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Mathematical Modeling of Heat and Mass Transfer Process Under Heat Treatment of Grain Materials in Dense Layer

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Summary. Postharvest processing of grain is an important step in the overall grain production process. It makes possible not only quantitative and qualitative preservation of the harvest, but also ensures maximum profit from its sale at the most favorable market conditions. Convective heat treatment (drying, cooling) guarantees commercial harvest conservation, prevents its loss, and in some cases improves the quality of the finished product.

The necessity of intensification and automation of technological processes of postharvest grain processing requires the development of methods of mathematical modeling of energy-intensive processes of convective heat treatment. The determination and substantiation of optimum modes and parameters of equipment operation to ensure the preservation of grain quality is possible only when applying mathematical modeling techniques.

In this work, a mathematical model of particulate material drying is presented through a system of differential equations in partial derivatives of which the variable in time and space relationship between heat and mass transfer processes in the material and a drying agent is reflected.

The aim of the research was to determine the dynamics of the interrelated fields of unsteady temperature and moisture content of the material and the drying agent on the basis of mathematical models of heat and mass transfer in the layer of particulate material in convective heat approach or heat retraction.

The implementation of the mathematical model proposed in the standard mathematical set allows analyzing efficiency of machines and equipment for the convective heat treatment of particulate agricultural materials in a dense layer, according the determinant technological parameters and operating modes.

Key words: heat and mass transfer, active ventilation, thermal drying, mathematical model, temperature field, field of moisture content.

INTRODUCTION

The increase in agricultural production on a background of expensive energy assets causes the demand of advanced energy saving technologies and equipment design. Major energy consumption in postharvest grain processing concerns with heat and mass transfer processes, including the processes of drying and cooling – which make up the main industrial methods of grain preservation [1]. But the actual problem does not consist only in the creation and implementation of new energy-saving technologies and equipment, but also in the reduction of financial costs of the development process itself [2 – 4].

The most common method of agricultural materials (grain harvest, forage raw materials, fruit and vegetable crop seeds) preservation is drying or cooling by filtration drying or cooling agent through unmixed layer of particulate material in a stationary (active ventilation for dehydration or cooling) and a moving state (moving product on the line or gravity columns).

Determination of the optimal modes of heat treatment of grain materials and their implementation by means of automation requires an adequate mathematical model that takes into account simultaneous modification of the parameters of the material and filtered gas in space and time. A promising direction of obtaining the dynamic characteristics of the process of treating particulate material in filtered layer is the use of computer modeling techniques based on mathematical description of interrelated non-stationary processes of heat and mass transfer.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

Since the analysis of processes of heat treatment and drying of particulate materials in stationary (unmoved) layer is an essential element of heat and mass transfer modeling in the dryer with cross movement of material and drying agent, a sufficient number of publications is devoted this issue [5-20]. General theory of heat and material transfer regarding the drying process is developed and generalized in the works [5 - 7]. Analytical solutions of the equations of heat and mass transfer for bodies of classical forms were received by O.V. Lykov. Analytical solution of equations for particles of the material in the layer where processes occur in alternating mode has not been received yet [8, 9]. Therefore, a simplified thermal mass transfer mechanism is used for the applied tasks solution for drying in a dense layer [10, 11]. A step numerical calculation method of heat and mass transfer processes in the grain mass is considered to be the most versatile and allows to obtain approximate solutions of the process dynamics [12], it provides a sufficient accuracy to solve some engineering problems, but requires considerable preparation of software settings for each task. In the research [15], during the numerical solution of equations of mathematical model of heat and mass transfer the impact of variable height settings layer of drying agent on the equilibrium moisture content of grain and the rate of drying was not considered. In the work [13] the analytical mathematical models of the dynamics of drying material developed based only on analysis of the mass transfer process regardless the influence of temperature is presented. In the works [16, 17] simplified mathematical models of the drying process of plant materials in a dense layer by means of active ventilation are given, but solutions of differential equations don't take into account the effect of drying agent temperature on the rate of material drying. In the issue [18] an analytical solution of the temperature distribution in the layer of particulate material in the gas filtration is presented.

Thus the problem of determining the dynamics of heat and mass transfer processes during particulate material filtration drying on the basis of cross-linking is not completely solved, but approximate solutions of heat or mass transfer equations in partial derivatives are known.

OBJECTIVES

The purpose of research consists in the determination of the dynamics of inter-related of unsteady temperature fields and moisture content of the material and drying agent based on mathematical models of heat and mass transfer in the layer of particulate material under the convective heat input.

THE MAIN RESULTS OF THE RESEARCH

Because of the lack of analytical solutions of differential equations describing the heat and mass transfer in a thick layer of material at alternating mode, simplified mechanisms of heat and mass transfer between the material and drying agent (moisture in the grain is in the liquid state, heat and mass exchange occurs between air and grain, temperature gradient within individual grains is too small, the heat exchange between air and grain is convective) are used for applied purposes. But even using simplified mechanisms causes describing the process of heat and mass transfer system of differential equations in partial derivatives (1-4) [11] analytical or numerical solution of which is associated with certain mathematical difficulties:

$$\frac{\partial t(\tau,x)}{\partial \tau} + 3600 \cdot V \frac{\partial t(\tau,x)}{\partial x} = -\frac{\gamma_m \cdot C_g}{\gamma_a \cdot C_a} \cdot \frac{\partial \theta(\tau,x)}{\varepsilon \cdot \partial \tau} - \frac{\gamma_m r'}{\gamma_a C_a \varepsilon} \frac{\partial U(\tau,x)}{\partial \tau} \cdot \frac{1}{100}, \tag{1}$$

$$\frac{\partial U(\tau,x)}{\partial \tau} = -\frac{\gamma_a}{10 \cdot \gamma_m} \cdot \left(\frac{\partial d(\tau,x)}{\partial \tau} + 3600 \cdot V \frac{\partial d(\tau,x)}{\partial x} \right), \tag{2}$$

$$\frac{\partial t(\tau,x)}{\partial \tau} + 3600 \cdot V \frac{\partial t(\tau,x)}{\partial x} = -\frac{\alpha_q \cdot \gamma_m}{\gamma_a C_a \cdot \varepsilon} \cdot \left(t(\tau,x) - \theta(\tau,x) \right), \tag{3}$$

$$\frac{\partial U(\tau,x)}{\partial \tau} = -K(t,\theta)\cdot\left(U(\tau,x)-U_r(t,d)\right), \tag{4}$$

where:

t – drying agent temperature, C; d – drying agent moisture content, g/kg of dry air; U – grain moisture content, %; θ – grain temperature, ^{0}C; V– drying agent speed, m/s; C_g, C_a – heat capacity of grain and air, kJ/kg·^{0}C; ε – pore volume of a grain layer; $r^{/}$ – hidden heat of vaporization, kJ/kg; α_q – heat transfer coefficient, kcal/kg·h ^{0}C; γ_m – bulk weight of grain material, kg/m^3; γ_a – required mass of air, kg/m^3; K – drying coefficient; 1/h; U_r – equilibrium grain moisture content, g/kg of raw material; x – spatial coordinate, м; τ – time, h.

Equilibrium grain moisture content $U_r(t,d)$ depends on the drying agent parameters which, in its turn, changes when passing through a layer of grain material.

The dependence of equilibrium moisture content on drying agent temperature and moisture content can be determined using the empiric formula:

$$U_r(t,d) = 37{,}1985\cdot t^{-0{,}9603}\cdot d\,. \tag{5}$$

Figure 1 shows a graphic dependence of moisture equilibrium on temperature and moisture content of air obtained using the formula (5) and Henderson formula [12].

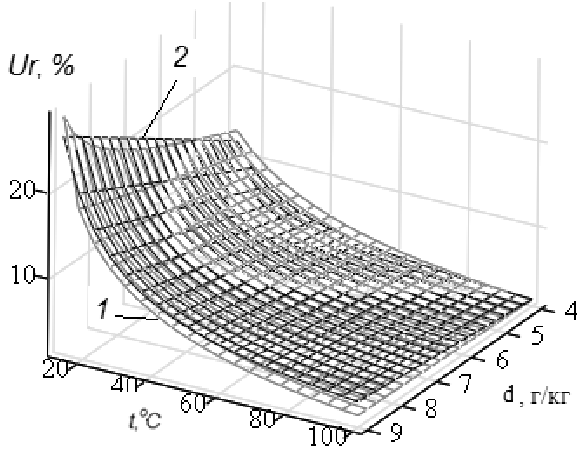

Fig. 1. Dependence of equilibrium grain moisture content on drying agent temperature and moisture content (1 – using the Henderson formula, 2 – using the formula (5))

The value of drying coefficient K in the equation (4) for a low-temperature heat treatment can be determined by empirical dependence [19]:

$$K(t,\theta) = 0{,}025\frac{\theta+t}{2}. \tag{6}$$

The development of temperature fields in a thick layer of particulate material with an internal negative source of heat (evaporation) with sufficient accuracy can be described by analytical models set out in the work [20]:

$$\theta = \theta_n + (t_n-\theta_n)b\cdot\exp(-Ax)\int_0^{\tau}\exp(-b\tau)I_0\left(2\sqrt{Abx\tau}\right)d\tau\,, \tag{7}$$

$$t = \theta_n + (t_n-\theta_n)\exp(-Ax)\times$$

$$\left[\exp(-b\tau)I_0\left(2\sqrt{Abx\tau}\right)+b\int_0^{\tau}\exp(-b\tau)I_0\left(2\sqrt{Abx\tau}\right)d\tau\right], \tag{8}$$

where:

$$A=\frac{\alpha F}{W_a\cdot h},\; b=\left(1+\frac{q_0}{\alpha\sigma\gamma_g}\right)\cdot\left(\frac{\alpha\sigma}{c_g}\right),$$

$\theta\,|_{\tau=0}=\theta_n$; $t\,|_{x=0}=t_n$; I_0 – Bessel function.

We decompose the Bessel function in a line [21] and take the two first members:

$$I_0\left(2\sqrt{Abx\tau}\right)\approx 1-Abx\tau\,. \tag{9}$$

Let's substitute (9) y (7) - (8) and after integration we'll obtain:

$$t = \theta_n + (t_n-\theta_n)\exp(-Ax)\left[1+Ax\cdot(\exp(-b\tau)-1)\right], \tag{10}$$

$$\theta = \theta_n + (t_n - \theta_n) \cdot \exp(-Ax - b\tau) \cdot [\exp(b\tau)(1 - Ax) + A(x + bx\tau) - 1]. \qquad (11)$$

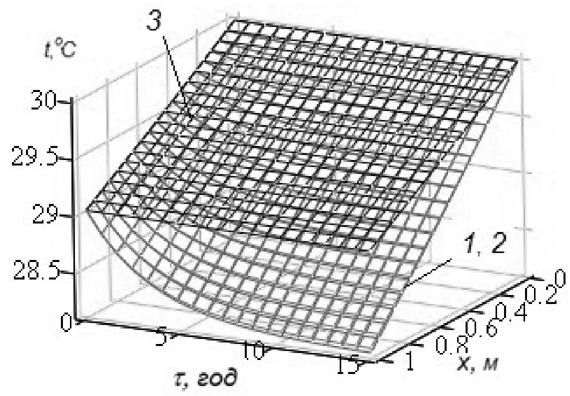

Fig. 2. Drying agent temperature field (1– according to the dependence (8), 2– according to the dependence (10), 3 – according to the dependence (8) at approximation $I_0\left(2\sqrt{Abx\tau}\right) \approx 1$)

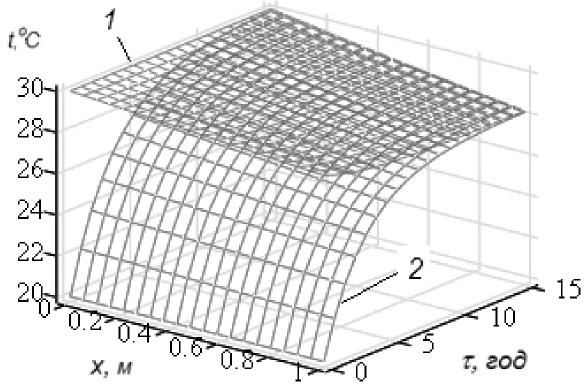

Fig. 3. Drying agent and grain material temperature field (1– according to the dependence (10), 2– according to the dependence (11))

Average effective coefficients *A, b* in equations (10) and (11) should be determined on the basis of experimental measurements of heat transfer agent and grains temperature at the exit of the layer in the mode $t_n = \text{const}$.

Let's substitute (10) - (11) into (5) - (6) and the dependences got in (4) and, thus, we'll obtain the system of equations:

$$\frac{\partial t(\tau,x)}{\partial \tau} + 3600 \cdot V \frac{\partial t(\tau,x)}{\partial x} = -\frac{\gamma_m \cdot C_g}{\gamma_a \cdot C_a} \cdot \frac{\partial \theta(\tau,x)}{\varepsilon \cdot \partial \tau} - \frac{\gamma_m r'}{\gamma_a C_a \varepsilon} \frac{\partial U(\tau,x)}{\partial \tau} \cdot \frac{1}{100}, \qquad (12)$$

$$\frac{\partial U(\tau,x)}{\partial \tau} = -\frac{\gamma_a}{10 \cdot \gamma_m} \cdot \left(\frac{\partial d(\tau,x)}{\partial \tau} + 3600 \cdot V \frac{\partial d(\tau,x)}{\partial x} \right), \qquad (13)$$

$$\frac{\partial t(\tau,x)}{\partial \tau} + 3600 \cdot V \frac{\partial t(\tau,x)}{\partial x} = -\frac{\alpha_q \cdot \gamma_m}{\gamma_a C_a \cdot \varepsilon} \cdot (t(\tau,x) - \theta(\tau,x)), \qquad (14)$$

$$\frac{\partial U(\tau,x)}{\partial \tau} = -0{,}025 \times$$

$$\frac{\exp(-Ax - B\tau)[2\exp(B\tau)(t_n - Axt_n + (Ax - 1 + \exp(Ax))\theta_n) + (t_n - \theta_n)(Ax(2 + B\tau) - 1)]}{2} \times$$

$$\left(U(\tau,x) - 37.1985 \cdot (\theta_n + (t_n - \theta_n)\exp(-Ax)[1 + Ax(\exp(-b\tau) - 1)])^{-0.9603} \cdot d(\tau,x)\right). \qquad (15)$$

The system of differential equations (12) - (15) with constant coefficients, unlike systems (1) - (4), is possible to be solved quantitatively in mathematical set Mathematica. The graphic interpretations of system solutions (12) - (15) in mathematical set Mathematica are presented in Fig.4 - 7.

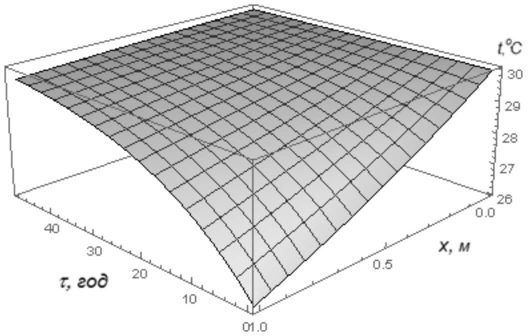

Fig. 4. Changes of air temperature in a garin layer in time and according to a layer height

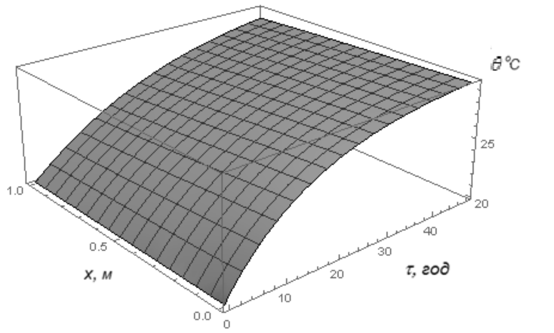

Fig. 6. Changes of grain temperature in time and according to a layer height

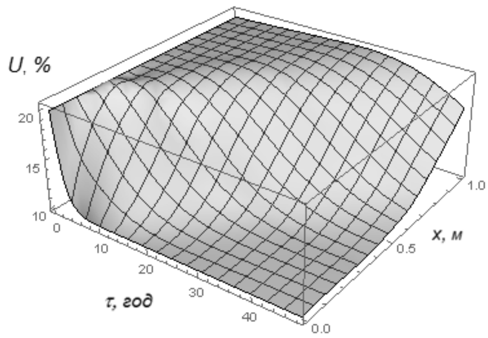

Fig. 5. Changes of grain moisture in time and according to a layer height

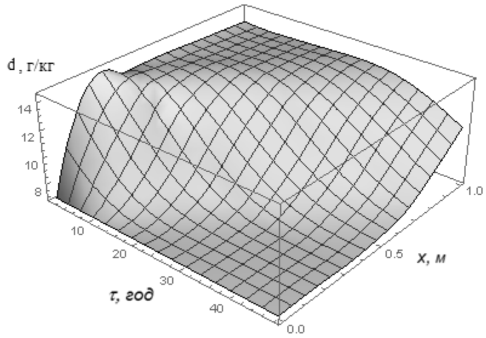

Fig. 7. Changes of air moisture content in the grain layer in time and according to a layer height

CONCLUSIONS

The proposed method for calculation of heat and mass transfer in a dense layer under convective heat treatment of grain allows the determination of interrelated development of unsteady temperature fields and moisture content of the grain and the drying agent. It makes it possible to substantiate rational modes and parameters of machines for grain material thermal processing for further conservation.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ТЕПЛОМАССООБМЕННЫХ ПРОЦЕССОВ ПРИ ТЕРМООБРАБОТКЕ ЗЕРНОВЫХ МАТЕРИАЛОВ В ПЛОТНОМ СЛОЕ

Аннотация. Послеуборочная обработка зерна является важным этапом в общем процессе производства зерна, позволяет не только количественно и качественно сохранить выращенный урожай, но и обеспечить получение максимальной прибыли от его реализации при наиболее выгодной конъюнктуре рынка. Конвективная термообработка (сушка, охлаждение) обеспечивает промышленную консервации собранного урожая, предотвращает его потери, а также в некоторых случаях повышает качество готового продукта.

Необходимость интенсификации и автоматизации технологических процессов послеуборочной обработки зерна требует развития методов математического моделирования энергоемких процессов конвективной термообработки. Определение и обоснование оптимальных режимов и параметров работы оборудования для обеспечения качественной консервации зерна возможно лишь с использованием методов математического моделирования.

Математическая модель сушки диспер-сного материала в работе представлена в виде системы дифференциальных уравнений в частных производных, в которой отражена переменная во времени и пространстве, взаимосвязь между тепловыми и массообменными процессами в материале и сушильном агенте.

Целью проведенных исследований было определение динамики развития взаимосвязанных нестационарных полей температуры и влагосодержания материала и сушильного агента на основе математических моделей процессов тепло- и массообмена в слое дисперсного материала при конвективном подводе или отводе теплоты.

Реализация рассматриваемой математической модели в стандартном математическом пакете позволяет анализировать производительность работы машин и оборудования для конвективной термообработки дисперсных сельскохозяйственных материалов в плотном слое в зависимости от определяющих технологических параметров и режимов работы.

Ключевые слова: тепло- и массообмен, активное вентилирование, термическая сушка, математическая модель, температурное поле, поле влагосодержания.