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# Iterated Local Search for Foundry Lotsizing and Scheduling Problem with Setup Costs

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## **Abstract**

The paper presents a novel Iterated Local Search (ILS) algorithm to solve multi-item multi-family capacitated lot-sizing problem with setup costs independent of the family sequence. The model has a direct application to real production planning in foundry industry, where the goal is to create the batches of manufactured castings and the sequence of the melted metal loads to prevent delays in delivery of goods to clients. We extended existing models by introducing minimal utilization of furnace capacity during preparing melted alloy. We developed simple and fast ILS algorithm with problem-specific operators that are responsible for the local search procedure. The computational experiments on ten instances of the problem showed that the presence of minimum furnace utilization constraint has great impact on economic and technological conditions of castings production. For all test instances the proposed heuristic is able to provide the results that are comparable to state-of-the art commercial solver.

Keywords: Application of information technology to the foundry industry, Scheduling, production planning, Local search heuristics

# 1. Introduction

In this paper we are dealing with multi-level, multi-item and multi-family lot-sizing problem which describes production scheduling in the foundries. Lot sizing and scheduling problem for foundries management has been rather rarely studied in the literature. In 2008 de Araujo et al. [1] analyzed the problem of lot-sizing and furnace scheduling in a small foundry in Brazil. Stawowy and Duda presented a series of papers extending this model for two furnaces, two molding lines [2] and core shop planning [3]. Most recently Li et al. [4] presented the production planning problem in a make-to-order foundry in which the total costs of production were being optimized. A review of classical mixed integer programming (MIP) approaches to solve the lot-sizing problem can be found in [5], while a review of various metaheuristics

applied to this problem has been done in [6]. In the latter case, the most popular metaheuristics are genetic algorithms that operate on the population of solutions [7].

In this paper we proposed a simple ILS algorithm that operates only on a single solution, improving it in subsequent iterations. The authors could not found any paper concerning application of ILS to the lot sizing and scheduling problem in foundries. Moreover, the review done by Jans and Degraeve [6] did not indicate the use of ILS algorithm to any of the lot-sizing and scheduling problems. Since then we have found only one paper applying a multi-start ILS for the problem of two-echelon distribution network for perishable products that was partially based on lot-sizing model [8]. This makes the research outlined in this article really the first attempt to develop an ILS algorithm for a standard lot-sizing and scheduling problem.

The particular aim of the paper is to provide an effective heuristic for production planning and scheduling in the single furnace-single casting line system, when setup costs and minimal furnace load are considered. Section 2 provides a MIP model for this problem. In Section 3, the details of proposed heuristic are given. The computational experiments are described in Section 4, and finally, the conclusions are drawn in Section 5.

# 2. Lot-sizing and scheduling model

The MIP model presented in this section is an extension of Araujo et al. [1] lot sizing and scheduling model for mid-size foundries. The extended model introduces minimal utilization of furnace capacity during preparing melted alloy what has not been taken into account yet in the studies of the presented problem.

We use the following notation:

i=1,...,K - produced items; k=1,...,K - produced alloys

t=1,...,N - working days; n=1,...,N - sub-periods

Parameters

 $d_{it}$  - demand for item i in day t

 $w_i$  - weight of item i

 $a_i^k = 1$ , if item i is produced from alloy k, otherwise 0

 $s_k$  - setup penalty for changing alloy to grade k

C - loading capacity of the furnace

 $u_{min}$  - minimum furnace utilization (0 - no restriction, 1 - 100%

 $h_{it}^-$ ,  $h_{it}^+$  - penalty for delaying (-) and storing (+) production of item i in day t

Variables

 $I_{it}^-$ ,  $I_{it}^+$  - number of items i delayed (-) and stored (+) at the end of

 $z_n^k = 1$ , if there is a setup (resulting from a change) of alloy k in sub-period n, otherwise 0

 $y_n^k = 1$ , if alloy k is produced in n in sub-period, otherwise 0  $x_{in}$  - number of items *i* produced in sub-period *n*.

Production planning problem is defined as follows:

Minimize 
$$\sum_{i=1}^{I} \sum_{t=1}^{T} (h_{it}^{-} I_{it}^{-} + h_{it}^{+} I_{it}^{+}) + \sum_{k=1}^{K} \sum_{n=1}^{N} (s_{k} z_{n}^{k})$$
 (1)

$$I_{i,t-1}^{+} - I_{i,t-1}^{-} + \sum_{n=1}^{N} \sum_{k=1}^{K} x_{in} a_{i}^{k} - I_{it}^{+} + I_{it}^{-} \ge d_{it}, \quad i = 1, ..., I, \quad t = 1, ..., T$$
 (2)

$$\sum_{i=1}^{l} w_i x_{in} a_i^k \le C y_n^k, \quad k = 1, ..., K, \quad n = 1, ..., N$$

$$\sum_{i=1}^{l} w_i x_{in} a_i^k \ge u_{\min} C y_n^k, \quad k = 1, ..., K, \quad n = 1, ..., N$$
(4)

$$\sum_{i=1}^{n} w_{i} x_{in} a_{i}^{k} \ge u_{\min} C y_{n}^{k}, \quad k = 1, ..., K, \quad n = 1, ..., N$$
(4)

$$\frac{\overline{i=1}}{z_n^k} \ge y_n^k - y_{n-1}^k, \quad k = 1, ..., K, \quad n = 1, ..., N$$
(5)

$$\sum_{k=1}^{K} y_{k}^{k} = 1, \quad n = 1, ..., N$$
 (6)

$$x_{in} \ge 0, \quad x_{in} \in \mathfrak{I}, \quad I_{in}^-, I_{in}^+ \in \mathfrak{I}, \quad i = 1, ..., I, \quad n = 1, ..., N$$
 (7)

The goal (1) is to find a schedule that minimizes the sum of the costs of delayed production, storage costs of finished goods and the setup cost, if the alloy is changed during furnace load.

Equation (2) balances inventories, delays and the volume of production of each item in each sub-period. Constraint (3) ensures that the furnace capacity is not exceeded in a single load. Constraint (4) restricts minimum furnace utilization to  $u_{min}$ . Constraint (5) sets variable  $z_n^k$  to 1, if there is a change in alloys in the subsequent sub-periods, while constraint (6) ensures that only one alloy is produced in each sub-period.

# 3. Proposed ILS heuristic

Iterated Local Search was introduced by Stützle in his PhD dissertation [9]. It is a simple metaheuristic that has been successfully applied to a wide range of combinatorial problems [10]. The outline of ILS is shown in Figure 1.

```
s0 = Generate initial solution
s* = Local search(s0)
REPEAT
          s' = Perturbation(s*,history)
          s*' = Local search(s')
          s* = Acceptance criterion(s*,s*',history)
UNTIL termination condition met
```

Fig. 1. Outline of ILS metaheuristic

The key elements of ILS are:

- a representation of the problem solution,
- a local search algorithm, producing local optima,
- a perturbation procedure that generates the initial solution for local search,
- an acceptance criterion that indicates whether to change the reference solution s\*,
- a termination condition.

The representation of solution, that is shown in Figure 2, consists of two parts (segments): vector a representing alloy types that are produced in the following sub-periods and table x representing the quantity of items that are produced in these sub-periods.

alloys	$k_1$	 $k_n$	 $k_N$
items	x 11	 $x_{1n}$	 $x_{IN}$
	$x_{i1}$	 x in	 $x_{iN}$
	$x_{I1}$	 $x_{In}$	 $x_{I\!N}$

Fig. 2. Solution representation used in proposed ILS

We used Evolutionary Based Heuristic (EBH) as a local search algorithm as was described in [3]. We employed six operators to generate a local neighbourhood:

Op1 - choose at random one of the sub-periods and alter alloy ID to another drawn from K available,

Op2 - choose at random two different sub-periods and exchange between them the quantities of produced items,

Op3 - like Op2 but the sub-periods' alloys are exchanged too

Op4 - like Op3, with second sub-period lying next to chosen,

Op5 - choose randomly one of the sub-periods and draw the produced quantities of all I items using normal distribution  $N(x_{in},$  $\sigma$ ), where  $x_{in}$  is quantity of item i from previous iteration,  $\sigma$  is parameter (standard deviation) responsible for operation intensity,



Op6 - choose randomly one of I items and change its quantities in all sub-periods using approach described in Op5.

After some preliminary experiments, we have chosen the following parameters of local search procedure:

- population size: 15 individuals,
- termination criterion: after 90 0000 unsuccessful operations (not improving the actual global optimum),
- exponential decreasing of operations intensity from  $\sigma = 15.0$  in first generation to  $\sigma = 0.5$  in the last generation.

The main difficulty with ILS is to make the perturbation gradual as too strong perturbation makes ILS a random search, while too weak - backs the search to the last local optimum. We decided to use an Op7 operation applied to the actual global optimum, i.e. choose at random I points in  $\mathbf{x}$  table and draw items quantity using approach described in Op5.

The acceptance criterion balances intensification and diversification; in this case we decided to accept only improving solutions. The algorithm is terminated after 3,000,000 function evaluations have been performed.

# 4. Computational experiments

### 4.1. Test problems

Experiments have been conducted using similar procedure that has been described in de Araujo et al. [1]. Ten different sets with 50 items made from 10 alloys have been prepared; in each experiment we examined seven level of minimum furnace utilization. The characteristic of the data is given in Table 1.

The values for demand, weight, setup penalty and delaying cost were determined using uniform distribution within a given range. The furnace capacity C was generated using formula corresponding to the total sum of the weights of ordered items:

$$C = \frac{\sum_{i=1}^{I} \sum_{i=1}^{T} d_{ii} w_i}{\sum_{i} M}$$
(8)

Table 1. Test problems characteristics

rest problems characteristics	
Parameter	Value
number of items (I), number of alloys (K)	(50,10)
number of days $(T)$	5
number of sub-periods (N)	10
furnace min load $u_{min}$ [% of $C$ ]	[0, 50, 60, 70, 80, 90, 95]
demand $(d_{ii})$ [items/sub-period]	[10, 60]
weight of item $(w_i)$ [kg]	[2, 50]
setup penalty $(s_k)$ [PLN]	[50, 200]
delaying cost $(h_i^-)$ [PLN/item]	[3.00, 9.00]
holding cost $(h_i^+)$ [PLN/item]	$w_i * 0.02 + 0.05$

### 4.2. Results of the experiments

ILS algorithm was run for 10 times for each instance of the problem. The parameters used in these experiments were defined in Section 3. In order to satisfy constraint (4) we used penalty

function: the value 100,000 was added to the goal function (1) for any sub-period where this constraint was not respected. The starting point was generated randomly taking into account a demand for the various alloys.

The aim of the computational analysis was twofold: firstly, the effect of the minimum capacity constraint on the goal costs and furnace utilization has been investigated, then, a comparison between the solutions obtained by means of proposed algorithm and CPLEX solver has been carried out to determine which procedure performs the better.

A single run for the algorithm took 5 minutes. The results are collected in Tables 2 and 3, where 'average costs' represents the average from 10 runs for the goal function (1) expressed in PLN, and 'furnace utilization' provides the ratio of sum of castings' weights scheduled to the overall furnace capacity in N subperiods.

Table 2. Results of the experiments for average costs

$\#/u_{min}$	0	50	60	70	80	90	95
1	8916.8	8974.2	8856.0	9013.7	8817.9	8977.4	10135.8
2	9714.8	9689.1	9950.6	10001.2	10107.1	10212.5	12258.7
3	8929.7	8897.4	8904.9	8948.3	9253.1	10473.1	15663.9
4	7348.0	7389.5	7425.4	7820.0	8246.0	13648.3	15304.6
5	9019.9	9036.5	9009.4	9000.4	8989.4	9127.8	9942.9
6	8209.8	8194.4	8212.1	8195.8	8276.4	8563.9	12372.9
7	9828.8	9907.7	10000.5	10020.3	10133.7	10542.0	12444.9
8	8021.4	7993.3	8016.1	8014.6	8065.5	8244.3	8786.1
9	10017.9	10028.8	10061.1	10001.4	10103.6	10587.2	11486.6
10	7932.8	7887.6	7963.1	7838.0	8014.2	8537.3	11657.9
Avg.	8794.0	8799.9	8839.9	8885.4	9000.7	9891.4	12005.4
St. d.	898.9	910.6	939.7	899.3	866.9	1605.2	2190.0

Table 3. Results of the experiments for furnace utilization

$\#/u_{min}$	0	50	60	70	80	90	95
1	0.98472	0.96918	0.97706	0.97678	0.97841	0.98266	0.98726
2	0.97445	0.97603	0.97832	0.98181	0.98495	0.98953	0.99100
3	0.96129	0.96615	0.95632	0.95612	0.96414	0.97454	0.98415
4	0.97005	0.96942	0.97011	0.97211	0.97418	0.97862	0.98540
5	0.98156	0.97917	0.98281	0.98345	0.98408	0.98680	0.98887
6	0.97041	0.97310	0.97285	0.97244	0.97332	0.98247	0.98656
7	0.96615	0.96882	0.96844	0.97354	0.97992	0.98736	0.98989
8	0.97733	0.97781	0.97722	0.97837	0.97858	0.98384	0.98669
9	0.98545	0.97684	0.97610	0.97788	0.98032	0.98622	0.99064
10	0.96541	0.96306	0.96462	0.96433	0.96777	0.97507	0.98625
Avg.	0.97368	0.97196	0.9723	0.9736	0.9765	0.9827	0.98767
St. d.	0.00842	0.00543	$0.007\bar{7}$	0.0082	0.0067	0.0051	0.00232

A detailed analysis of the results demonstrated that the increase in the lower limit increases the level of the furnace utilization, but the process economy is decreasing (goal costs increase). This is particularly evident for  $u_{min}$ =95% when compared to the other values of the lower limit: excessive focus on full use of the furnace causes unnecessary items to be produced, which in turn results in high costs of delaying, storing and setup.

The average utilization of the furnace is very high (above 97%) regardless of accepted lower limit, but if there is no limit, then in some sub-periods the furnace is used only in 40%. So it is worth taking this restriction to ensure economic and technological conditions of castings' production.

The one-way analysis of variance (ANOVA) have been performed to investigate the influence of the furnace minimum utilization; the test level of significance  $\alpha$  has been set equal to 0.05. The ANOVA shows that the differences in the goal costs and the furnace utilization are affected by the minimum capacity constraint (critical value for F test with 6 and 63 degrees of freedom at  $\alpha = 0.05$  is 2.246, while obtained F statistics had values greater than 8.2).

Finally, we made a multiple paired sample t test at the alphalevel 0.05 for the average results achieved by ILS at different minimum load levels. The results, not presented here, prove that for  $u_{min} = 0$ , 50, 60, 70, 80 there are no statistically significant differences in average costs and furnace utilization, while differences between this group and  $u_{min} = 90$ , 95 are statistically significant. Therefore,  $u_{min}$  was settled equal to 50% when comparing the proposed heuristic to CPLEX solver as the goal costs for this  $u_{min}$  are little lower than for the other values of considered parameter.

The results achieved for all 10 instances are presented in Table 4. Solution in MIP column provides the results for CPLEX solver (with time limit equal to 10 minutes). Column RH provides the results for the rolling horizon relax and fix approach proposed by de Araujo et al. [1], and ILS represents the results of proposed heuristic in the following order: the best result out of ten runs, average result and the standard deviation of the solutions.

Table 4. Results of the experiments for  $u_{min} = 50\%$ 

#	MIP	RH	ILS <sub>best</sub>	$ILS_{avg}$	$ILS_{sd}$
1	9014.1	8106.9	8675.5	8974.2	274.14
2	9873.0	9051.1	9531.0	9689.1	130.62
3	9121.0	8298.0	8742.1	8897.4	101.95
4	8449.8	6484.3	7245.9	7389.5	112.95
5	10758.8	8239.7	8959.4	9036.5	88.68
6	8435.8	7583.5	8048.0	8194.4	105.06
7	9840.9	8895.5	9751.8	9907.7	129.85
8	8512.2	7464.3	7876.6	7993.3	92.49
9	10090.5	9233.3	9731.0	10028.8	179.11
10	8006.2	7464.7	7679.3	7887.6	129.83
Avg.	9210.2	8082.1	8624.0	8799.9	
St. d.	846.84	853.89	876.03	910.59	

The proposed ILS heuristic performed on average significantly better than CPLEX solver. Rolling horizon approach performed definitely the best. The difference between RH and ILS best solution was 6.7% on average and 8.9% when average ILS solution was taken into account. However, it is worth noticing that the difference varied depending on the data structure and for fourth instance it reached 14.0%. When not counting this worst case, the difference between RH and the best ILS solutions drops to 6.3%, while for average ILS solutions drops to 8.4%.

#### 5. Conclusions

The scheduling problem for automated foundry has been investigated and modeled as a lot-sizing and scheduling problem with sequence-independent set-up costs and lower limit of furnace utilization. The performances of proposed iterated search heuristic

and the CPLEX solver have been investigated. The obtained results show that the ILS algorithm outperforms the commercial tool for all examined instances. This is due to the use of the evolutionary based heuristic at the local search stage, and the use of the effective perturbation procedure. Investigating the effects of the minimum furnace load on the goal costs completed the study. Six different levels of the lower limit have been considered and compared with a configuration assuming no minimum furnace utilization constraint. The analysis shows that the presence of this constraint has great impact on economic and technological conditions of foundry production.

There are certain opportunities to further improve the proposed approach. First, new ways of generating local neighborhoods using a specific knowledge of the problem can be developed. Second, some effective local search heuristics may be used instead of EBH during local optimization stage.

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