

## **Modeling Macro-Financial Linkages: Combined Impulse Response Functions in SVAR Models**

Dobromił Serwa,\* Piotr Wdowiński†

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### **Abstract**

We estimated a structural vector autoregressive (SVAR) model describing the links between a banking sector and a real economy. We proposed a new method to verify robustness of impulse-response functions to the ordering of variables in an SVAR model. This method applies permutations of orderings of variables and uses the Cholesky decomposition of the error covariance matrix to identify parameters. Impulse response functions are computed and combined for all permutations. We explored the method in practice by analyzing the macro-financial linkages in the Polish economy. Our results indicate that the combined impulse response functions are more uncertain than those from a single model specification with a given ordering of variables, but some findings remain robust. It is evident that macroeconomic aggregate shocks and interest rate shocks have a significant impact on banking variables.

**Keywords:** vector autoregression, Cholesky decomposition, combined impulse response, banking sector, real economy

**JEL Classification:** C32, C51, C52, C87, E44, E58

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## 1 Introduction

We analyze the linkages between a banking sector and a real economy within a structural vector autoregressive framework (SVAR). There is an ongoing debate on the appropriate structure of SVAR models containing banking and real variables to identify the shocks affecting the real economy through the credit channel as an alternative to the interest rate channel. Various identification methods of SVAR models are used. These methods include short-term zero restrictions (e.g., Sims, 1980; Bernanke, 1986), long-term restrictions (e.g., Blanchard and Quah, 1989; Pagan and Pesaran, 2008; Caporale *et al.*, 2014), both short- and long-run restrictions (King *et al.*, 1991), sign restrictions (e.g., Canova and De Nicoló, 2002; Uhlig, 2005; Meeks, 2012) in SVAR models, long-term identifying restrictions in structural vector error correction models (SVEC) (e.g., Gonzalo and Ng, 2001; Iacoviello and Minetti, 2008), identification through changes in the volatility of residuals (Rigobon, 2003; Rigobon and Sack, 2003; Lanne and Lütkepohl, 2008; Lanne *et al.*, 2010), as well as shock variables identified outside the VAR (e.g., shocks estimated using lending survey data; Ciccarelli *et al.*, 2015).

An application of the short-term zero restrictions is the most common approach due to its relative simplicity and mild assumptions on the contemporaneous relationships among the variables of the SVAR system. These mild restrictions leave a large space for the effects driven by economic data. On the other hand, economic assumptions in such models introduce the risk of misspecified restrictions and assumption-driven results.

In this article, we propose a simple robustness analysis for SVAR models with short-term zero restrictions. A popular approach to deal with uncertainty surrounding an economic structure of the model is to use the Cholesky decomposition of the error covariance matrix and to orthogonalize the structural shocks. This method depends on the ordering of variables in a VAR model. In the Cholesky decomposition, the variables placed first affect other variables immediately and the other variables affect those placed first only with a lag. Accordingly, the ordering of variables may have a crucial impact on the impulse response functions in the estimated SVAR model. Indeed, the contemporaneous responses to shocks are usually the strongest and they tend to die out through time.

Our approach is to account for the differences in impulse response functions depending on the ordering of variables (see e.g., Amisano and Giannini, 1997; Lütkepohl, 2007 for a discussion on SVAR models under different orderings of variables). We propose a method to mix impulse responses from different model specifications and to build a “combined” impulse response function robust to the ordering of variables. This approach is similar to the generalized impulse response functions (GIRF) analyzed by Koop, Pesaran, and Potter (1996) for nonlinear models and proposed by Pesaran and Shin (1998) for linear vector autoregression models. However, our “combined” impulse response functions do not share the property of GIRFs to generate remarkable reactions to all shocks as if each shock was generated by the variable ordered first in the

Cholesky decomposition approach—a strong assumption in practice (cf. Proposition 3.1 in Pesaran and Shin, 1998; Kim, 2012). Our approach is also similar to the method of imposing sign restrictions on the realizations of impulse response functions (e.g., Fry and Pagan, 2011). However, it does not use any out-of-sample information imposed by sign restrictions and it is simpler to implement because it considers only a finite number of impulse responses.

Our empirical results reveal that some shock effects identified using the traditional recursive method are based on strong assumptions and are not robust to changing model specifications. In turn, the effects measured by using GIRFs of Pesaran and Shin (1998) tend to identify too many macro-financial linkages. The “combined” impulse response analysis identifies much fewer links between the real and financial sectors than do the standard approaches. It turned out that the interest rate affects banking and real variables, whereas the credit market conditions have no statistically significant impact on the macroeconomic variables.

The paper is structured as follows. We explain the links between the banking and real sectors in Section 2. The econometric method is presented in Section 3. Section 4 contains empirical results. We end with conclusions.

## 2 Dependence between banking and real sectors

Banks provide various services to the financial and real sectors of the economy. Channeling financial resources between savers and borrowers through deposit and credit intermediation, i.e., creating liquidity in the economy, is the most important role of banks. Other major economic functions of banks include credit quality assessment and improvement, settlement of payments, and managing the maturity mismatch between assets and liabilities. All these functions generate wealth effects for households and corporations in the long run. The short-run effects are also intense, as the banking sector influences the economy through the interest rate and credit channels. These effects are managed by changing the interest rates or by adjusting the lending and borrowing volumes, respectively. Such adjustments affect both consumption and investment. In turn, the real sector should also have a strong impact on financial sector activities through the aggregate growth and unemployment, as it affects the demand for loans and supply of deposits, the quality of loans, asset prices, and hence the value of collateral. It is evident that the financial and real sectors are interconnected.

The most popular tool to analyze the linkages between business and financial cycles in the short and medium run is a vector autoregressive model (VAR). Most studies utilizing VARs aim at measuring the response of macroeconomic variables to shocks in the financial sector, including credit supply and demand shocks, interest rate shocks, and asset price shocks. The two prevailing tools used in these investigations are impulse response analysis and forecast error variance decomposition. They are often accompanied by analyses of causality between the real and financial variables.

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A few studies present historical decompositions of macroeconomic aggregates, most importantly GDP, to observe the changing factors influencing these aggregates through time. Tables 1 and 2 present selected research.

The typical variables used in these analyses are: (1) macroeconomic aggregates such as GDP, price indices, and unemployment; (2) banking sector measures including credit or deposit aggregates, interest rate spreads, measures of loan quality and financial position of banks; (3) policy instruments, e.g., the exchange rate and the short-term market interest rate. The variables are investigated either in log-levels or log-differences.

Most research analyzes the impact of banking sectors on real sectors through two channels (apart from the analyses of the interest rate channel not necessarily linked to the role of the banking sector), namely the bank lending channel (i.e., credit view) and the balance sheet channel (i.e., balance sheet view). The credit view assumes that credit supply shocks, directly affecting consumption and investment in the real economy, are caused by factors related to the financial situation of banks. These factors include changes to lending policies of banks, adjustments in the regulatory framework, modifications of monetary policies, as well as funding shocks to banks, or even banking crises. In line with the balance sheet view, financial conditions of households and corporations affect their ability to borrow depending on the value of their eligible collateral, credit risk, monitoring costs for banks, price of loans, and other similar factors.

The economic identification of the above-mentioned shocks is of crucial importance in SVAR models. Economic theories are often suitable and enable researchers to impose short-term or long-term restrictions on parameters in VAR models. When well-established economic theories are unavailable, an ad-hoc approach is to use recursive restrictions. This is done by using the Cholesky decomposition of the error covariance matrix to identify structural shocks in SVAR models (e.g., Bernanke, 1986; Gilchrist and Zakrajšek, 2012). Importantly, analysts often consider alternative restriction schemes to assess robustness of their results to different model specifications. Unfortunately, the choice of alternative identifying restrictions is arbitrary and the number of alternative models considered by practitioners is usually limited. Hence, this leaves some room for model misspecification. Other identification methods include sign restrictions in Bayesian VAR models, cointegrating restrictions in vector error correction models (VEC), and measures of shocks constructed outside the VAR model (e.g., by using financial instruments or survey data) (Chrystal and Mizen, 2002; Meeks, 2012; Bassett *et al.*, 2014, and other research listed in Tables 1 and 2).

Identified impulse responses demonstrate relationships between the endogenous variables in a VAR model. The results obtained so far in the literature suggest that credit shocks have a strong influence on real economic growth, especially during financial crises. Depending on the study, credit shocks were responsible for 10–20% of a decrease in GDP in the euro zone, the UK, and the United States, 30–50% of

Table 1: Analyzing the linkages between banking and real sectors with VAR models

Study	Model	Causality	Identifying restrictions	C	I	F	H	Other issues
Barnett, Thomas (2013)	SVAR		identifying shocks	+	+		+	
Bassett <i>et al.</i> (2014)	SVAR		economic identification			+		stationary variables
Berkelmans (2005)	SVAR		economic restrictions			+	+	variables in levels (nonstationary)
Bermanke (1986)	SVAR		economic identification			+	+	variables in levels (nonstationary) (log) and growth rates
Bezemer, Grydaki (2014)	VAR	+						stationary variables
Caporale <i>et al.</i> (2014)	SVAR	+	long-run restrictions			+		stationary variables
Chrystal, Mizen (2002)	SVECM					+		variables in levels (nonstationary) (log)
Ciccarelli <i>et al.</i> (2015)	BVAR		recursive restrictions, alternative specifications			+		
Elbourne (2008)	SVAR		testing overidentifying restrictions	+		+		variables in levels (nonstationary), analyses of alternative scenarios
Finlay, Jääskelä (2014)	BVAR		sign restrictions			+	+	variables: mainly growth rates, demand and supply credit shocks identified
Gambetti, Musso (2012)	TVP-VAR		sign restrictions			+		alternative scenarios
Gilchrist, Zakrajšek (2012)	SVAR		recursive restrictions			+	+	growth rate variables
Halvorsen, Jacobsen (2014)	SVAR		sign restrictions, alternative specifications			+		stationary variables

Note: Respective studies are presented in separate rows. The “+” sign indicates that a given analysis has been considered in the respective study.

C – Cointegrating relation; I – Impulse response analysis; F – Forecast error variance decomposition; H – Historical decomposition

Table 2: Analyzing the linkages between banking and real sectors with VAR models

Study	Model	Causality	Identifying restrictions	C	I	F	H	Other issues
Iacoviello, Minetti (2008)	VAR		recursive restrictions, short- and long-run restrictions					
Karfakis (2013)	VAR	+						stationary variables
Kim, Rousseau (2012)	VAR, VEC	+						
Lown, Morgan (2006)	VAR	+						variables in levels (nonstationary)
Magkonis, Tsopanakis (2014)	SVAR		recursive restrictions, sign restrictions, testing overidentifying restrictions					
Meeks (2012)	BVAR		sign restrictions					variables in levels (nonstationary) (log)
Milcheva (2013)	SVAR							variables in levels (nonstationary), model simulations
Mumtaz <i>et al.</i> (2015)	SVAR, DSGE		recursive restrictions, sign restrictions, moment equations					variables in levels (nonstationary), simulating VAR and DSGE models
Musso, Neri, Stracca (2011)	SVAR		recursive restrictions					variables in levels (nonstationary) (log)
Safaei, Cameron (2003)	SVAR		economic identification					stationary variables
Tamási, Világi (2011)	BSVAR		sign restrictions and zero restrictions					
Walentin (2014)	SVAR							variables in levels (nonstationary)

Note: Respective studies are presented in separate rows. The “+” sign indicates that a given analysis has been considered in the respective study. C – Cointegrating relation; I – Impulse response analysis; F – Forecast error variance decomposition; H – Historical decomposition

production slowdown in Austria, Canada, and the UK, and up to a 60% fall in real output in the United States during the recent global financial crisis (Bernanke, 1986; Berkelmans, 2005; Gambetti and Musso, 2012; Meeks, 2012; Bezemer and Grydaki, 2014; Finlay and Jääskelä, 2014; Halvorsen and Jacobsen, 2014). Financial shocks caused up to 50% of volatility in GDP growth in the United States and in the G7 countries (Jermann and Quadrini, 2012; Magkonis and Tsopanakis, 2014). The identification of banking channels responsible for real effects revealed that the credit channel was active in Canada, Finland, the UK, and in the euro zone. In turn, the balance sheet channel was found important in the United States and Germany (Chrystal and Mizen, 2002; Safaei and Cameron, 2003; Lown and Morgan, 2006; Iacoviello and Minetti, 2008; Tamási and Világi, 2011; Musso *et al.*, 2011; Ciccarelli *et al.*, 2015).

It is important to precisely specify the banking variables to be considered. We found that the measures of credit rationing better explained real output than credit spreads. On the other hand, default risk affected credit spreads and influenced the economy (Hall, 2011; Bassett *et al.*, 2014; Caporalle *et al.*, 2014). Several studies found lending market activity (measured with credit spread) to lead or to “predict” the real business cycle (Balke, 2000; Gilchrist *et al.*, 2009; Gilchrist and Zakrajšek, 2012; Karfakis, 2013). The interactions between the banking and real sectors in Poland have been rarely investigated with SVAR models (e.g., Wdowiński, 2013). Many investigations focused mainly on the role of monetary policy and its effects on the real economy, but the role of banking variables has remained unexplored (Brzoza-Brzezina, 2002; Waszkowski and Czech, 2012; Haug *et al.*, 2013; Kapuściński *et al.*, 2014; Bogusz *et al.*, 2015). The need to explore the role of banking variables further motivates our research.

### 3 Combining results from SVAR models

The identification methods for SVAR models include approaches utilizing out-of-sample economic information, e.g., sign restrictions, short and long-term restrictions, and approaches incorporating additional in-sample statistical information, e.g., the identification-through-heteroscedasticity methods and methods utilizing non-gaussian error distributions. Each of these approaches has strengths and weaknesses (e.g., Lanne and Lütkepohl, 2010; Fry and Pagan, 2011; Kilian, 2013; Gouriéroux and Monfort, 2014; Lanne, Meitz, and Saikkonen, 2015; Lütkepohl and Netsunajev, 2015). As we want to show how economic assumptions about specific macro-financial linkages affect results, we concentrate on the most popular approach, i.e., short-term zero restrictions. Moreover, the total number of possible short-term just-identifying zero restrictions in a medium-size VAR model is so large that it prohibits investigating all of them in practice. Therefore, we consider only recursive identification schemes and limit the number of investigated specifications in this way.

The recursive method is usually used when economic theory does not provide

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a clear view of a structural model. In this article, we assume no preference regarding the economic structure of our model of macro-financial linkages. Permuting the ordering of variables in the recursive method enables verifying robustness of dependencies between economic variables. This can be done by verifying some extreme restrictions (when linkages between the first and the last variable are analyzed) and milder restrictions (when linkages between two neighboring variables are analyzed). Moreover, combining recursive restrictions is a useful procedure when the aim is to search for significant linkages between economic variables rather than to identify specific economic shocks (e.g., Diebold and Yilmaz, 2009; Klößner and Wagner, 2014). In this article, we do not identify any specific economic or financial shocks, but instead we search for linkages between banking and macroeconomic variables.

A typical VAR model explaining the linkages between the banking and real sectors in a small open economy contains three sets of variables. The first set includes aggregate macroeconomic variables, such as GDP or components of final demand (e.g., consumption and investments), and a price index. The second set is composed of financial variables, such as the monetary aggregate, the value of banking loans, the interest rate spread, and other measures of banking sector activity. The third set consists of financial market variables usually related to monetary policy instruments, e.g., the exchange rate and the short-term interest rate. This set may also consider instruments of macroprudential policy, e.g., regulatory capital buffers, liquidity measures, and leverage.

The identifying short-run restrictions are usually imposed in the form of zero restrictions under certain ordering of variables. The typical ordering is that the variables from the first set (macroeconomic aggregates) immediately affect all other variables in the model and the variables from the second set (banking sector variables) affect the variables from the third set (policy variables). However, the effects in the opposite direction are only possible with a lag. The identifying conditions are imposed by zero recursive restrictions in the form of Cholesky decomposition of the error covariance matrix.

Our aim is to assess the robustness of a given causal dependence between macroeconomic and financial variables by combining results from many impulse response functions depending on different specifications of the SVAR model. We follow two approaches. First, we consider all possible orderings of endogenous variables that are specified in a given VAR model. Second, we fix the order of selected variables and consider all orderings of the remaining variables. In either case, we identify the model by using the Cholesky decomposition and calculate impulse responses. The respective impulse responses from different permutations (orderings of variables) are then combined into the augmented impulse response.

In this approach, some orderings of variables may seem economically implausible, but they are observationally equivalent and as such are included in the augmented impulse response function. This corresponds to the situation where a researcher has no prior knowledge of the dependencies between the real and financial variables.

The combination of impulse responses is then used to identify the most invincible links between the real and financial sectors. By permuting the order of variables, we gather additional information on the robustness of the analyzed impulse responses. We introduce our method below. Let us consider the vector autoregressive model (Lütkepohl, 2007, pp. 18–40):

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{i=1}^p \boldsymbol{\Phi}_i \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t, \quad t = 1, 2, \dots, T \quad (1)$$

where  $\mathbf{y}_t = (\mathbf{w}'_t, \mathbf{x}'_t, \mathbf{z}'_t)'$  is a three-block vector ( $m \times 1$ ), where  $\mathbf{w}_t$  is a vector of macroeconomic aggregates and prices,  $\mathbf{x}_t$  is a vector of banking sector variables, and  $\mathbf{z}_t$  is a vector of financial market variables,  $\boldsymbol{\Phi}_i$  are fixed ( $m \times m$ ) coefficient matrices,  $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \dots, \varepsilon_{mt})'$  is a Gaussian white noise process,  $E(\boldsymbol{\varepsilon}_t) = 0$ ,  $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t) = \boldsymbol{\Sigma}_\varepsilon$ ,  $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_s) = 0$  for  $t \neq s$ , and  $\boldsymbol{\Sigma}_\varepsilon$  is the covariance matrix of the error term.

Assuming that the VAR model (1) is stable, it has the following moving average representation:

$$\mathbf{y}_t = \mathbf{c} + \sum_{i=0}^{\infty} \mathbf{A}_i \boldsymbol{\varepsilon}_{t-i} \quad (2)$$

The coefficient matrices  $\mathbf{A}_i$  can be obtained from the following recursive formula:

$$\mathbf{A}_i = \sum_{j=1}^i \mathbf{A}_{i-j} \boldsymbol{\Phi}_j, \quad i = 1, 2, \dots \quad (3)$$

with  $\mathbf{A}_0 = \mathbf{I}_m$  and  $\boldsymbol{\Phi}_j = 0$  for  $j > p$ . The constant term can be obtained from  $\mathbf{c} = (\mathbf{I}_m - \boldsymbol{\Phi}_1 - \dots - \boldsymbol{\Phi}_p)^{-1} \boldsymbol{\mu}$ .

The traditional approach to compute impulse response functions has been suggested by Sims (1980). The impulse response function (IRF) of a one standard deviation structural shock to the  $i$ th variable in  $\mathbf{y}_t$  on the  $j$ th variable in  $\mathbf{y}_{t+n}$  is given by:

$$\psi_{ji}(n) = \mathbf{e}'_j \mathbf{A}_n \mathbf{P} \mathbf{e}_i, \quad n = 0, 1, 2, \dots \quad (4)$$

where  $\mathbf{e}_i$  is a column selection vector with unity as the  $i$ th element and zeros otherwise,  $\mathbf{P}$  is a lower triangular matrix obtained by decomposing the covariance matrix  $\boldsymbol{\Sigma}_\varepsilon$  using the Cholesky method so that  $\mathbf{P} \mathbf{P}' = \boldsymbol{\Sigma}_\varepsilon$ .

In turn, the generalized impulse response function (GIRF) suggested by Pesaran and Shin (1998) is given by:

$$\psi_{ji}^g(n) = \mathbf{e}'_j \sigma_{ii}^{-1/2} \mathbf{A}_n \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i, \quad n = 0, 1, 2, \dots \quad (5)$$

where  $\sigma_{ii}$  is the  $ii$ th element of  $\boldsymbol{\Sigma}_\varepsilon$  (see also Koop, Pesaran, and Potter, 1996). One problem with impulse response functions calculated using the Cholesky decomposition is that their values may heavily depend on the order of equations (and hence variables)

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in the SVAR model and in the covariance matrix  $\Sigma_\varepsilon$ . An important advantage of GIRF over a standard impulse response function is that the former is invariant to the ordering of equations in the VAR. One disadvantage is that the method treats all the shock variables as if they were ordered first in a VAR. In practice, GIRFs generate responses that are larger and more frequently statistically significant than ordinary IRFs. Therefore, using GIRFs may result in misleading inferences caused by their extreme identification schemes (Kim, 2012).

We propose an alternative approach to obtain impulse response functions invariant to the ordering of variables. In this approach, we combine impulse response functions from all permutations of SVAR orderings. In an SVAR model with  $m$  endogenous variables, the number of all orderings of variables is equal to the number of permutations, i.e.,  $m!$  ( $m$  factorial). The approach of combining impulse responses is similar to the one considered by Diebold and Yilmaz (2009) and Klößner and Wagner (2014) who computed average generalized forecast error variance decompositions to calculate spillover effects between economic variables, e.g., asset returns. Other algorithms to find the correct identification structure in an SVAR model include the automated general-to-specific model selection procedures and the graph-theoretic causal search algorithm (e.g., Krolzig, 2003; Hoover, 2005).

Let  $\{k\}$  denote the  $k$ th variable ordering in the  $m$ -variable SVAR ( $k = 1, 2, \dots, m!$ ) and  $\psi_{ji}^{\{k\}}(n)$  be the impulse response function of a one standard deviation shock to the  $i$ th element  $\mathbf{y}_t$  on the  $j$ th element of  $\mathbf{y}_{t+n}$ . The combined impulse response function is defined as:

$$\bar{\psi}_{ji}(n) = \frac{1}{m!} \sum_{k=1}^{m!} \psi_{ji}^{\{k\}}(n), \quad n = 0, 1, 2, \dots \quad (6)$$

As we assume no prior knowledge on the ordering of variables in a given SVAR model, we can only use the statistical inference about the likelihood of different specifications of the model. However, for the just-identified SVAR model, i.e., under the Cholesky decomposition, the likelihood function has the same value under each permutation because all orderings are observationally equivalent. Therefore, each impulse response function  $\psi_{ji}^{\{k\}}(n)$  has weight equal to  $\frac{1}{m!}$  in equation (6).

When in fact we use some prior knowledge and recognize that some orderings have no economic interpretation, we can rule out certain permutations. For example, we can assume that real shocks to  $\mathbf{w}_t$  may affect all other variables instantaneously, and banking and financial variables in  $\mathbf{x}_t$  and in  $\mathbf{z}_t$  can affect  $\mathbf{w}_t$  only with a lag. In such a case, the number of Cholesky decompositions is significantly reduced and equals  $m^* = (m_x + m_z)!$ , where  $m_w$ ,  $m_x$ , and  $m_z$  are the numbers of variables in vectors  $\mathbf{w}_t$ ,  $\mathbf{x}_t$ , and  $\mathbf{z}_t$ , respectively. Then the combined impulse response function is given by the following:

$$\bar{\psi}_{ji}^{wxz}(n) = \frac{1}{m^*} \sum_{k=1}^{m^*} \psi_{ji}^{\{k\}}(n), \quad n = 0, 1, 2, \dots \quad (7)$$

where  $\{k\}$  denotes the  $k$ th ordering of variables in the  $m$ -variable SVAR with variables in  $\mathbf{w}_t$  always preceding variables in  $\mathbf{x}_t$  and  $\mathbf{z}_t$ .

In practice, the coefficients in matrices  $\Phi_i$  and the elements of the covariance matrix  $\Sigma_\varepsilon$  are unknown and have to be estimated. Therefore, the values of the impulse response functions need to be estimated as well. Lütkepohl (1990) provides asymptotic distributions of impulse response function estimates under the assumption of normal disturbances in a VAR. Pesaran and Shin (1998) present asymptotic distributions of GIRF estimates.

Let  $\mu_{ji}^{\{k\}}(n)$  be the mean estimate of the impulse response function  $\psi_{ji}^{\{k\}}(n)$  in the  $k$ th variable ordering and  $\sigma_{ji}^{\{k\}}(n)$  be its estimated variance. We can obtain a mean estimate of the combined impulse response function  $\bar{\psi}_{ji}(n)$  defined in (6) by considering a mixture of normally distributed estimates of  $\psi_{ji}^{\{k\}}(n)$  for all  $k = 1, 2, \dots, m!$ . The mean of the normal mixture equals:

$$\bar{\mu}_{ji}(n) = \frac{1}{m!} \sum_{k=1}^{m!} \mu_{ji}^{\{k\}}(n), \quad n = 0, 1, 2, \dots \quad (8)$$

The variance of the mixture is given by:

$$\bar{\sigma}_{ji}(n) = \frac{1}{m!} \sum_{k=1}^{m!} \sigma_{ji}^{\{k\}}(n) + \frac{1}{m!} \sum_{k=1}^{m!} \left( \mu_{ji}^{\{k\}}(n) - \bar{\mu}_{ji}(n) \right)^2, \quad n = 0, 1, 2, \dots \quad (9)$$

Similarly,  $\bar{\psi}_{ji}^{wxz}(n)$  can be approximated with a mixture of normally distributed estimates of  $\psi_{ji}^{\{k\}}(n)$  for these permutations ( $k = 1, 2, \dots, m^*$ ) where variables in  $\mathbf{w}_t$  precede those in  $\mathbf{x}_t$  and variables in  $\mathbf{x}_t$  precede those in  $\mathbf{z}_t$ . The mean of this mixture equals:

$$\bar{\mu}_{ji}^{wxz}(n) = \frac{1}{m^*} \sum_{k=1}^{m^*} \mu_{ji}^{\{k\}}(n), \quad n = 0, 1, 2, \dots \quad (10)$$

The respective variance is given by:

$$\bar{\sigma}_{ji}^{wxz}(n) = \frac{1}{m^*} \sum_{k=1}^{m^*} \sigma_{ji}^{\{k\}}(n) + \frac{1}{m^*} \sum_{k=1}^{m^*} \left( \mu_{ji}^{\{k\}}(n) - \bar{\mu}_{ji}(n) \right)^2, \quad n = 0, 1, 2, \dots \quad (11)$$

After combining impulse responses, we can proceed with two results. First, we decompose the joint uncertainty  $\bar{\sigma}_{ji}(n)$  of the combined impulse response into two components presented in equation (9). The first component  $\frac{1}{m!} \sum_{k=1}^{m!} \sigma_{ji}^{\{k\}}(n)$  describes the average uncertainty of estimated model parameters. The second component  $\frac{1}{m!} \sum_{k=1}^{m!} \left( \mu_{ji}^{\{k\}}(n) - \bar{\mu}_{ji}(n) \right)^2$  is related to the dispersion of individual impulse responses in different variable orderings (i.e., permutations). The same

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interpretation applies to the variance defined in equation (11).

Second, the joint uncertainty makes it possible to assess statistically significant impulse responses to orthogonal shocks. We verify the statistical significance of combined impulse response functions. For a normal distribution, the two-sigma confidence interval  $\langle \bar{\mu} - 2\bar{\sigma}; \bar{\mu} + 2\bar{\sigma} \rangle$  includes 95.5% of observations. Even if the distribution is not known, at least 75% of observations lie inside this interval according to Chebyshev's inequality. In general, it is also possible to calculate quantiles of a mixture of normal distributions. As the closed-form expression for the quantile function is not available, a non-linear optimization problem must be solved to compute these quantiles (e.g., Gilchrist, 2000; Rahman *et al.*, 2004). In our empirical analysis, we used the two-sigma interval for the estimated combined impulse response functions to assess their uncertainty.

## 4 Empirical results

In this section, we present results from our empirical analysis. We estimated SVAR models describing the linkages between the banking and real sectors in Poland. The Polish banking sector is interesting to investigate because of its moderate size and simple structure, typical for emerging and less developed economies. It contains around 70 commercial banks and branches of foreign banks. Banking assets account for 86% of GDP and they have been growing rapidly in recent years (Polish Financial Supervision Authority, PFSA, 2014). The analysis of the banking sector in Poland is facilitated by the fact that banking activities are traditional. The banks concentrate mainly on lending to local companies and households. This may indicate that links between bank activities and macroeconomic developments are much more straightforward than in other developed banking sectors. Another important characteristic of the Polish banking sector during the past 15 years has been its unique robustness against financial crises and bank defaults. Therefore, we may avoid the risk of major structural shocks and nonlinearities caused by crises and other turbulences in the banking sectors of more developed economies by analyzing the Polish economy.

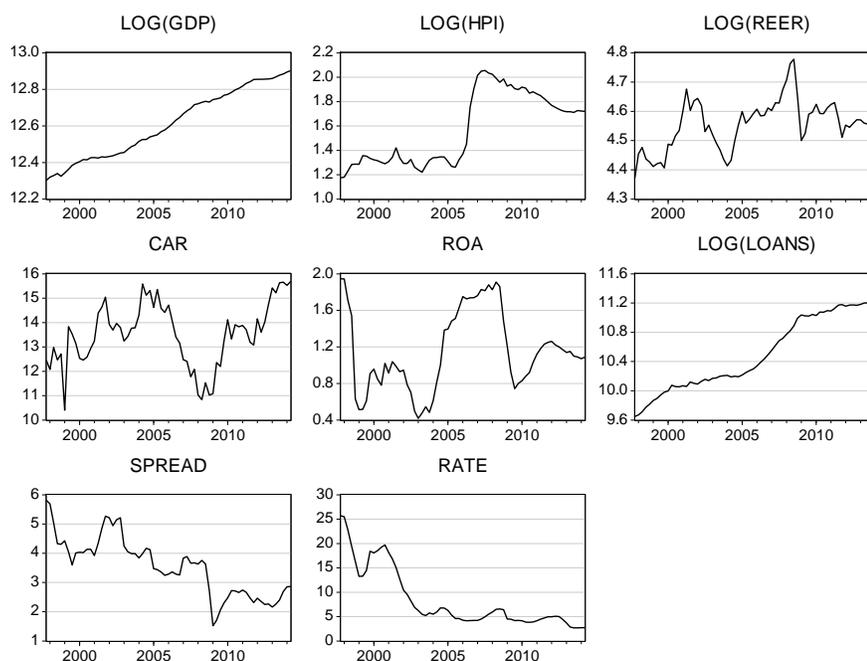
### 4.1 Data

We have utilized eight variables describing the real and financial processes in the Polish economy. Real output (seasonally adjusted GDP at constant prices, million zloty) and real housing prices (seasonally adjusted HPI index deflated with the consumer price index) describe the developments in the non-financial sector. The variables representing activity of the banking sector include aggregate loan supply to the non-financial sector (LOANS, deflated with the consumer price index, million zloty), return on bank assets (ROA, percent), capital adequacy ratio (CAR, percent) aggregated over the whole sector, and the spread between the lending and deposit

rates of interest (SPREAD, percent). The short-term money market rate (RATE, percent), and the real effective exchange rate (REER, index) control for the monetary policy and external shocks, respectively.

We have used quarterly data in the period Q4, 1997 – Q2, 2014 from Eurostat (GDP), Narodowy Bank Polski (loan aggregate, return on assets, capital adequacy ratio, interest rate spread, money market rate, housing price index, and consumer price index), and from the Bank for International Settlements (real effective exchange rate). The GDP, loans, housing prices, and exchange rate are expressed in natural logarithms, and all other variables (the interest rate, spread, bank return on assets, and capital adequacy ratio) are in levels. In Figure 1 we present plots of variables.

Figure 1: Plots of variables in VAR model



## 4.2 Estimation

We estimated several VAR models for lag orders up to four and selected the optimal lag-length based on the Schwarz information criterion and the model stability condition. We also calculated the probabilities based on Schwarz (BIC) and Akaike (AIC) weights, measuring the degree of belief that a certain model is the “true” data generating model (e.g., Wagenmakers and Farrell, 2004). Numerous articles show

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that under general conditions the Schwarz and Hannan-Quinn information criteria can be used in both stationary and non-stationary autoregressions (Hannan, 1980; Quinn, 1980; Tsay, 1984; Paulsen, 1984; Tjøstheim and Paulsen, 1985; Pötscher, 1989; Nielsen, 2006). Tsay (1984) has shown that for a univariate  $AR(p)$  nonstationary process with i.i.d errors, the asymptotic distribution of the AIC criterion derived by Shibata (1976) under normality and stationarity assumption continues to hold and that the BIC criterion is weakly consistent. Paulsen (1984) has shown that under general conditions the AIC and BIC information criteria hold for  $I(1)$  processes in vector  $AR(p)$  models.

Table 3 presents the main results from different lag-length specifications of the VAR model. We decided to use VAR(1), i.e., the model with one lag, and we called it an optimal VAR model. In order to avoid the specification problems of the VAR model one may reduce the number of the parameters that have to be estimated (Brüggemann, 2004). It can be done by reducing the number of variables ( $m$ ). However, this way of modeling seems to be constrained by the fact that the choice of the vector process  $\mathbf{y}_t$  is driven by economic theory. Hence, the proper choice of order  $p$  is of crucial importance. We tried to obtain the lag order with different model selection criteria. Finally, we used the order specified by the Schwarz criterion. It is known from the literature that the Akaike AIC suggests the largest order, and the Schwarz BIC chooses the smallest order (Lütkepohl, 2007, p. 151). We used a sample of 67 observations with 8 variables. Hence, we estimated 9 parameters (including a constant term) in each equation. We decided to use the most parsimonious model available, which is VAR(1), in order not to lose too many degrees of freedom. We also noticed that the variables we used are highly persistent with the information content centred at the first lag. With more information available from the credit market, it will be possible to test for robustness of the results to  $p$ -order of the VAR model. However, as pointed out by Gonzalo and Pitarakis (2002), in moderate samples all selection criteria of the model lag length  $p$  tend to point toward low orders as the system dimension increases.

In the next step, we considered a structural identification of impulse responses in the optimal VAR model. Hence, we identified the structural model parameters by using the Cholesky decomposition of the error covariance matrix. The initial order of variables in the model determined the sequence of structural shocks and their effects on other endogenous variables. This initial order was the following:  $\log(\text{GDP})$ ,  $\log(\text{HPI})$ ,  $\log(\text{REER})$ , ROA, CAR,  $\log(\text{LOANS})$ , SPREAD, and RATE. The VAR model also included dummies as exogenous variables, adjusting for any remaining seasonal effects and outliers. Dummies were included to control those outlier observations where errors exceeded two standard deviations.

In line with the majority of empirical studies, we assumed that shocks to GDP affect all other variables instantaneously. Shocks to housing prices affect immediately all variables except GDP. We also assumed that the real exchange rate affects immediately the value of loans (a large portion of loans in Poland is indexed to

Table 3: Summary statistics of VAR models

Model	VAR(1)	VAR(2)	VAR(3)	VAR(4)
LogL	678.00	754.54	829.61	949.04
AIC	-17.15	-17.80	-18.43	-20.48
BIC	-13.44	-11.91	-10.33	-10.14
w(AIC)	0.08	0.14	0.21	0.57
w(BIC)	0.54	0.25	0.11	0.10
Stability	yes	yes	yes	no

Note: LogL is the log-value of the likelihood function in the estimated VAR model. AIC and BIC are Akaike and Schwarz information criteria, respectively. The symbols: w(AIC) and w(BIC) denote the relative probabilities that given specifications are the best ones. These probabilities were computed with so-called Akaike and Schwarz weights, respectively (Wagenmakers & Farrell, 2004). “Stability” is set to “yes” if the VAR model is stable and “no” otherwise.

foreign currencies, mainly CHF and EUR), the values of ROA and CAR (through the balance sheet value of assets), and the value of interest rate spread. We further assumed that shocks to bank returns, loans, capital ratio, and spread, respectively, affect the market interest rate directly. Hence, by assumption, the financial market variables responded to news more rapidly than the other macroeconomic variables and they influenced the economic aggregates only with a lag.

In Tables 4–7, we present an inter-sectoral “map” of statistically significant impulse responses in the model. The cells denoted with a “+” sign represent positive shock reactions, the cells denoted with a “–” sign represent negative reactions, and the cells denoted with both “+” and “–” signs represent a combination of positive and negative reactions. The integers in the cells represent numbers of periods when the reaction to the shock was statistically significant, i.e., the mean response function was at least two standard deviations above or below zero. The fractional number represents the share of observations with statistically significant reaction values. The idea of this map is to visualize all combinations of reactions to shocks in a single table or figure. The results we have obtained may seem plausible (cf. Table 4). A positive macroeconomic output shock raises housing prices (through increased demand for housing), increases the value of loans and bank returns (e.g., through an improved financial situation of borrowers), and decreases interest rate spreads (e.g., through the channel of diminishing credit risk and increased collateral value). Similarly, growing housing prices lead to a rise in loans (due to increased values of mortgages and collateral) and boost aggregate demand. The values of ROA and CAR are reduced by the housing shock, most likely due to an increase in total assets (the denominator part of CAR and ROA). In turn, a shock strengthening the currency reduces the value of loans and improves ROA.

We also observed some interesting effects of shocks to banking variables. A positive shock to aggregate loans had a negative impact on bank returns and on the interest

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Table 4: Impulse responses of endogenous variables to orthogonal shocks under a single Cholesky decomposition

Variables	log(GDP)	log(HPI)	log(REER)	CAR	ROA	log(LOANS)	SPREAD	RATE
log(GDP)	(+); 18; 0.9	(+); 3; 0.15						(-); 18; 0.9
log(HPI)	(+); 9; 0.45	(+); 8; 0.4			(+); 6; 0.3		(-); 3; 0.15	(-); 15; 0.75
log(REER)	(-); 1; 0.05		(+); 4; 0.2					(-); 2; 0.1
CAR		(-); 7; 0.35		(+); 3; 0.15	(-); 11; 0.55	(+); 3; 0.15		(+); 8; 0.4
ROA	(+); 9; 0.45	(-); 2; 0.1	(+); 3; 0.15		(+); 6; 0.3	(-); 8; 0.4		(+); 1; 0.05; (-); 7; 0.35
log(LOANS)	(+); 20; 1	(+); 9; 0.45	(-); 2; 0.1	(-); 3; 0.15	(-); 4; 0.2	(+); 5; 0.25		(-); 11; 0.55
SPREAD	(-); 5; 0.25		(+); 3; 0.15	(+); 3; 0.15		(-); 4; 0.2	(+); 3; 0.15	(+); 1; 0.05
RATE		(-); 1; 0.05	(-); 6; 0.3		(+); 1; 0.05			(+); 8; 0.4

Note: The names in columns denote "shocking" variables and the shocked variables are presented in rows. The respective symbols are separated with semicolons in cells. The symbol "(+)" denotes a statistically significant positive effect of a one unit positive shock and "(-)" denotes a negative effect. An integer next to the "(+)" and "(-)" signs denotes the number of periods for which the reaction to a shock is statistically significantly different from zero. The fractional numbers next to integers denote the fraction of the horizon (20 quarters) where the reaction to a shock is statistically significant. An empty cell denotes no significant reaction to a shock.

Table 5: Generalized impulse responses of endogenous variables

Variables	log(GDP)	log(HPI)	log(REER)	CAR	ROA	log(LOANS)	SPREAD	RATE
log(GDP)	(+); 18; 0.9	(+); 3; 0.15	(-); 6; 0.3		(+); 2; 0.1	(+); 5; 0.25	(-); 2; 0.1	(-); 16; 0.8
log(HPI)	(+); 9; 0.45	(+); 8; 0.4						(-); 4; 0.2
log(REER)	(-); 1; 0.05		(+); 4; 0.2			(-); 4; 0.2	(+); 2; 0.1	
CAR		(-); 7; 0.35		(+); 3; 0.15		(-); 2; 0.1		
ROA	(+); 9; 0.45	(-); 3; 0.15	(+); 2; 0.1		(+); 6; 0.3	(-); 4; 0.2		(+); 2; 0.1
log(LOANS)	(+); 20; 1	(+); 11; 0.55	(-); 2; 0.1	(-); 4; 0.2	(-); 2; 0.1	(+); 5; 0.25	(-); 3; 0.15	(-); 8; 0.4
SPREAD	(-); 5; 0.25		(+); 3; 0.15	(+); 4; 0.2		(-); 4; 0.2	(+); 4; 0.2	
RATE		(-); 1; 0.05	(-); 7; 0.35	(-); 5; 0.25	(+); 1; 0.05	(+); 7; 0.35		(+); 6; 0.3
RATE		(-); 1; 0.05	(-); 6; 0.3		(+); 1; 0.05			(+); 8; 0.4

Note: The names in columns denote "shocking" variables and the shocked variables are presented in rows. The respective symbols are separated with semicolons in cells. The symbol "(+)" denotes a statistically significant positive effect of a one unit positive shock and "(-)" denotes a negative effect. An integer next to the "(+)" and "(-)" signs denotes the number of periods for which the reaction to a shock is statistically significantly different from zero. The fractional numbers next to integers denote the fraction of the horizon (20 quarters) where the reaction to a shock is statistically significant. An empty cell denotes no significant reaction to a shock.

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rate spread, but surprisingly it had a positive effect on the bank capital ratio. As we discuss in due course, the latter effect is not robust to the model specification. There was also no reaction of macroeconomic variables to increased loan supply. As expected, a shock increasing CAR reduced the amount of loans and increased the interest rate spread. However, a shock to increase ROA reduced the values of supplied loans and the capital ratio in subsequent periods and it caused housing prices to increase. Again, these above-mentioned effects are not robust to the model specification. An increase in ROA also had a positive short-lived effect on the market interest rate. Finally, a shock to the interest rate spread had a negative effect on housing prices, reflecting the working channel of loan supply.

We should notice that the market interest rate turned out to be one of the most important variables in the model as it affected all other variables. A positive shock to the market interest rate reduced output, housing prices, as well as aggregate loans. It also influenced currency depreciation and increased spread in the short-run.

As a further robustness check, we computed generalized impulse responses using the formula (5) as an alternative to traditional impulse responses given in (4) (cf. Table 5). Nevertheless, the new results are similar to those presented above. For example, the results are the same for shocks to GDP and HPI, which suggests that macroeconomic shocks generate responses robust to model specifications. For the exchange rate, the only additional significant effect in comparison to the traditional impulse responses was the negative reaction of GDP to currency appreciation, possibly due to weakening terms-of-trade conditions and a drop in exports.

In the case of banking variables, a positive shock to loans had a positive effect on GDP and on the interest rate, and a negative effect on the exchange rate (zloty depreciation), spread, as well as CAR and ROA. The contradicting reactions of the market rate and the spread seem implausible, but they could suggest a strong correlation of deposit rate and market rate after shocks in the loan market. In comparison to the results of traditional impulse responses, the generalized responses to shocks in CAR indicate an additional negative reaction of the market rate, and the generalized impulses to shocks in ROA indicate a positive reaction of GDP instead of HPI and no reaction of CAR. A positive shock to the spread had a negative effect on GDP and loan supply. It also had a positive effect on the real exchange rate, but no effect on housing prices.

Surprisingly, the effects of market rate shocks are not as widespread under generalized impulse responses as they are under traditional responses. The difference is the lack of significant reactions of REER, CAR, and spread, as well as a short-lived positive reaction of ROA.

The economic theory on macro-financial linkages as yet does not provide any clear view on the momentum of specific shock effects. Therefore, looking for the ordering of variables in a VAR is crucial to understand the nature of responses to shocks. The proposed robustness check with the combined impulse responses may help assess vulnerability of the main results to different model specifications. Tables 6 and 7

Table 6: Combined impulse responses of endogenous variables to orthogonal shocks under Cholesky decompositions of all permuted variables

Variables	log(GDP)	log(HPI)	log(REER)	CAR	ROA	log(LOANS)	SPREAD	RATE
log(GDP)	(+); 20; 1							(-); 17; 0.85
log(HPI)		(+); 8; 0.4						(-); 10; 0.5
log(REER)			(+); 4; 0.2					
CAR		(-); 8; 0.4		(+); 3; 0.15				
ROA	(+); 5; 0.25	(-); 4; 0.2	(-); 2; 0.1		(+); 2; 0.1	(-); 3; 0.15		(-); 2; 0.1
log(LOANS)	(+); 15; 0.75	(+); 10; 0.5				(+); 4; 0.2		(-); 9; 0.45
SPREAD				(+); 3; 0.15			(+); 3; 0.15	
RATE			(-); 5; 0.25					(+); 7; 0.35

Note: The names in columns denote "shocking" variables and the shocked variables are presented in rows. The respective symbols are separated with semicolons in cells. The symbol "(+)" denotes a statistically significant positive effect of a one unit positive shock and "(-)" denotes a negative effect. An integer next to the "(+)" and "(-)" signs denotes the number of periods for which the reaction to a shock is statistically significantly different from zero. The fractional numbers next to integers denote the fraction of the horizon (20 quarters) where the reaction to a shock is statistically significant. An empty cell denotes no significant reaction to a shock.

Table 7: Combined impulse responses of endogenous variables to orthogonal shocks under Cholesky decompositions of selected permuted variables

Variable	log(GDP)	log(HPI)	log(REER)	CAR	ROA	log(LOANS)	SPREAD	RATE
log(GDP)	(+); 18; 0.9	(+); 3; 0.15						(-); 19; 0.95
log(HPI)	(+); 9; 0.45	(+); 8; 0.4						(-); 12; 0.6
log(REER)	(-); 1; 0.05		(+); 4; 0.2					
CAR		(-); 7; 0.35		(+); 3; 0.15				(+); 6; 0.3
ROA	(+); 9; 0.45	(-); 2; 0.1	(-); 3; 0.15		(+); 2; 0.1	(-); 4; 0.2		(-); 6; 0.3
log(LOANS)	(+); 20; 1	(+); 9; 0.45				(+); 3; 0.15		(-); 10; 0.5
SPREAD	(-); 5; 0.25			(+); 3; 0.15			(+); 3; 0.15	(+); 1; 0.05
RATE		(-); 1; 0.05	(-); 5; 0.25					(+); 8; 0.4

Note: The names in columns denote "shocking" variables and the shocked variables are presented in rows. The respective symbols are separated with semicolons in cells. The symbol "(+)" denotes a statistically significant positive effect of a one unit positive shock and "(-)" denotes a negative effect. An integer next to the "(+)" and "(-)" signs denotes the number of periods for which the reaction to a shock is statistically significantly different from zero. The fractional numbers next to integers denote the fraction of the horizon (20 quarters) where the reaction to a shock is statistically significant. An empty cell denotes no significant reaction to a shock.

contain the results concerning combined impulse responses calculated by using all permutations of orderings ( $m! = 8! = 40320$ ) and using only permutations of selected variables ( $m^* = 6! = 720$ ), respectively. In the latter case, the permuted variables are  $\log(\text{REER})$ ,  $\text{CAR}$ ,  $\text{ROA}$ ,  $\log(\text{LOANS})$ ,  $\text{SPREAD}$ , and  $\text{RATE}$ , respectively, whereas the variables  $\log(\text{GDP})$  and  $\log(\text{HPI})$  are kept at their fixed positions (they are not permuted) and they precede the other variables in the VAR.

The main difference between Tables 6 and 7, and the previously described Tables 4 and 5 is in the much less evident reactions to shocks in banking sector variables. A shock to loans had only a negative effect on ROA and a shock to CAR had a positive effect on the interest rate spread. GDP and HPI did not react to banking variables and they only reacted to interest rate shocks. GDP influenced ROA and loans with a positive sign. A shock to housing prices increased the value of loans and decreased ROA. The appreciating currency had a negative effect on ROA, which is at odds with the evidence concerning this relationship from traditional impulse response and generalized impulse response analyses. The impact of REER shocks on the value of loans was not statistically significant. The impact of interest rate shocks on macroeconomic variables and loans was again significant and negative. In Table 6, there is also evidence of interest rate shocks affecting ROA negatively. In Table 7, interest rate shocks affect all variables except REER.

In our opinion, Table 7 provides the most reliable results because it both accounts for possible misspecification (e.g., in the ordering of variables) among the banking sector variables and assumes the leading role of aggregate macroeconomic shocks in line with the literature. Therefore, we also present more detailed results from this analysis, namely the graphs of all impulse response functions in Figures 2a to 2h. In each graph, the inner line represents the mean reaction function, the shaded area is the confidence region of the size up to two standard deviations around the mean, and the darker border lines represent the size of response uncertainty associated solely with the parameter estimation errors. In turn, the shaded area beyond the dark lines is related to the dispersion of individual impulse responses over different orderings (permutations) of variables. There is no dispersion of impulse responses depending on model permutations in Figures 2a and 2b because the impulse responses are invariant under permutations of six variables when the ordering of shock variables GDP and HPI is fixed (i.e., GDP and HPI always precede other variables in the VAR model).

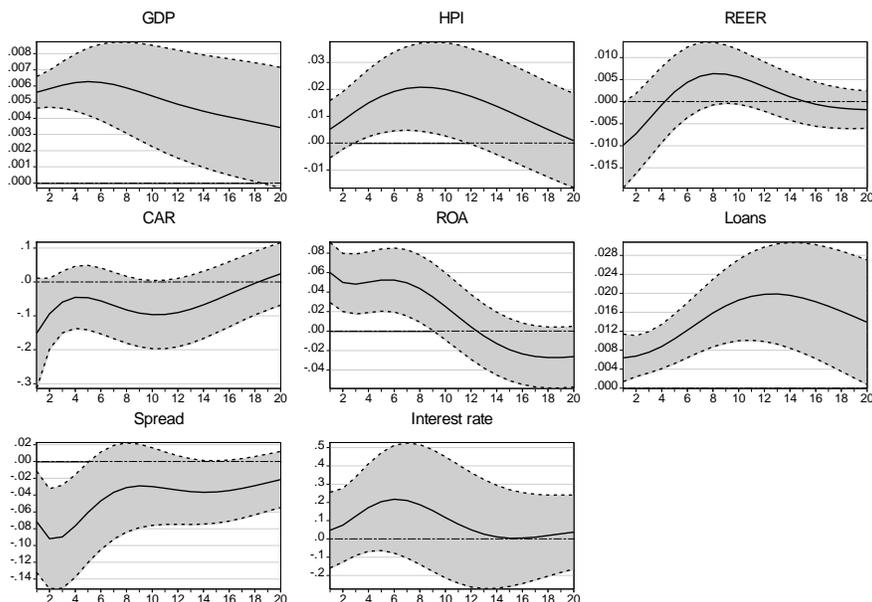
We explain this result in the following way. The impulse response function  $\psi_{ji}(n)$  depends on elements from matrices  $\mathbf{A}_n = [a_{ji}(n)]_{m \times m}$  and on elements from matrices  $\mathbf{P} = [p_{ji}]_{m \times m}$  according to formula (4), i.e.,

$$\psi_{ji}(n) = \begin{bmatrix} a_{j1}(n) & a_{j2}(n) & \dots & a_{jm}(n) \end{bmatrix} \cdot \begin{bmatrix} p_{1i} & p_{2i} & \dots & p_{mi} \end{bmatrix}'.$$

In different permutations of variables in a VAR, the elements of matrix  $\mathbf{A}_n$  are adjusted suitably (i.e., they are also permuted) and the impulse response function is not affected by these adjustments. However, all impulse responses to shocks in GDP and HPI also depend on the first two columns of matrix  $\mathbf{P}$ . The response of the

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Figure 2a: Reactions to shocks in GDP

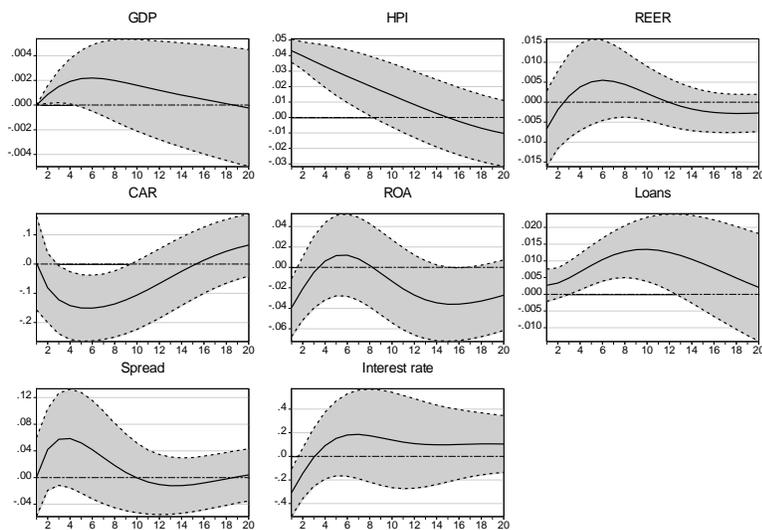


Note: The titles of the graphs indicate the variables reacting to the shock. In each graph, the solid line represents the mean reaction function, the shaded area is the confidence region of the size equal to two standard deviations around the mean, and the dotted border lines represent the size of response uncertainty associated solely with the parameter estimation errors. In turn, the shaded area beyond the dotted lines (if present) is related to the dispersion of individual impulse responses in different orderings of variables (permutations).

$r$ th variable to the shock in the first variable (i.e., GDP) depends on all  $p_{r1} = \frac{\sigma_{r1}}{\sqrt{\sigma_{11}}}$ ,  $r = 1, \dots, m$ , which in turn depend on covariances  $\sigma_{r1}$ ,  $r = 1, \dots, m$ , and do not depend on any other covariances in  $\Sigma_{\varepsilon}$ . This is important because all  $\sigma_{r1}$ ,  $r = 1, \dots, m$ , adjust appropriately (they are permuted) under permutations of variables in a VAR. Thus, all  $p_{r1}$  for  $r = 1, \dots, m$ , also adjust accordingly, and there is no dispersion of responses to shocks in GDP under considered permutations.

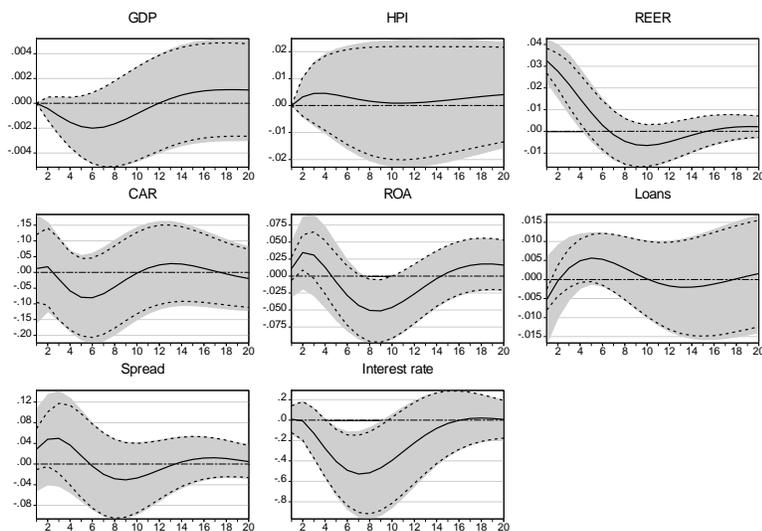
Similarly, the response of the  $r$ th variable to the shock in the second variable (e.g., HPI) depends on all  $p_{r2}$ ,  $r = 1, \dots, m$ , where  $p_{r2} = \frac{\sigma_{r2} - \sigma_{r1}(\sigma_{21}/\sigma_{11})}{\sqrt{\sigma_{22} - \sigma_{21}^2/\sigma_{11}}}$  for  $r \geq 2$  and  $p_{r2} = 0$  for  $r = 1$ . This means that the response function depends on the covariances  $\sigma_{r1}$ ,  $\sigma_{r2}$  for  $r = 1, \dots, m$ , and does not depend on any other covariances in  $\Sigma_{\varepsilon}$ . Because the covariances  $\sigma_{11}$ ,  $\sigma_{21}$ , and  $\sigma_{22}$  are fixed and the other covariances  $\sigma_{r1}$ ,  $\sigma_{r2}$  for  $r = 3, \dots, m$ , adjust suitably to any permutations of variables, then the elements  $p_{r2}$ ,  $r = 1, \dots, m$ , also adjust accordingly, and the impulse responses to shocks in HPI do not change value under considered permutations. If the ordering of GDP and HPI changed, the formulas for  $p_{r1}$  and  $p_{r2}$  would be different for different

Figure 2b: Reactions to shocks in HPI



Note: same as for the Figure 2a.

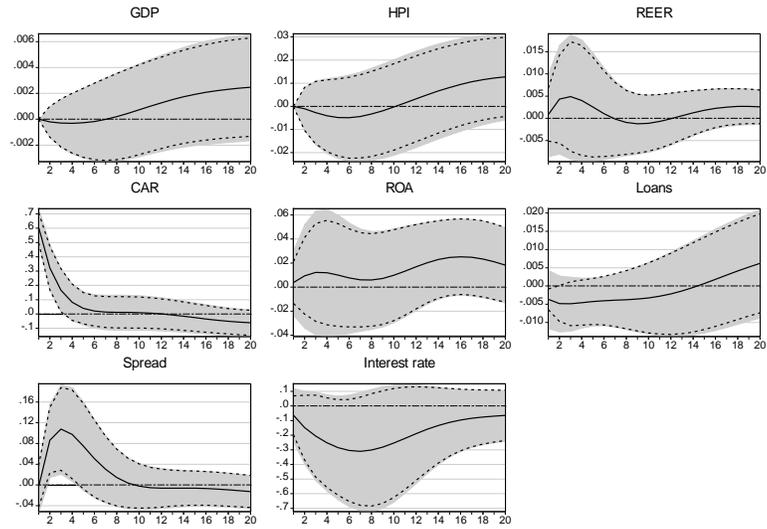
Figure 2c: Reactions to shocks in REER



Note: same as for the Figure 2a.

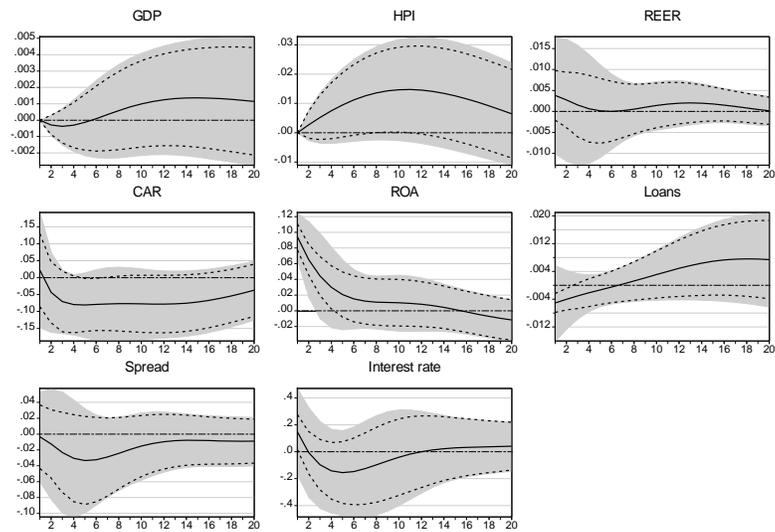
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Figure 2d: Reactions to shocks in CAR



Note: same as for the Figure 2a.

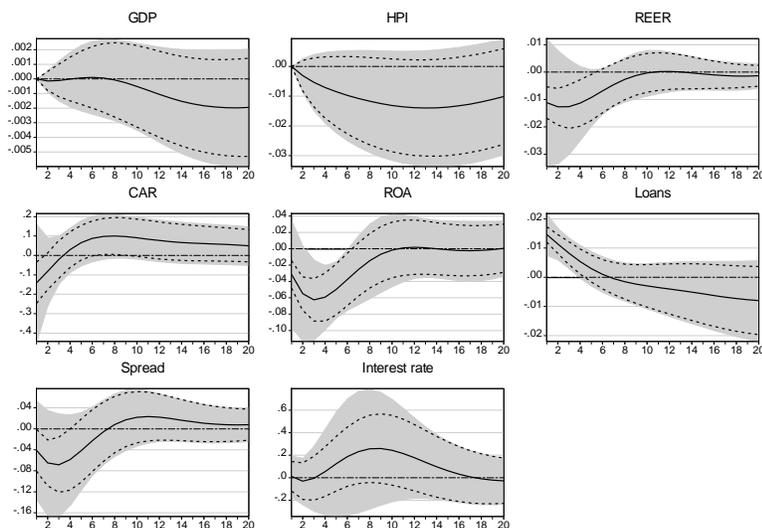
Figure 2e: Reactions to shocks in ROA



Note: same as for the Figure 2a.

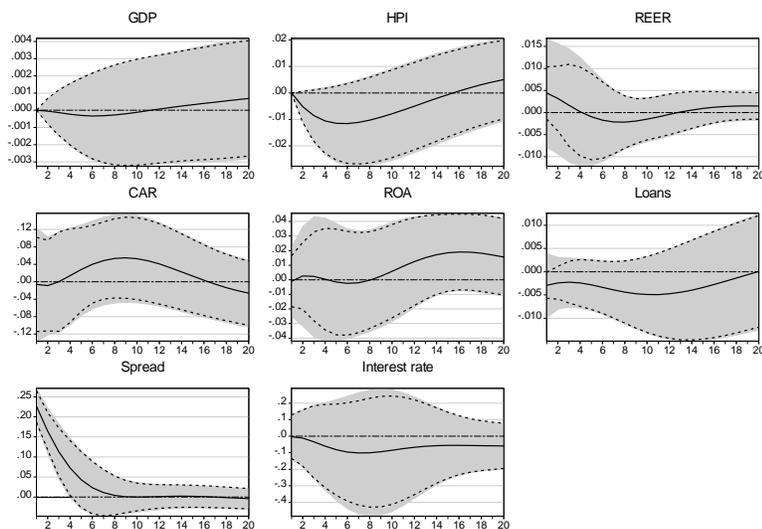
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Figure 2f: Reactions to shocks in Loans



Note: same as for the Figure 2a.

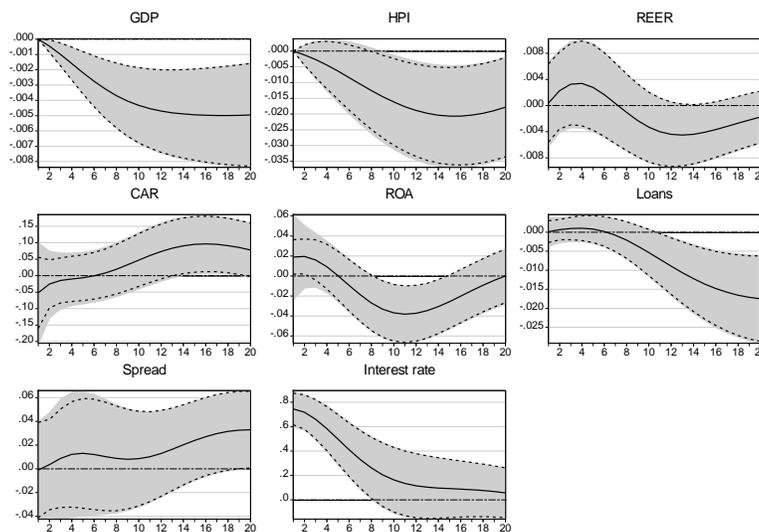
Figure 2g: Reactions to shocks in Spread



Note: same as for the Figure 2a.

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Figure 2h: Reactions to shocks in Interest rate



Note: same as for the Figure 2a.

permutations, and the impulse responses would likely display some dispersion.

We conclude that the ordering of response variables does not affect the values of respective impulse responses to shocks in GDP and HPI when these latter two variables are ordered first in our VAR model.

In Figures 2c to 2h, the dispersion of impulse responses depending on model permutations plays a more significant role. The additional uncertainty generated by the dispersion of mean IRFs in permutations reduces the number of significant response values, especially in the first periods after a shock. For example, the shock to loans has no statistically significant effects on REER, CAR, or spread due to an increased dispersion of responses in the initial periods after the shock in Figure 2f. This result is caused by the uncertainty associated with a correct model specification because the dispersion caused by the parameter uncertainty is relatively low.

In general, we confirmed the strong positive impact of macroeconomic conditions and housing prices on the performance of the loan market in Poland. In contrast to earlier studies relying on single VAR specifications, we have not found an unequivocal effect of a lending market on output growth because the banking variables did not cause any statistically significant reactions of macroeconomic variables. As no information about specific restrictions on shocks is assumed in our method, the combined positive and negative responses tend to give less significant results than impulse responses from individual specifications. It must be noted that a more precise assessment of specific credit market channels (supply vs. demand shocks; liquidity, credit risk, or portfolio rebalancing shocks) would be possible with the use of other identification

methods, e.g., sign restrictions in a VAR model. In turn, the interest rate channel drives developments in both the real and banking sectors.

## Conclusions

In this article, we proposed a new method to verify the robustness of impulse response functions in a structural VAR model under Cholesky's decomposition of the error covariance matrix. The method applies permutations of the variable ordering in a structural model. For all permutations impulse response functions are estimated and combined accordingly. In order to explore the method in practice, we estimated a structural VAR model describing the linkages between the banking sector and the real economy of Poland. Our results indicate that the combined impulse responses are more uncertain than those from a single specification, but some findings remain robust. For example, macroeconomic aggregate shocks and interest rate shocks have a significant impact on banking variables. This result is further confirmed by the outcomes from generalized impulse responses proposed by Pesaran and Shin (1998). Future studies may further explore the idea of combining other important statistics in SVAR models, including forecast error variance decompositions and historical decompositions. The studies may also elaborate more on the properties of distributions of combined impulse responses. The idea of combining impulse response functions seems to be particularly interesting for SVAR and SVEC models where the number of dependent variables is limited and analyzing all permutations is not computationally intensive. Extending the number of combined impulse responses is also worth considering, should just-identifying restrictions, other than the Cholesky decompositions or over-identifying restrictions prove relevant.

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## Appendix

### Combining impulse responses from different model specifications

Let us consider all  $m!$  structural specifications with orthogonalized impulse responses of the  $m$ -variable VAR model. These specifications are obtained by considering all possible orderings  $k = 1, \dots, m!$  of explained variables in the VAR model and the respective Cholesky decompositions of the error covariance matrix  $\mathbf{P}_k \mathbf{P}'_k = \boldsymbol{\Sigma}_\varepsilon^{\{k\}}$ .

We assign the probability  $Pr(\mathcal{M}_k)$  to each model specification  $\mathcal{M}_k$  that this specification is the correct one, where  $k = 1, \dots, m!$ . Selecting one model specification means excluding all other specifications because all specifications are mutually exclusive. When no prior knowledge about the correct specification exists and assuming that the correct specification exists in the group of considered specifications, the probability  $Pr(\mathcal{M}_k)$  is equal for each model specification, i.e.,  $Pr(\mathcal{M}_k) = \frac{1}{m!}$  for all  $k = 1, \dots, m!$ . Therefore, the variable responsible for selecting the model specification can be defined as a random variable uniformly distributed over the set of all considered model specifications  $\mathcal{M}_k$ . One can also assign different probabilities to different model specifications. For example, if some specifications are implausible, their probabilities may be set to zero and the sum of probabilities of all plausible specifications must be set to 1 accordingly.

We compute values of the impulse responses conditional on the choice of the model specification  $\mathcal{M}_k$ . The impulse response function (IRF) measuring the effect of a one

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standard deviation shock to the  $i$ th variable in  $\mathbf{y}_t$  on the  $j$ th variable in  $\mathbf{y}_{t+n}$  is given by:

$$\psi_{ji, \mathcal{M}_k}(n) = \mathbf{e}'_j \mathbf{A}_n \mathbf{P}_k \mathbf{e}_i, \quad n = 0, 1, 2, \dots \quad (12)$$

where  $k = 1, \dots, m!$  is the index of the model specification  $\mathcal{M}_k$ .

In practice, the calculated size of a given structural shock (e.g., credit shock) may differ to some extent depending on the ordering of variables in a VAR model. This is due to the construction of matrix  $\mathbf{P}_k = [p_{ji}]_{m \times m}$ . For example, the size of a shock to the variable ordered first in a VAR equals  $p_{11} = \sqrt{\sigma_{11}}$  (i.e., one standard deviation of the respective error term) and the shock to the second variable equals  $p_{22} = \sqrt{\sigma_{22} - \sigma_{21}^2 / \sigma_{11}}$  (i.e., less than one standard deviation of the respective error term). As the resulting response functions from different permutations may be difficult to compare, structural shocks of a unit size may be considered. Then the formula (12) will become  $\psi_{ji, \mathcal{M}_k}(n) = \mathbf{e}'_j \mathbf{A}_n \mathbf{P}_k \mathbf{e}_i p_{ii}^{-1}$ . The size of a given structural shock is usually very similar across permutations.

Lütkepohl (1990) showed that IRF estimates converge at the  $\sqrt{T}$ -rate to the normal distribution, i.e.,  $\sqrt{T} \left( \hat{\psi}_{ji, \mathcal{M}_k}(n) - \psi_{ji, \mathcal{M}_k}(n) \right) \xrightarrow{d} N \left( 0, V_{\psi_{ji, \mathcal{M}_k}} \right)$ , under some standard regularity conditions (Proposition 1, p. 118;  $\xrightarrow{d}$  denotes convergence in distribution). Therefore, we assume that  $\hat{\psi}_{ji, \mathcal{M}_k}(n)$  is approximately normally distributed,  $\hat{\psi}_{ji, \mathcal{M}_k}(n) \sim g \left( \hat{\psi}_{ji}(n) | \mathcal{M}_k \right)$ , for a given VAR specification. Let  $\mu_{ji}^{\{k\}}(n)$  be the expected value of the IRF estimate at time  $t+n$  in specification  $\mathcal{M}_k$  and  $\sigma_{ji}^{\{k\}}(n)$  denote the variance of the respective IRF estimate. The approximate unconditional probability density function of the IRF estimate can be obtained by marginalizing over the discrete distribution of the variable responsible for selecting a model specification:

$$f \left( \hat{\psi}_{ji}(n) \right) = \sum_{k=1}^{m!} g \left( \hat{\psi}_{ji}(n) | \mathcal{M}_k \right) \cdot Pr \left( \mathcal{M}_k \right) \quad (13)$$

The resulting density function  $f \left( \hat{\psi}_{ji}(n) \right)$  is a mixture of normal density functions and its mean and variance can be calculated using the following formulas:

$$\bar{\mu}_{ji}(n) = \frac{1}{m!} \sum_{k=1}^{m!} \mu_{ji}^{\{k\}}(n), \quad n = 0, 1, 2, \dots \quad (14)$$

$$\bar{\sigma}_{ji}(n) = \frac{1}{m!} \sum_{k=1}^{m!} \sigma_{ji}^{\{k\}}(n) + \frac{1}{m!} \sum_{k=1}^{m!} \left( \mu_{ji}^{\{k\}}(n) - \bar{\mu}_{ji}(n) \right)^2, \quad n = 0, 1, 2, \dots \quad (15)$$

Note that  $\hat{\psi}_{ji}(n)$  measures effects of a shock to the  $i$ th variable in  $\mathbf{y}_t$  on the  $j$ th variable in  $\mathbf{y}_{t+n}$  after controlling for different orderings of variables in different

specifications.

It should be noted that different permutations of variables in the VAR model may lead to the same values of respective impulse responses. Similarly, when ordering of some variables is fixed then it is possible that all impulse responses of some specific variable under different permutations of variables in the model are equal. In both cases, the formulas (13) to (15) are still valid. In the latter case, there is no uncertainty regarding a model specification, but there remains the uncertainty due to precision of estimation. Impulse responses depend not only on parameter estimates of a VAR model but also on the Cholesky matrix  $\mathbf{P}$  whose elements are known functions of covariances between error terms in that VAR model. Permutations of variables in the VAR affect matrix  $\mathbf{P}$ . Therefore, it seems possible to analytically derive how certain permutations of variables (e.g., moving a single variable to another position in a VAR) affect impulse responses of specific variables in the SVAR. This would allow for obtaining more precise analytical formulas of combined impulse responses  $\psi_{ji}(n)$ . We leave this topic for future investigations.