps is described by a geometric Brownian motion, while the arrival rate of jumps is described by a homogeneous Poisson process, $\{N_t\}$, and jump magnitudes – by $\{\xi_i\}$.
The Merton model (Merton 1976) is presumably the most famous jump-diffusion structure. The Bayesian version of a discrete version of this model was given e.g. by Kostrzewski (2014) in the framework of Bayesian JD(M)J model. The discrete version of DEJD model (also called DEJD) is similar to the JD(M)J specyfication for $M = 1$. The difference resides in different distributions of jump values. In the case of the DEJD model the double exponential distribution $DE\left(p_D, \eta_D, p_U, \eta_U\right)$ is assumed.

Hence the distribution of a jump value ξ_i is given by the density:

$$f_{\xi_i}(x) = p_D \frac{1}{\eta_D} \exp\left(\frac{1}{\eta_D} x\right) \mathbb{I}_{(-\infty,0)}(x) + p_U \frac{1}{\eta_U} \exp\left(-\frac{1}{\eta_U} x\right) \mathbb{I}_{[0,\infty)}(x),$$
$$p_D \geq 0,\ p_U \geq 0,\ p_D + p_U = 1,\ \eta_D > 0,\ \eta_U > 0.$$

Hence, negative and positive jumps follow exponential distributions with different values of parameters. The main motivation behind such a structure is a presumption of the asymmetry of frequency and values of jumps. Additional properties of the DEJD structure are presented in Kostrzewski (2015).
The DEJD structure might be criticized for the assumption of independence of returns and a constant value of the volatility parameter. Because of these disadvantages, a new model is suggested so as to incorporate, additionally, a stochastic volatility component and correlation between returns and volatility, usually referred to as the leverage effect. Let us first write it in a continuous time scale:

$$d(\ln(S_t)) = \mu dt + \sqrt{\exp(h_t)} dW_t^{(1)} + d\left(\sum_{i=1}^{N_t} \xi_i\right)$$
$$dh_t = \kappa_h(\theta_h - h_t) dt + \sigma_h dW_t^{(2)},\ \ d\left\langle W^{(1)}, W^{(2)}\right\rangle_t = \rho dt,$$

where $\{\xi_i\} \sim iid\ DE(p_D,\eta_D,p_U,\eta_U)$ and $\left\{W_t^{(1)}\right\},\left\{W_t^{(2)}\right\}$ are Wiener processes with an instantaneous correlation ρ. This new specification incorporates continuous and small changes of values of the process $\ln(S_t)$, jumps (with the double exponential distribution) as well as stochastic volatility and correlation between returns and volatility. The process $\sum_{i=1}^{N_t} \xi_i$ is a compound Poisson process with jump intensity $\lambda > 0$ and the double exponential distribution of jump values. In what follows research the discrete version (the Euler-Maruyama approximation) of the model is considered:

$$\begin{aligned} y_{i+1} = &\quad y_i + \mu + \sqrt{\exp(h_i)} \varepsilon_{i+1}^{(1)} + J_{i+1}, \\ h_{i+1} = &\quad h_i + \kappa_h(\theta_h - h_i) + \sigma_h\left(\rho\varepsilon_{i+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{i+1}^{(2)}\right), \end{aligned} \tag{1}$$

where $y_i = \ln(S_i)$, $\left\{\varepsilon_i^{(1)}\right\} \sim iid\ N(0,1)$,$\left\{\varepsilon_i^{(2)}\right\} \sim iid\ N(0,1)$, $\varepsilon_i^{(1)}$ and $\varepsilon_j^{(2)}$ are independent, and the distribution of a jump component value J_i is given by the density:

$$f_{J_i}(x) = p_D \frac{1}{\eta_D} \exp\left(\frac{1}{\eta_D} x\right) \mathbb{I}_{(-\infty,0)}(x) + p_0 \delta_{(0)}(x) + p_U \frac{1}{\eta_U} \exp\left(-\frac{1}{\eta_U} x\right) \mathbb{I}_{(0,\infty)}(x),$$

where $\delta_{(0)}$ is the Kronecker delta.
An additional restriction of $0 < \kappa_h < 2$ is assumed in order to ensure the stationarity of the log-volatility process h_i. Parameter $\mu \in \mathbb{R}$ represents the drift of

$Z_{i+1} = y_{i+1} - y_i$. Note that $E\left(h_i | h_1\right) \to \theta_h$ for $i \to \infty$. The value of $|1-\kappa_h|$ is responsible for the speed of mean reversion of the log-volatility process h_i towards its mean θ_h (which is called the long run equlibrium value or the mean reversion level of h_i (Brigo and Mercurio 2001)) and σ_h is the volatility parameter of h_i. The value of $\frac{1}{\kappa_h}$ represents the mean reversion time. The parameter $\rho \in (-1,1)$ is interpreted as (instantaneous) correlation coefficient between return shocks $\varepsilon_i^{(1)}$ and volatility shocks $\rho\varepsilon_i^{(1)} + \sqrt{1-\rho^2}\varepsilon_i^{(2)}$. If $\rho < 0$ or $\rho > 0$, then the leverage effect or the inverse leverage effect appears, respectively. The parameters p_D, p_0 and p_U are interpreted as the probabilities of a negative jump, no jump and a positive jump, respectively. Moreover, η_D and η_U are means of negative and positive jumps, correspondingly.
Let us note that:

1. $E\left(Z_{i+1} | h_i\right) = \mu - \eta_D p_D + \eta_U p_U$.
2. $E\left(Z_{i+1} | h_i, q_{i+1} = -1\right) = \mu - \eta_D$, $E\left(Z_{i+1} | h_i, q_{i+1} = 1\right) = \mu + \eta_U$, $E\left(Z_{i+1} | h_i, q_{i+1} = 0\right) = \mu$.
3. $Var\left(Z_{i+1} | h_i\right) = \exp\left(h_i\right) + \eta_D^2 p_D\left(1-p_D\right) + \eta_U^2 p_U\left(1-p_U\right) + 2\eta_D\eta_U p_D p_U$.
4. $Z_{i+1} | h_i$ is a mixture of continuous distributions:
   (a) $Z_{i+1} | h_i, q_{i+1} = 0 \sim N\left(\mu, \exp\left(h_i\right)\right)$.
   (b) $$p\left(Z_{i+1} | h_i, q_{i+1} = -1\right) = \frac{1 - \Phi_{0,1}\left(\frac{Z_{i+1} - (1-\exp(h_i)/\eta_D)}{\exp(h_i/2)}\right)}{\eta_D} \cdot$$
   $$\cdot \exp\left(\frac{1}{2}\left(\exp\left(-\frac{1}{2}h_i\right) - \frac{1}{\eta_D}\exp\left(\frac{1}{2}h_i\right)\right)^2 - \frac{1}{2}\exp\left(-h_i\right)\mu + Z_{i+1}/\eta_D\right),$$
   where $\Phi_{0,1}$ stands for a standard normal cumulative density function.
   (c) $$p\left(Z_{i+1} | h_i, q_{i+1} = 1\right) = \frac{1 - \Phi_{0,1}\left(\frac{Z_{i+1} - (1+\exp(h_i)/\eta_U)}{\exp(h_i/2)}\right)}{\eta_U} \cdot$$
   $$\cdot \exp\left(\frac{1}{2}\left(\exp\left(-\frac{1}{2}h_i\right) + \frac{1}{\eta_U}\exp\left(\frac{1}{2}h_i\right)\right)^2 - \frac{1}{2}\exp\left(-h_i\right)\mu - Z_{i+1}/\eta_U\right).$$
5. $Z_{i+1} | h_{i+1}, h_i, J_{i+1} \sim$
   $$N\left(\mu + J_{i+1} + \frac{\sqrt{\exp\left(h_i\right)}}{\sigma_h}\rho\left(h_{i+1} - h_i + \kappa_h\left(\theta_h - h_i\right)\right), \left(1-\rho^2\right)\exp\left(h_i\right)\right).$$
6. If $h_{i+1} = h_i \equiv \ln\left(\sigma^2\right)$ (e.g. $\kappa_h = \sigma_h = 0$ or $\rho = \theta_h = 0$ and $\kappa_h = 1$), where $\sigma^2 > 0$ and $J_i \equiv 0$ then $\{y_i\}$ is a Gaussian random walk with a drift μ and $\{Z_i\} \sim iid\ N\left(\mu, \sigma^2\right)$.
7. If $J_i \equiv 0$ then $\{Z_i - \mu\}$ is a stochastic volatility process.

8. If $h_{i+1} = h_i \equiv \ln\left(\sigma^2\right)$, where $\sigma^2 > 0$ then $\{y_i\}$ is a jump-diffusion process with a double exponential distribution of jump values.
9. If $\kappa_h \in (0, 2)$ then $\{h_i\}$ is an AR(1) process and $h_i \sim N\left(\theta_h, \dfrac{\sigma_h^2}{(2+\kappa_h)\,\kappa_h}\right)$.

Let justify the interpretation of ρ. Assume that $\zeta_{i+1} = \ln\left(\frac{S_{i+1}}{S_i}\right) - \mu - J_{i+1}$ and $\gamma = \kappa_h\theta_h$ and $\phi = 1 - \kappa_h$. Therefore

$$\begin{aligned} \zeta_{i+1} = &\quad \sqrt{\exp\left(h_i\right)}\varepsilon_{i+1}^{(1)}, \\ h_{i+1} = &\quad \gamma + \phi h_i + \sigma_h\left(\rho\varepsilon_{i+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{i+1}^{(2)}\right), \end{aligned}$$

where $\varepsilon_i^{(1)} \perp \varepsilon_j^{(2)}$ (stochastically independent) for all i and j, and $\left\{\varepsilon_i^{(1)}\right\}$, $\left\{\varepsilon_i^{(2)}\right\} \sim iid\ N(0,1)$ and $|\phi| < 1$.
Note that

$$\begin{aligned} E\left(h_{i+1}\middle|\,\varepsilon_{i+1}^{(1)}, h_i\right) &= \gamma + \phi h_i + \sigma_h E\left(\rho\varepsilon_{i+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{i+1}^{(2)}\middle|\,\varepsilon_{i+1}^{(1)}, h_i\right) \\ &= \gamma + \phi h_i + \sigma_h\rho\varepsilon_{i+1}^{(1)}. \end{aligned}$$

As $|\phi| < 1$, then $h_i \sim N\left(\frac{\gamma}{1-\phi}, \frac{\sigma_h^2}{1-\phi^2}\right)$ and

$$\begin{aligned} E\left(h_{i+1}\middle|\,\varepsilon_{i+1}^{(1)}\right) &= E\left(E\left(h_{i+1}\middle|\,\varepsilon_{i+1}^{(1)}, h_i\right)\middle|\,\varepsilon_{i+1}^{(1)}\right) \\ &= E\left(\gamma + \phi h_i + \sigma_h\rho\varepsilon_{i+1}^{(1)}\middle|\,\varepsilon_{i+1}^{(1)}\right) \\ &= \gamma + \phi E\left(h_i\middle|\,\varepsilon_{i+1}^{(1)}\right) + \sigma_h\rho\varepsilon_{i+1}^{(1)} \\ &= \gamma + \phi\frac{\gamma}{1-\phi} + \sigma_h\rho\varepsilon_{i+1}^{(1)}. \end{aligned}$$

Note that $sign\left(\zeta_{i+1}\right) = sign\left(\varepsilon_{i+1}^{(1)}\right)$. In order to interpret ρ, assume for an instant that $\rho < 0$. If the value of h_i is constant and values of ζ_{i+1} decrease, then $\varepsilon_{i+1}^{(1)}$ decreases and volatility increases i.e. the conditional mean of h_{i+1} (i.e. $E\left(h_{i+1}\middle|\,\varepsilon_{i+1}^{(1)}\right)$) increases, to be exact. Note that if $J_{i+1} = 0$ and $\mu = 0$, then $\zeta_{i+1} < 0$ means negative logarithmic return. For these reason, ρ can be interpreted as the leverage effect (Yu, 2005).
The above discrete time structure (1) is called the stochastic volatility model with a leverage effect and double exponential jumps (SVLEDEJ, in short). Further considerations are limited to this structure. Note that the log-volatility process h_i (and not h_{i+1}as in Jacquier, Polson, and Rossi 2004) appears in the formula for y_{i+1}, which is in line with the idea presented by Yu (2005) (and also used in Omori, Chib, Shephard, and Nakajima 2007, Li, Wells, and Yu 2008, Johannes and Polson 2010).

2.2 The Bayesian SVLEDEJ model

While analysing a time series which is believed to be a trajectory of some stochastic process, we do not actually know if a given data point has been generated by a pure diffusion, or a pure SV process, or a jump component, or any combination of them. To manage the problem, latent variables q_i, ξ_i^D and ξ_i^U are introduced. The variable q_i takes three values -1, 0 or 1. The first value corresponds with a negative jump, the second one with no jump and the last one with a positive jump. Formally, an occurence of a jump is equivalent to an event $q_i \neq 0$. Unfortunately, the values of variables q_i are not observed, but the posterior probability $P(q_i \neq 0| y)$ can be assessed. Let us assume that a jump occurs at i if the posterior probability of a jump exceeds an arbitrarily chosen value of 0.5. Additionally, the value of a negative jump equals $-\xi_{i+1}^D \cdot \mathbb{I}(q_{i+1} = -1)$, and a positive one $\xi_{i+1}^U \cdot \mathbb{I}(q_{i+1} = 1)$. It leads to the following specification:

$$
\begin{aligned}
y_{i+1} = \quad & y_i + \mu + \sqrt{\exp(h_i)}\varepsilon_{i+1}^{(1)} + J_{i+1}, \\
h_{i+1} = \quad & h_i + \kappa_h(\theta_h - h_i) + \sigma_h\left(\rho\varepsilon_{i+1}^{(1)} + \sqrt{1-\rho^2}\varepsilon_{i+1}^{(2)}\right), \\
J_{i+1} = \quad & -\xi_{i+1}^D \cdot \mathbb{I}(q_{i+1} = -1) + \xi_{i+1}^U \cdot \mathbb{I}(q_{i+1} = 1),
\end{aligned}
$$

where $y_i = \ln(S_i)$, $\left\{\varepsilon_i^{(1)}\right\}$, $\left\{\varepsilon_i^{(2)}\right\} \sim iid\ N(0,1)$, $\varepsilon_i^{(1)}$ and $\varepsilon_j^{(2)}$, are independent, $\left\{\xi_i^D\right\} \sim iid\ Exp(\eta_D)$, $\left\{\xi_i^U\right\} \sim iid\ Exp(\eta_U)$, $q_i \in \{-1,0,1\}$, $P(q_i = -1) = p_D$, $P(q_i = 0) = p_0$, $P(q_i = 1) = p_U$. The Bayesian SVLEDEJ model is defined with extended parameter space including also the latent variables.

Let us assume $Z_i = y_i - y_{i-1}$, where $i = 2, ..., n$. The random variables Z_i stand for the logarithmic growth rates of S_i. Moreover,

$$
\begin{aligned}
Z_i &= \mu + \sqrt{\exp(h_{i-1})}\varepsilon_i^{(1)} + J_i, \\
h_i &= h_{i-1} + \kappa_h(\theta_h - h_{i-1}) + \sigma_h\varepsilon_i^{(3)}.
\end{aligned}
$$

The vector of all unknown quantities is defined as:
$(\theta, h, q, \xi) = \left(\mu, \kappa_h, \theta_h, \sigma_h, \rho, \eta_D, \eta_U, p_D, p_0, p_U, h_1, ..., h_n, q_2, ..., q_n, \xi_2^D, ..., \xi_n^D, \xi_2^U, ..., \xi_n^U\right)$, where $\mu \in \mathbb{R}$, $0 < \kappa_h < 2$, $\theta_h \in \mathbb{R}$, $\sigma_h > 0$, $\rho \in (-1,1)$, $\eta_D > 0$, $\eta_U > 0$, $p_D \geq 0$, $p_0 \geq 0$, $p_U \geq 0$, $p_D + p_0 + p_U = 1$, $h_i \in \mathbb{R}$, $q_i \in \{-1,0,1\}$, $\xi_i^D \in (0,\infty)$, $\xi_i^U \in (0,\infty)$.
According to Jacquier, Polson, and Rossi (2004), we reparametrize the model using $\phi_h = \sigma_h\rho$ and $\omega_h = \sigma_h^2\left(1-\rho^2\right)$ instead of σ_h and ρ.
For all unknown quantities of the model, standard proper prior distributions reflecting prior uncertainty are assumed. The prior structure is as follows:

$$
\begin{aligned}
p(\theta, h, q, \xi) = \quad & p(\mu)\, p(\kappa_h)\, p(\theta_h)\, p(\phi_h, \omega_h)\, p(\eta_D)\, p(\eta_U)\, p(p_D, p_0, p_U) \cdot \\
& \cdot p(h_1) \prod_{i=2}^{n} p(h_i | h_{i-1}, \theta) \prod_{i=2}^{n} p(q_i | \theta) \prod_{i=2}^{n} p\left(\xi_i^D \middle| \theta\right) \prod_{i=2}^{n} p\left(\xi_i^U \middle| \theta\right),
\end{aligned}
$$

where

$\mu \sim N(m_\mu, w_\mu)$,

$\kappa_h \sim N\left(m_{\kappa_h}, w_{\kappa_h}\right)\mathbb{I}_{(0,2)}$ (a truncated normal distribution),
$\theta_h \sim N\left(m_{\theta_h}, w_{\theta_h}\right)$,
$\omega_h \sim IG\left(a_{\omega_h}, b_{\omega_h}\right)$ (an inverse gamma distribution), where
$p\left(\omega_h\right) \propto \omega_h^{-a_{\omega_h}-1}\exp\left(-\frac{b_{\omega_h}}{\omega_h}\right)$,
$\phi_h\left|\omega_h\right. \sim N\left(0, \frac{1}{2}\omega_h\right)$,
$\eta_D \sim IG\left(a_{\eta_D}, b_{\eta_D}\right)$,
$\eta_U \sim IG\left(a_{\eta_U}, b_{\eta_U}\right)$,
$\left(p_D, p_0, p_U\right) \sim Dirichlet\left(d_D, d_0, d_U\right)$, where $p\left(p_D, p_0, p_U\right) \propto p_D^{d_D-1}p_0^{d_0-1}p_U^{d_U-1}$.
The prior assumptions for latent variables are as follows:
$p\left(q_i = -1\left|\theta\right.\right) = p_D$, $p\left(q_i = 0\left|\theta\right.\right) = p_0$, $p\left(q_i = 1\left|\theta\right.\right) = p_U$, $\xi_i^D\left|\theta\right. \sim Exp\left(\eta_D\right)$, $\xi_i^U\left|\theta\right. \sim Exp\left(\eta_U\right)$, where $p\left(\xi_i^D\left|\theta\right.\right) = \frac{1}{\eta_D}\exp\left(-\frac{\xi_i^D}{\eta_D}\right)$. Moreover, we assume $h_i\left|\, h_{i-1}, \theta\right. \sim N\left(\kappa_h\theta_h + \left(1-\kappa_h\right)h_{i-1}, \sigma_h^2\right)$ and a uniform distribution for $h_1 \sim U\left(-A_{h_1}, A_{h_1}\right)$, where A_{h_1} is a "large" number (e.g. 10^4) belonging to the interval $(0, \infty]$.
Finally, the Bayesian SVLEDEJ model we define as the joint density

$$p\left(Z, \theta, h, q, \xi\right) := p\left(\theta, h_1, q, \xi\right)\prod_{i=2}^{n} p\left(Z_i, h_i\left|\, h_{i-1}, q_i, \xi_i^D, \xi_i^U, \theta\right.\right),$$

where $p\left(Z_i, h_i\left|\, h_{i-1}, q_i, \xi_i^D, \xi_i^U, \theta\right.\right) \propto \prod_{i=2}^{n}\exp\left(-\frac{1}{2(1-\rho^2)}\left[\left(\varepsilon_i^{(1)}\right)^2 - 2\rho\varepsilon_i^{(1)}\varepsilon_i^{(3)} + \left(\varepsilon_i^{(3)}\right)^2\right]\right)$.

The Gibbs sampler

Combining the Gibbs sampler and the independent Metropolis-Hastings algorithm facilitates the Bayesian inference under the SVLEDEJ model. These techniques are numerical methods of multidimensional integration, which belong to the Markov Chain Monte Carlo (MCMC) methods (Gamerman and Lopes 2006). In what follows the applied in practise algorithm is described.
The algorithm of generating a sample from the posterior conditional distribution of h_i employed by Jacquier, Polson and Rossi (2004) is applied. In particular, the property of time reversibility of the AR(1) is used in generating h_1. Some tedious, yet fairly simple calculations yield the following formulae of relevant posterior conditionals, making the Gibbs sampler ready to use:

1. μ :
   $p\left(\mu|\theta_{\setminus(\mu)}, h, q, \xi, y\right) \propto \exp\left(-\frac{1}{2}C_\mu\left(\mu - \frac{D_\mu}{C_\mu}\right)^2\right)$, where

$$C_\mu = \frac{1}{w_\mu} + \frac{1}{1-\rho^2} \sum_{i=2}^n \exp\left(-h_{i-1}\right),$$
$$D_\mu = \frac{m_\mu}{w_\mu} + \frac{1}{1-\rho^2} \sum_{i=2}^n \frac{Z_i - J_i}{\exp\left(h_{i-1}\right)} - \frac{\rho}{1-\rho^2} \sum_{i=2}^n \frac{\varepsilon_i^{(3)}}{\exp\left(h_{i-1}\right)},$$
$$\varepsilon_i^{(3)} = \frac{h_i - h_{i-1} - \kappa_h\left(\theta_h - h_{i-1}\right)}{\sigma_h}.$$

2. κ_h :

$$p\left(\kappa_h|\theta_{\setminus(\kappa_h)}, h, q, \xi, y\right) \propto \exp\left(-\tfrac{1}{2}C_{\kappa_h}\left(\kappa_h - \tfrac{D_{\kappa_h}}{C_{\kappa_h}}\right)^2\right) \cdot \mathbb{I}\left(\kappa_h \in (0,2)\right), \text{ where}$$
$$C_{\kappa_h} = \frac{1}{w_{\kappa_h}} + \frac{1}{1-\rho^2} \sum_{i=2}^n \frac{\left(\theta_h - h_{i-1}\right)^2}{\sigma_h^2},$$
$$D_{\kappa_h} = \frac{m_{\kappa_h}}{w_{\kappa_h}} + \frac{1}{\sigma_h^2\left(1-\rho^2\right)} \sum_{i=2}^n \left(-\rho\varepsilon_i^{(1)}\sigma_h\left(\theta_h - h_{i-1}\right) + \left(h_i - h_{i-1}\right)\left(\theta_h - h_{i-1}\right)\right),$$
$$\varepsilon_i^{(1)} = \frac{y_i - y_{i-1} - \mu - J_i}{\sqrt{\exp\left(h_{i-1}\right)}}.$$

3. θ_h :

$$p\left(\theta_h|\theta_{\setminus(\theta_h)}, h, q, \xi, y\right) \propto \exp\left(-\tfrac{1}{2}C_{\theta_h}\left(\theta_h - \tfrac{D_{\theta_h}}{C_{\theta_h}}\right)^2\right), \text{ where}$$
$$C_{\theta_h} = \frac{1}{w_{\theta_h}} + \frac{1}{1-\rho^2} \frac{(n-1)\,\kappa_h^2}{\sigma_h^2},$$
$$D_{\theta_h} = \frac{m_{\theta_h}}{w_{\theta_h}} + \frac{\kappa_h}{\sigma_h^2\left(1-\rho^2\right)} \left[\sum_{i=2}^n \left(h_i - h_{i-1} + \kappa_h h_{i-1} - \rho\sigma_h\varepsilon_i^{(1)}\right)\right],$$

4. (ϕ_h, ω_h) :

$$p\left(\phi_h, \omega_h|\theta_{\setminus(\phi_h,\omega_h)}, h, q, \xi, y\right) = p\left(\phi_h|\theta_{\setminus(\phi_h,\omega_h)}, h, q, \xi\right) p\left(\omega_h|\theta_{\setminus(\phi_h,\omega_h)}, h, q, \xi\right),$$

where

$$p\left(\phi_h|\theta_{\setminus(\phi_h,\omega_h)}, h, q, \xi, y\right) \propto$$
$$\exp\left(-\frac{1}{2}\frac{2+\sum_{i=2}^n\left(\varepsilon_i^{(1)}\right)^2}{\omega_h}\left(\phi_h - \frac{1}{\left(2+\sum_{i=2}^n\left(\varepsilon_i^{(1)}\right)^2\right)}\sum_{i=2}^n \varepsilon_i^{(1)}\left[h_i - h_{i-1} - \kappa_h\left(\theta_h - h_{i-1}\right)\right]\right)^2\right),$$

so that $\phi_h|\theta_{\setminus(\phi_h)}, h, q, \xi, y \sim N\left(\frac{D_{\phi_h}}{C_{\phi_h}}, \frac{1}{C_{\phi_h}}\right)$, where

$C_{\phi_h} = \frac{1}{\omega_h}\left(2 + \sum_{i=2}^n\left(\varepsilon_i^{(1)}\right)^2\right)$, $D_{\phi_h} = \frac{1}{\omega_h}\sum_{i=2}^n \varepsilon_i^{(1)}\left[h_i - h_{i-1} - \kappa_h\left(\theta_h - h_{i-1}\right)\right]$,

$p\left(\omega_h|\theta_{\backslash(\phi_h,\omega_h)},h,q,\xi,y\right)\propto$

$\left(\frac{1}{\omega_h}\right)^{n/2+d+1}\exp\left(-\frac{1}{\omega_h}\left[b_{\omega_h}+\frac{1}{2}\sum_{i=2}^{n}\left[h_i-h_{i-1}-\kappa_h\left(\theta_h-h_{i-1}\right)\right]^2\right]\right)\cdot$

$\cdot\exp\left(-\frac{1}{\omega_h}\left[-\frac{1}{2}\frac{1}{\left(2+\sum_{i=2}^{n}\left(\varepsilon_i^{(1)}\right)^2\right)}\left(\sum_{i=2}^{n}\varepsilon_i^{(1)}\left[h_i-h_{i-1}-\kappa_h\left(\theta_h-h_{i-1}\right)\right]\right)^2\right]\right),$

so that $\omega_h|\theta_{\backslash(\phi_h,\omega_h)},h,q,\xi,y\sim IG\left(A_{\omega_h},B_{\omega_h}\right)$, where

$A_{\omega_h}=n/2+a_{\omega_h},$

$B_{\omega_h}=b_{\omega_h}+\frac{1}{2}\sum_{i=2}^{n}\left[h_i-h_{i-1}-\kappa_h\theta_h-h_{i-1}\right]^2+$

$-\frac{1}{2}\frac{1}{\left(2+\sum_{i=2}^{n}\left(\varepsilon_i^{(1)}\right)^2\right)}\left(\sum_{i=2}^{n}\varepsilon_i^{(1)}\left[h_i-h_{i-1}-\kappa_h\left(\theta_h-h_{i-1}\right)\right]\right)^2.$

1. h_1 :

$$p\left(h_1|\theta,h_{\backslash(h_1)},q,\xi,y\right)\propto\quad\frac{1}{\sqrt{\exp\left(h_1\right)}}\exp\left(\frac{\left[Z_2-\mu-J_2-\sqrt{\exp\left(h_1\right)}\rho\varepsilon_2^{(3)}\right]^2}{-2\cdot\left(1-\rho^2\right)\exp\left(h_1\right)}\right)\cdot$$
$$\cdot\exp\left(\frac{\left[h_1-h_2\left(1-\kappa_h\right)-\kappa_h\theta_h\right]^2}{-2\sigma_h^2}\right).$$

In the empirical research a Metropolis-Hastings (MH) step with the proposal density of the normal distribution $N\left(h_2\left(1-\kappa_h\right)+\kappa_h\theta_h,\sigma_h^2\right)$ is applied.

2. h_n :

$$h_n|\theta,h_{\backslash(h_n)},q,\xi,y\sim N\left(\frac{\rho\sigma_h\left(Z_n-\mu-J_n\right)}{\sqrt{\exp\left(h_{n-1}\right)}}+h_{n-1}+\kappa_h\left(\theta_h-h_{n-1}\right),\sigma_h^2\left(1-\rho^2\right)\right).$$

3. $h_i,\ i\in\{2,...,n-1\}$:

$p\left(h_i|\theta,h_{\backslash(h_i)},q,\xi,y\right)\propto\exp\left(-\frac{1}{2}\frac{1}{1-\rho^2}\frac{1}{\exp(h_{i-1})}\left[Z_i-\mu-J_i-\sqrt{\exp\left(h_{i-1}\right)}\varepsilon_i^{(3)}\rho\right]^2\right)\cdot$

$\cdot\frac{1}{\sqrt{\exp\left(h_i\right)}}\exp\left(-\frac{1}{2}\frac{1}{1-\rho^2}\frac{1}{\exp\left(h_i\right)}\left[Z_{i+1}-\mu-J_{i+1}-\sqrt{\exp\left(h_i\right)}\varepsilon_{i+1}^{(3)}\rho\right]^2\right)\cdot$

$\cdot\exp\left(-\frac{1}{2}\left(\varepsilon_i^{(3)}\right)^2\right)\cdot\exp\left(-\frac{1}{2}\left(\varepsilon_{i+1}^{(3)}\right)^2\right)$. In the empirical research a MH step with the proposal density of

$$N\left(\frac{\left(1-\kappa_h\right)\left(h_{i-1}+h_{i+1}\right)+\kappa_h^2\theta_h}{1+\left(1-\kappa_h\right)^2},\frac{\sigma_h^2}{1+\left(1-\kappa_h\right)^2}\right)$$

is applied.

1. p_D, p_0, p_U :
$$p\left(p_D, p_0, p_U|\theta_{\backslash(p_D,p_0,p_U)}, q, \xi, y\right) \propto (p_D)^{d_D+\sum_{i=2}^{n}\mathbb{I}(q_i=-1)-1} \cdot (p_0)^{d_0+\sum_{i=2}^{n}\mathbb{I}(q_i=0)-1} \cdot (p_U)^{d_U+\sum_{i=2}^{n}\mathbb{I}(q_i=1)-1}.$$

2. q :
$$p\left(q_i|\theta, q_{\backslash(q_i)}, h, \xi, y\right) \propto p_D^{\mathbb{I}(q_i=-1)} \cdot p_0^{\mathbb{I}(q_i=0)} \cdot p_U^{\mathbb{I}(q_i=1)} \cdot \exp\left(-\frac{1}{2\left(1-\rho^2\right)\exp\left(h_{i-1}\right)}\left[Z_i - \mu - J_i - \rho\varepsilon_{3h}\exp\left(h_{i-1}\right)\right]^2\right),$$

3. η_D, η_U :
$$p\left(\eta_D|\theta_{\backslash(\eta_D)}, q, h, \xi, y\right) \propto \left(\tfrac{1}{\eta_D}\right)^{a_{\eta_D}+n}\exp\left(-\tfrac{1}{\eta_D}\left[b_{\eta_D} + \textstyle\sum_{i=2}^{n}\xi_i^D\right]\right),$$
$$p\left(\eta_U|\theta_{\backslash(\eta_U)}, q, h, \xi, y\right) \propto \left(\tfrac{1}{\eta_U}\right)^{a_{\eta_U}+n}\exp\left(-\tfrac{1}{\eta_U}\left[b_{\eta_U} + \textstyle\sum_{i=2}^{n}\xi_i^U\right]\right).$$

4. ξ_i^D, ξ_i^U :
$$p\left(\xi_i^D|\theta, q, \xi_{\backslash\left(\xi_i^D\right)}, h, y\right) \propto$$
$$\propto \begin{cases} \exp\left(-\frac{1}{2}\frac{1}{D_{\xi,i}}\left(\xi_i^D - \left(-C_{\xi,i} - \frac{D_{\xi,i}}{\eta_D}\right)\right)^2\right)\cdot\mathbb{I}\left(\xi_i^D > 0\right), & q_i = -1, \\ \exp\left(-\frac{\xi_i^D}{\eta_D}\right)\cdot\mathbb{I}\left(\xi_i^D > 0\right), & q_i \in \{0,1\}, \end{cases}$$
$$p\left(\xi_i^U|\theta, q, \xi_{\backslash\left(\xi_i^U\right)}, h, y\right) \propto$$
$$\propto \begin{cases} \exp\left(-\frac{\xi_i^U}{\eta_U}\right)\cdot\mathbb{I}\left(\xi_i^U > 0\right) & q_i \in \{-1,0\}, \\ \exp\left(-\frac{1}{2}\frac{1}{D_i}\left(\xi_i^U - \left(C_{\xi,i} - \frac{D_{\xi,i}}{\eta_U}\right)\right)^2\right)\cdot\mathbb{I}\left(\xi_i^U > 0\right), & q_i = 1 \end{cases}$$

where $C_{\xi,i} = Z_i - \mu - \rho\varepsilon_i^{(3)}\sqrt{\exp\left(h_{i-1}\right)}$, $D_{\xi,i} = \left(1-\rho^2\right)\exp\left(h_{i-1}\right)$.

Modification of the Bayesian structure

It is possible to define the Bayesian structure introducing latent variables $J_1, \ldots, J_n$ which directly describe jump values components, without defining ξ_i^D, ξ_i^U and q_i. In effect, we obtain a more parsimonious model (i.e. with a lower number of unknown latent variables). Moreover, calculation of the conditional posterior densities for the Gibbs sampler, under the modified specification, is based on the same idea as in the former version. In practice, the Markov chains generated by the Gibbs sampler in the modified model seem to be more stable, which is indicative of a faster convergence. However, the results of Bayesian inference for both structures are very similar. Hereafter, in the empirical research, the outcomes calculated under the original version are presented.

3 Empirical study

3.1 Data presentation

Natural gas, oil and coal are important energy sources. They were employed to production of 87% of total world energy consumption and 80% of Europe and Eurasia consumption in 2012, 2013 and 2014 (BP 2014, BP 2015). The natural gas is crucial for both electricity production and heating. Prices of gas contracts play critical role for commodity markets. One of the main, easily noticeable characteristics of these prices are sharp upward and downward movements. For this reason, it appears well-justified to analyse such data by means of models incorporating jumps.

At a virtual trading hub, injected into transmission network gas is traded. It is not important, at a virtual hub, which entry or exit is chosen for the natural gas injection or withdrawal. The time series analysed in this study comprises daily prices of one month gas contract prices (€/MWh) coming from the Dutch Title Transfer Facility (TTF) virtual trading hub. The data is provided by the London Energy Brokers' Association (`https://www.leba.org.uk/`). The daily prices ranges from January 21, 2008 to April 22, 2015. The number of observations equals 1840. The left panel of Figure 1 presents contract prices, while the logarithmic growth rates are shown on the right.

Figure 1: The contract prices (left) and the logarithmic growth rates (right)

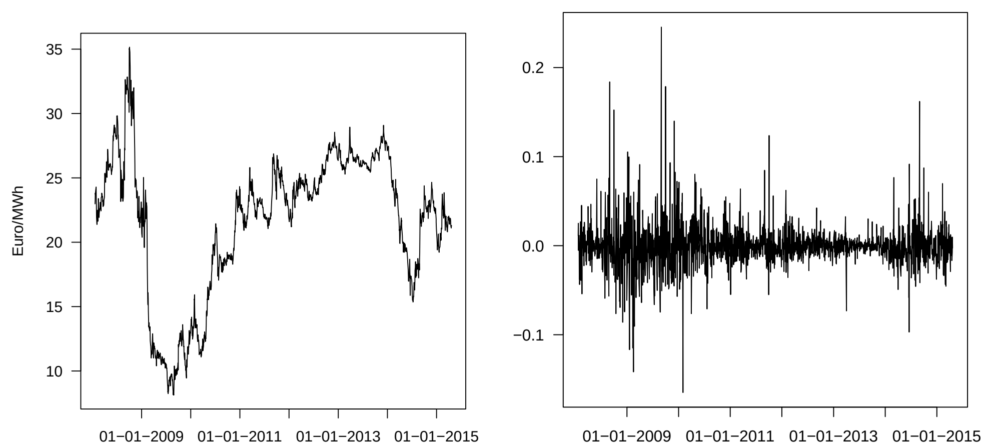


Table 1 shows descriptive statistics calculated for the analysed logarithmic growth rates on forward contract prices. A large value of kurtosis and a positive value of skewness suggest a nonnormal right-skewed distribution of the data.

Table 1: Descriptive statistics of the logarithmic growth rates

Min	-0.16488
Max	0.24543
Median	-0.000941
Mean	-0.00004
Kurtosis	17.97714
Standard deviation	0.02430
Skewness	1.27031
Number of observations	1839

3.2 Numerical methods

Posterior characteristics of the unknown quantities are calculated via the MCMC methods, combining the Gibbs sampler and the independent Metropolis-Hastings algorithm. The MH acceptance rates are high and range from 78.2% to 95.81% for $h_1, \ldots, h_{n-1}$. The convergence of ergodic means and standard deviations as well as convergence of the CUMSUM statistics are observed. The presented results are based on 1.000.000 MCMC draws, preceded by 500.000 burn-in cycles. In order to analyse the posterior characteristics of the latent variables the final 100.000 draws were taken into account.

3.3 The Bayesian estimation results

The values of hyperparameters of the prior structure are given in Table 2. The values of hyperparameters for μ, κ_h, θ_h, ω_h are the same as in Szerszen (2009). Prior mean and mode for η_D (and η_U) equal 0.5 and 0.15, respectively. The prior probability of values of η_D and η_U located close to zero is not "high". It expresses a prior expectation of a "large" jump. In such case "small" changes of values should be modelled by the (pure) SV part. The values of hyperparameters d_D, d_0 and d_U imply $E(p_D) = E(p_0) = E(p_U) = \frac{1}{3}$, so the prior means of the probability of a negative jump, no jump and positive jump are the same. Note, that the marginal distribution of p_D (and p_U) is the Beta(1,2) distribution (Frühwirth-Schnatter 2006, Kwiatkowski 2015) with $P(p_D < 0.5) > 0.5$ (and $P(p_U < 0.5) > 0.5$). The prior specification favours "lower" (closer to zero) values of p_D and p_U rather than values close to one.

Table 2: The hyperparameters of the prior structure

m_μ	m_{θ_h}	w_μ	w_{θ_h}	m_{κ_h}
0	0	10	10	1
w_{κ_h}	a_{ω_h}	b_{ω_h}	a_{η_D}	a_{η_U}
6	3	1/20	1.86	1.86
b_{η_D}	b_{η_U}	d_D	d_0	d_U
0.43	0.43	1	1	1

Table 3 presents the posterior means, $E(\theta|\,y)$, and standard deviations, $D(\theta|\,y)$, calculated for the parameters of the SVLEDEJ model. The histograms, depicted in Figure 2, represent the posterior distributions of the parameters, with solid lines representing the prior densities. The posterior distribution of κ_h is concentrated close to zero which implies strong volatility persistence thereby validating introducing the SV component into the model structure. The posterior mean of the long run equilibrium value of the log-volatility process equals -6.44. The mean reversion time equals about 167 days, so the log-volatility process h_i needs (on average) about 167 days to return to the long run equlibrium value. Zero is located in the 76% highest posterior density interval for ρ, so, accordingly to Lindley's test, the parameter is not significantly different from zero. Hence, the results suggest that the leverage effect is negligible. The posterior probability of upward jumps, p_U, is higher than that of downward movements, p_D. Additionally, the posterior mean of the probability of no jump is about 0.96. The results are in line with our prior expectations - jumps are sporadic events, and upper jumps are more frequent than lower ones, which is a typical feature of commodity series but not financial ones. Moreover, the posterior means of negative and positive jumps equal -0.1202 and 0.0564, respectively, so the negative jumps are on average twice as strong as the positive ones (in the absolute value terms).

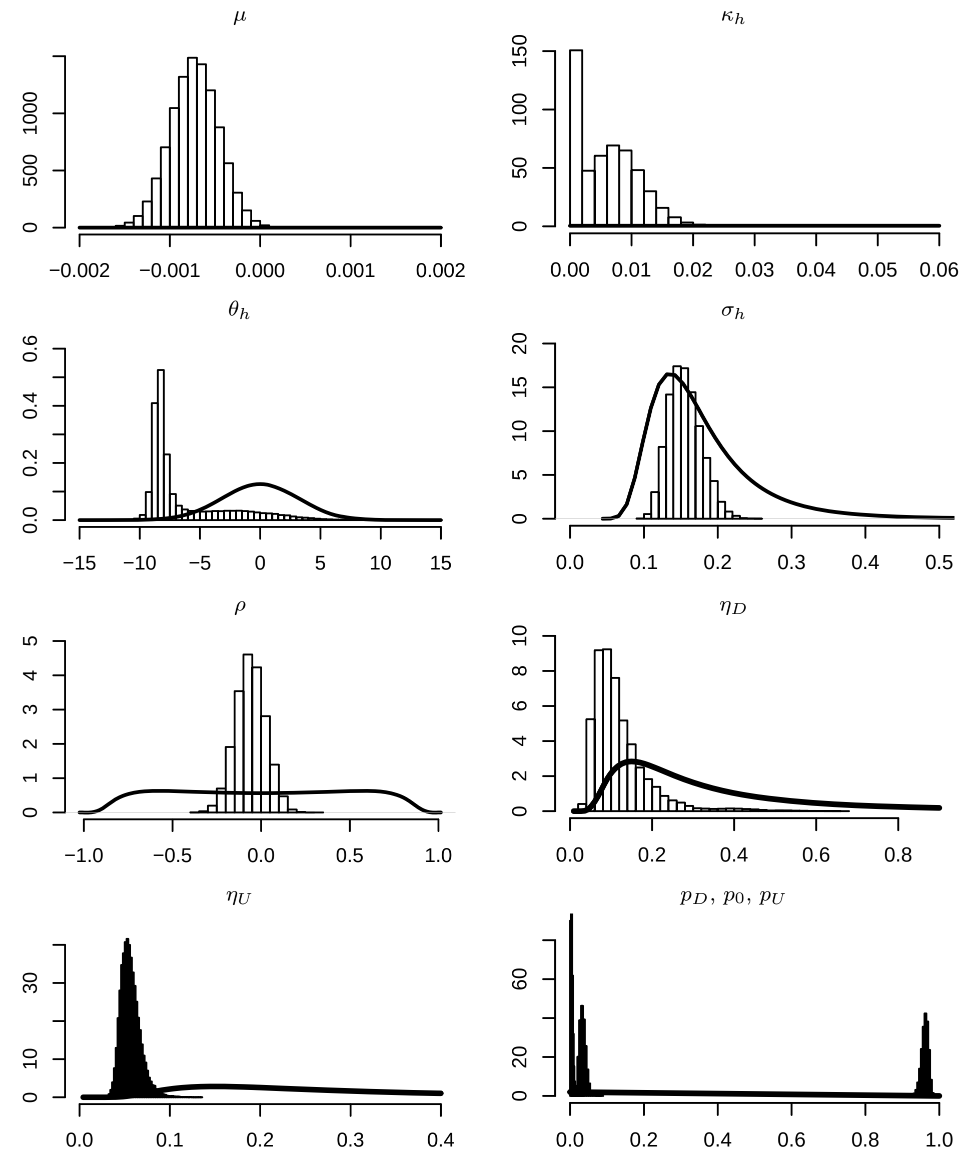
Table 3: The posterior means and standard deviations of the model parameters

θ	$E(\theta\|\,y)$	$D(\theta\|\,y)$
μ	-0.000727	0.000268
κ_h	0.005991	0.004802
θ_h	-6.439377	3.062736
σ_h	0.155582	0.022145
ρ	-0.059986	0.084033
η_D	0.120194	0.070058
η_U	0.056425	0.010924
p_D	0.004585	0.003079
p_0	0.960854	0.009587
p_U	0.035616	0.008762

Figure 3 shows the posterior probabilities of jumps, that is the values of a function $i \to P(q_i \neq 0|\,y) \cdot sign(y_i)$. Upward jumps are notably more frequent. The SV component explains many sharp movements of the series. However, in some cases, the probability of a jump is close to one. It proves that the SV component does not explain the whole dynamics, thereby justifying the need for a jump component to be included in the model structure. Note that the number of jumps depends on the arbitrarily chosen threshold of the posterior probability (here 0.5).

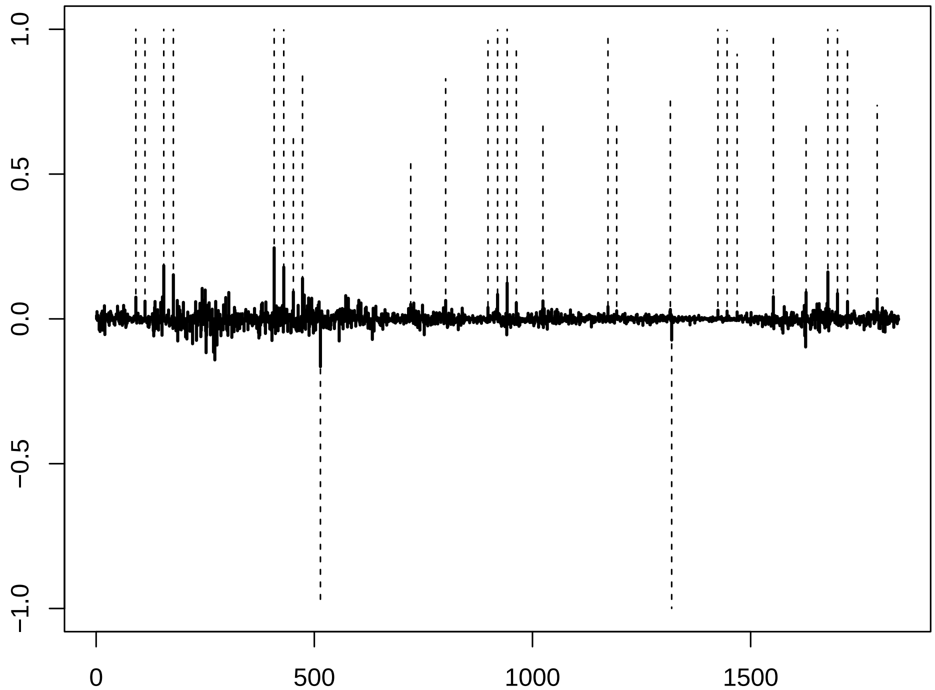
Figure 4 shows the posterior means of the jumps and the posterior means of stochastic volatility and their mirror reflection along the horizontal axis. The volatility tends to cluster into periods of higher and lower logarithmic growth rates variability. The jumps found by the presented technique do not correspond to values below or above

Figure 2: The posterior histograms and prior densities of the model parameters



some fixed thresholds, i.e. outliers. There do exist values classified as jumps whose sizes are lower than some other observations which were not classified as jumps. The highest return featuring a positive jump and the lowest return featuring a negative

Figure 3: Logarithmic growth rates (solid line) and probabilities of jumps times the signs of the logarithmic growth rates (dashed lines)

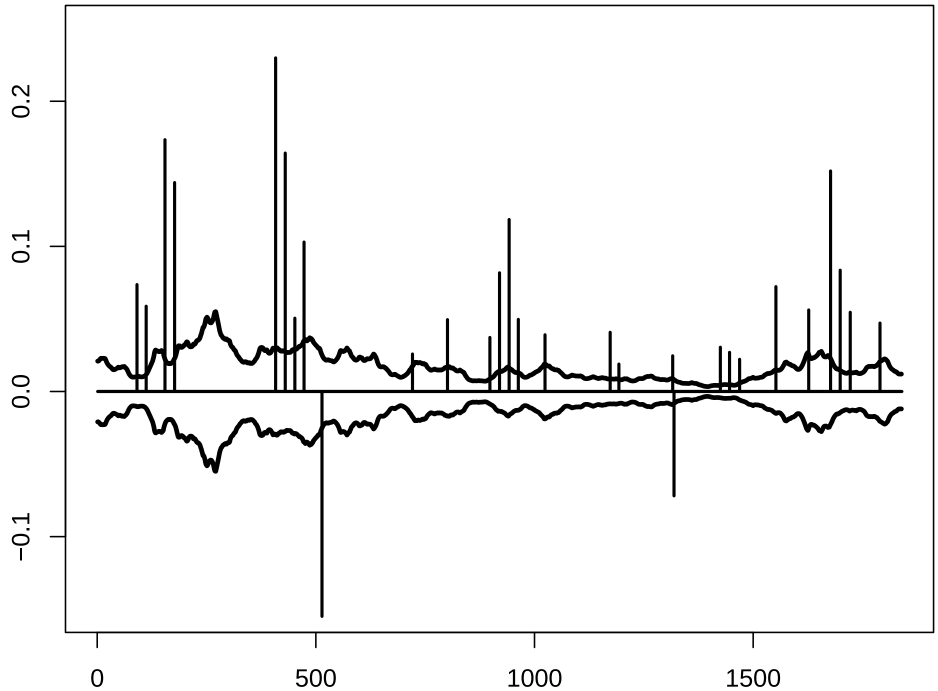


jump (cf. Figure 4) appear on 01.09.2009 (Tuesday) and 01.02.2010 (Monday), respectively. Note that they appear on the first days of the months after trading breaks. An extensive analysis of frequency of jumps and a study of their moments of appearances we defer for further research.

4 Final remarks and conclusions

The Bayesian stochastic volatility model with a leverage effect, normal errors and jump component with the double exponential distribution of a jump value is presented. The double exponential distribution of jumps corresponds with our expectation of asymmetrical (around zero) jump sizes. Moreover, the structure facilitates testing of the volatility persistence and a leverage effect. The Bayesian approach to the model in question is largely facilitated by introducing latent variables, and computationally feasible by means of MCMC methods. The proposed algorithm, combining the Gibbs sampler and the Metropolis-Hastings procedure, is an effective tool of numerical calculations.

Figure 4: Posterior means of jump values and stochastic volatility (continuous line) and their mirror reflection along the horizontal axis



The described theory is exemplified in the empirical study. The results confirm strong persistence of volatility. The methodology proves itself capable of detecting jumps. The SV component explains many sharp movements in the series. However, the SV component does not explain the whole dynamics and the role of the jump component is crucial for the investigated dataset. Detected jumps do not correspond to values below or above some fixed thresholds. This result is in a contradiction to assumptions made in some other papers, where jumps are defined in that fashion (e.g. Janczura, Truck, Weron, and Wolff 2013). The frequency of positive jumps is much higher than negative jumps, and the jump value distribution is nonsymmetric around zero, which is in line with typical empirical features of commodity time series. The leverage effect is negligible and might be omitted from the model.

The SVLEDEJ model seems to be a promising and useful tool in modelling commodity time series. Future research will concern comparison of DEJD, SVLEDEJ and some pure SV structures, especially, the comparison with a fat-tailed SV model e.g. FCSV model (Pajor 2003) seems interesting. Moreover, the analysis of jumps frequency and linking the times of jumps with calendar effects will be conducted.

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