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B – data collection
C – statistical analysis
D – data interpretation
E – manuscript preparation
F – literature search

Mathematical modelling of filtration processes in drainage systems using conformal mapping

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Abstract

The situation when groundwater considerably rises above the "normal" level, water intake, lowering of groundwater levels and other relevant practical tasks require the drainage facilities. The most effective techniques of numerical studies of the corresponding boundary problems at present time are methods of dealing with inverse boundary value problems (conformal and quasi-conformal mappings). As basis of this research we used the case of combining the fictitious domain methods with quasi-conformal mappings of the solution of nonlinear boundary value problems for the calculation of filtration regimes in environments with free boundary areas (depression curves) and zones of "mountainous" areas.

This paper reviews the stationary issue of flat-vertical stationary non-pressure liquid filtration to horizontal symmetric drainage. In the paper a practical methodology for solving boundary value problems on conformal mappings is suggested for the calculation of the filtration process in the horizontal symmetrical drainage.

The idea of block iterative methods was used during the creation of the corresponding algorithm which is based on the alternating "freeze" of the anticipated conformance parameter, the internal and boundary connections of the curvilinear area.

The results of the conducted numerical calculations confirmed the effectiveness of the suggested problem formulations and algorithms of their numerical solution and the possibility of their use in the modelling of nonlinear filtration processes occurring in horizontal drainage systems, as well as in the design of drainage facilities and optimizing other hydrosystems. Therefore these results are of great importance.

Key words: *conformal mapping, drainage systems, filtration processes, flood protection, mathematical modelling*

INTRODUCTION

Groundwater flood protection of territories and municipalities, when groundwater considerably rises above the "normal" level, water intake and lowering of groundwater levels, as well as other relevant practical tasks, are all associated with the need for drainage facilities. This issue is especially important for amelioration, where drainage is the key element of any amelioration system [MARTYNYUK 2015; ROKOCHINSKIY *et al.* 2017; SMEDEMA 2011]. Nowadays, the methods of dealing with inverse boundary value problems (conformal and quasi-conformal mappings) are the most effective techniques of numerical studies of the corresponding boundary problems [BERESLAVSKII, LIKHA-

CHEVA 2011; CARABINEANU 2015; ORLOV *et al.* 2016]. In particular, the previous studies [BOMBA *et al.* 2007a] examine the case of combining the fictitious domain methods and quasi-conformal mappings of the solution of nonlinear boundary value problems for the calculation of filtration regimes in environments with free boundary areas (depression curves) and zones of „mountainous" areas. At the same time, the other study [BOMBA *et al.* 2008] suggests an algorithm for a numerical solution of model nonlinear boundary value problems through quasi-conformal mappings in the areas bounded by two equipotential lines and two flow lines, when one of the boundary fragments is an unknown (free) curve with fixed and free ends. This paper reviews the stationary issue of flat-vertical stationary non-pressure liquid filtration to horizontal symmetric drainage.

MATERIALS AND METHODS

Statement of problem. Let's consider the process of filtration to horizontal drainage. Due to the symmetry of the flow pattern, we shall study only one fragment of such a system, namely a single-connected hexagonal curvilinear area (porous layer) $\partial G_z = ABC^*C_*CD$ $(z = x + iy)$ (see Fig. 1) limited by two lines of the current

$$AD = \{z: x = 0, -h_1 + r \le y \le 0\}$$
$$BC^* = \{z: x = l, -h_1 - r - h_2 \le y \le 0\}$$
$$C_*C^* = \{z: y = -h_1 - r - h_2, 0 \le x \le l\}$$
$$CC_* = \{z: x = 0, -h_1 - r - h_2 \le y \le -h_1 - r\}$$

as well as two equipotential lines

$$CD = \left\{z: x - \sqrt{r^2 - (y + h_1)^2} = 0, -h_1 - r \le y \le -h_1 + r\right\}$$

and $AB = \{z: y = 0, 0 \le x \le l\}$

Similarly as BOMBA *et al.* [2007b; 2008], we shall describe it by the flow equation $\vec{v} = k\,\mathrm{grad}\varphi$ (Darcy's law) and continuity equation $\mathrm{div}\vec{v} = 0$, where $\vec{v} = [v_x(x, y) + iv_y(x, y)]$ is the filtration rate, k is the coefficient of conductivity, piecewise constant in the area G_z, so that it is $k = k_1$ at $(-h_1 < y \le h_1 + 0.3)$ and $(x <= t_f)$, $(x, y) \in G_z$, $k = k_2$ whenever $(-h_1 < y \le h_1 + 0.3)$ and $(t_f <= x <= b_{tp})$, $(x, y) \in G_z$, while for the rest of the area $G_z - k = k_3$. The classical conjugation conditions are used at the boundary of the section [BOMBA *et al.* 2007b] $\varphi = \varphi(x, y)$ is potential at the point (x, y), so that $\varphi|_{AB} = 0$ (is equal to the pressure on the horizon), $\varphi|_{CD} = \varphi^*(h_1) = h_1$ (the average value of the pressure in the drain, which is equal to the depth of its laying), $\left.\frac{\partial\varphi}{\partial\vec{n}}\right|_{BC^*C_*C} = \left.\frac{\partial\varphi}{\partial\vec{n}}\right|_{AD} = 0$, $\vec{n}$ is the outward normal on the corresponding range of the area boundary.

The conforming problem, if it is viewed through conformal mapping $\omega = \omega(z) = \varphi(x, y) + i\psi(x, y)$ of the considered area G_z to the corresponding sector of the complex potential $G_\omega = \{\omega: \varphi_* < \varphi < h_1, 0 < \psi < Q\}$ with unknown discharge Q shall look as follows [BOMBA *et al.* 2007a, 2008]:

$$k\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y}, \qquad k\frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial y} \tag{1}$$

$$\varphi|_{AB} = 0,\ \ \varphi|_{CD} = h_1,\ \ \psi|_{AD} = 0,\ \ \psi|_{BC^*C_*C} = Q \tag{2}$$

The inverse problem of equations (1) – (2) on the conformal mapping $z = z(\omega) = x(\varphi, \psi) + iy(\varphi, \psi)$ in the area G_ω at G_z with unknown discharge Q shall be presented as:

$$\begin{cases} k\dfrac{\partial y}{\partial\psi} = \dfrac{\partial x}{\partial\varphi}, \\ k\dfrac{\partial x}{\partial\psi} = -\dfrac{\partial y}{\partial\varphi}, \end{cases} (\varphi, \psi) \in G_\omega \tag{3}$$

$$\begin{cases} y(\varphi_*, \psi) = 0,\ \ 0 \le \psi \le Q \\ x(h_1, \psi) - \sqrt{r^2 - (y(h_1, \psi) + h_1)} = 0,\ \ 0 \le \psi \le Q \\ x(\varphi, Q) = l,\ \ 0 \le \varphi \le \varphi_1 \\ y(\varphi, Q) = -h_1 - r - h_2,\ \ \varphi_1 \le \varphi \le \varphi_2 \\ x(\varphi, Q) = 0,\ \ \varphi_2 \le \varphi \le h_1 \\ x(\varphi, 0) = 0,\ \ 0 \le \varphi \le h_1 \\ g(y(\varphi, \psi)) = \varphi,\ \ 0 \le \psi \le Q, 0 \le \varphi \le h_1 \end{cases} \tag{4}$$

The corresponding second order equations used in order to find functions $x = x(\varphi, \psi)$ and $y = y(\varphi, \psi)$ in a divergent form shall look as follows:

$$\begin{aligned} \frac{\partial}{\partial\varphi}\left(\frac{1}{k}\frac{\partial x}{\partial\varphi}\right) + \frac{\partial}{\partial\psi}\left(k\frac{\partial x}{\partial\psi}\right) = 0 \\ \frac{\partial}{\partial\varphi}\left(\frac{1}{k}\frac{\partial y}{\partial\varphi}\right) + \frac{\partial}{\partial\psi}\left(k\frac{\partial y}{\partial\psi}\right) = 0 \end{aligned} \tag{5}$$

Algorithm for numerical solution of the problem. The difference analogue of equations (5), boundary conditions (4), near-boundary orthogonality conditions and conditions of so-called "conformal similarity in the small" of the corresponding quadrilaterals, in the relevant uniform grid area

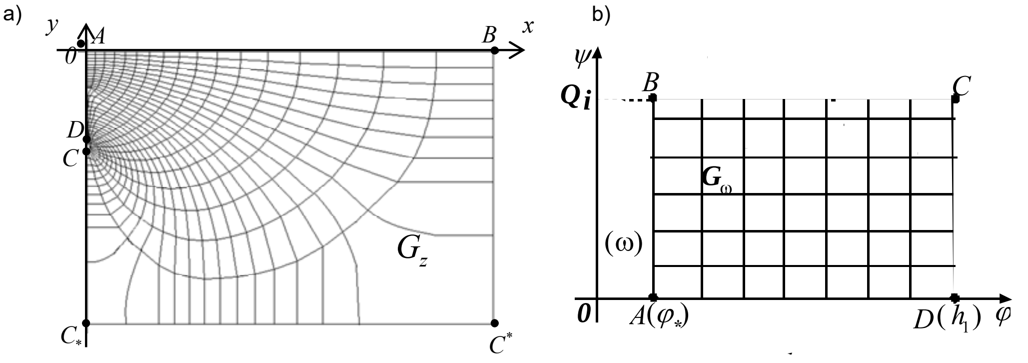

Fig. 1. Filtration area G_z (a) and corresponding complex potential area G_ω (b); source: own elaboration

$G_{\omega}^{\gamma} = \left\{(\varphi_i, \psi_j): \varphi_i = \Delta\varphi \cdot i, i = \overline{0, m+1}; \psi_j = \Delta\psi \cdot j, j = \overline{0, n+1}; \Delta\varphi = \frac{h_1}{m+1}, \Delta\psi = \frac{Q}{n+1}, \gamma = \frac{\Delta\varphi}{\Delta\psi}, m, m_1, m_2, n \in \mathrm{N}\right\}$ we shall write as follows [BOMBA *et al.* 2008; MARCHUK 1980; SAMARSKIY 1977]:

$$\begin{cases} y_{0,j} = 0, \;\; j = \overline{0, n+1} \\ x_{m+1,j} - \sqrt{r^2 - (y_{m+1,j} + h_1)} = 0, \;\; j = \overline{0, n+1} \\ x_{i,n+1} = l, \;\; i = \overline{0, m_1} \\ y_{i,n+1} = -h_1 - r - h_2, \;\; i = \overline{m_1, m_2} \\ x_{i,n+1} = 0, \;\; i = \overline{m_2, m+1} \\ x_{i,0} = 0, \;\; i = \overline{0, m+1} \end{cases} \tag{6}$$

$$\begin{cases} \sigma\left[a_{i+1,j+1}x_{i+1,j+1} - (a_{i+1,j+1} + a_{i,j+1})x_{i,j+1} + a_{i,j+1}x_{i-1,j+1}\right] + \\ +(1-2\sigma)\left[a_{i+1,j}x_{i+1,j} - (a_{i+1,j} + a_{i,j})x_{i,j} + a_{i,j}x_{i-1,j}\right] + \\ +\sigma\left[a_{i+1,j-1}x_{i+1,j-1} - (a_{i+1,j-1} + a_{i,j-1})x_{i,j-1} + a_{i,j-1}x_{i-1,j-1}\right] + \\ +\gamma^2\{\sigma\left[b_{i+1,j+1}x_{i+1,j+1} - (b_{i+1,j+1} + b_{i+1,j})x_{i+1,j} + b_{i+1,j}x_{i+1,j-1}\right] + \\ +(1-2\sigma)\left[b_{i,j+1}x_{i,j+1} - (b_{i,j+1} + b_{i,j})x_{i,j} + b_{i,j}x_{i,j-1}\right] + \\ +\sigma\left[b_{i-1,j+1}x_{i-1,j+1} - (b_{i-1,j+1} + b_{i-1,j})x_{i-1,j} + b_{i-1,j}x_{i-1,j-1}\right]\} = 0 \\ \sigma\left[a_{i+1,j+1}y_{i+1,j+1} - (a_{i+1,j+1} + a_{i,j+1})y_{i,j+1} + a_{i,j+1}y_{i-1,j+1}\right] + \\ +(1-2\sigma)\left[a_{i+1,j}y_{i+1,j} - (a_{i+1,j} + a_{i,j})y_{i,j} + a_{i,j}y_{i-1,j}\right] + \\ +\sigma\left[a_{i+1,j-1}y_{i+1,j-1} - (a_{i+1,j-1} + a_{i,j-1})y_{i,j-1} + a_{i,j-1}y_{i-1,j-1}\right] + \\ +\gamma^2\{\sigma\left[b_{i+1,j+1}y_{i+1,j+1} - (b_{i+1,j+1} + b_{i+1,j})y_{i+1,j} + b_{i+1,j}y_{i+1,j-1}\right] + \\ +(1-2\sigma)\left[b_{i,j+1}y_{i,j+1} - (b_{i,j+1} + b_{i,j})y_{i,j} + b_{i,j}y_{i,j-1}\right] + \\ +\sigma\left[b_{i-1,j+1}y_{i-1,j+1} - (b_{i-1,j+1} + b_{i-1,j})y_{i-1,j} + b_{i-1,j}y_{i-1,j-1}\right]\} = 0 \\ i = \overline{1, m}, \;\; j = \overline{1, n} \end{cases} \tag{7}$$

$$\begin{cases} (x_{1,j} - x_{0,j}) = 0, \;\; j = \overline{0, n+1} \\ \sqrt{r^2 - (y_{m+1,j})}(y_{m,j} - y_{m+1,j}) - (y_{m+1,j} + h_1)(x_{m,j} - x_{m+1,j}) = 0, \;\; j = \overline{0, n+1} \\ (y_{1,n} - y_{i,n+1}) = 0, \;\; i = \overline{0, m_1} \\ (x_{i,n} - x_{i,n+1}) = 0, \;\; i = \overline{m_1, m_2} \\ (y_{i,n} - y_{i,n+1}) = 0, \;\; i = \overline{m_2, m+1} \\ (y_{i,1} - y_{i,0}) = 0, \;\; i = \overline{0, m+1} \end{cases} \tag{8}$$

$$\gamma = \frac{1}{(m+1)(n+1} \sum_{i,j=0}^{m,n} \frac{1}{k_{i+\frac{1}{2}, j+\frac{1}{2}}} \gamma_{i,j} \tag{9}$$

$$\gamma_{i,j} = \frac{\sqrt{\left(x_{i+1,j} - x_{i,j}\right)^2 + \left(y_{i+1,j} - y_{i,j}\right)^2} + \sqrt{\left(x_{i+1,j+1} - x_{i,j+1}\right)^2 + \left(y_{i+1,j+1} - y_{i,j+1}\right)^2}}{\sqrt{\left(x_{i,j+1} - x_{i,j}\right)^2 + \left(y_{i,j+1} - y_{i,j}\right)^2} + \sqrt{\left(x_{i+1,j+1} - x_{i+1,j}\right)^2 + \left(y_{i+1,j+1} - y_{i+1,j}\right)^2}} \tag{10}$$

where: $a_{i,j} = 2/(k_{i,j} + k_{i-1,j})$, $b_{i,j} = (k_{i,j-1} + k_{i,j})/2$, $x_{i,j} = x(\varphi_i, \psi_j)$, $y_{i,j} = y(\varphi_i, \psi_j)$, $k_{i+1/2,\, j+1/2} = (k_{i,j} + k_{i+1,j} + k_{i,j+1} + k_{i+1,j+1})/4$, $\sigma \in [0, 0.5]$ are weighting factors.

The solution of the difference problem (6) – (9) should be presented as [BOMBA *et al.* 2007b: 2008]. We set the number of m and n joint connections of the grid area G_ω partition, parameter ε which characterises the approximation accuracy of the corresponding difference problem solution. We set the initial approximation of the set of values. Namely: the initial approximation of the coordinates of the boundary connections $x_{0,j}^{(0)}$, $y_{0,j}^{(0)}$, $x_{m+1,j}^{(0)}$, $y_{m+1,j}^{(0)}$, $x_{i,n+1}^{(0)}$, $y_{i,n+1}^{(0)}$, $x_{i,0}^{(0)}$, $y_{i,0}^{(0)}$ (so that it satisfies the equalities set in the equation (7), as well as the initial approximation of the coordinates of the internal connections $\left(x_{i,j}^{(0)}, y_{i,j}^{(0)}\right)$, $i = \overline{1, m}, j = \overline{1, n}$. We shall set the initial approximation of a conformal invariant γ according to the Equation (9) which uses the newly set initial values of the coordinates of the internal connections, i.e. $\gamma^{(0)} = \gamma\left(x_{i,j}^{(0)}, y_{i,j}^{(0)}\right)$. Then we make a refinement of the following: the internal connections $\left(x_{i,j}^{(k+1)}, y_{i,j}^{(k+1)}\right)$ (k = 0, 1, …) is the number of the iteration step) using the Gauss–Seidel method according to the formulas obtained by solving the equation (6) with respect to $x_{i,j}$ and $y_{i,j}$; γ value according to the Equation (9), the coordinates of the boundary connections, for instance, by solving the system of nonlinear equations (7) and (8). Subsequently, we check if all terms of finishing the computational process are satisfied, for example, in line with the following equations:

$$\max_{x_{i,j},\, y_{i,j} \in \partial G_z} \left(\left| x_{i,j}^{(k+1)} - x_{i,j}^{(k)} \right|, \left| y_{i,j}^{(k+1)} - y_{i,j}^{(k)} \right| \right) < \varepsilon \qquad (11)$$
$$\left| Q^{(k+1)} - Q^{(k)} \right| < \varepsilon,\ \left| D^{(k+1)} - D^{(k)} \right| < \varepsilon,$$

where

$$D = \frac{1}{(m+1)(n+1)} \sum_{i,j=0}^{m,n} \frac{\sqrt{\left(x_{i+1,j+1} - x_{i,j}\right)^2 + \left(y_{i+1,j+1} - y_{i,j}\right)^2}}{\sqrt{\left(x_{i,j+1} - x_{i+1,j}\right)^2 + \left(y_{i,j+1} - y_{i+1,j}\right)^2}}$$

is the average value of the ratio of the lengths of the diagonals of the curvilinear quadrilaterals of the grid area G_z^{γ}.

If the conditions set in (11) are not satisfied, then we come back to the step of the refinement of the internal connections and so on. In the opposite case, we calculate the closure error of the resulting grid conformance according to the equation $\varepsilon_* = |1 - D|$. Its value characterises the deviation of the resulting curvilinear quadrilaterals from the corresponding rectangles (since the ratio of the lengths of the diagonals in a rectangle is equal to one, and the existence of right angles is the fundamental orthogonality condition).

In case if, for instance, only one of the conditions (11) is not satisfied, we reconcile the relationship between accuracy ε_* and the given number of partition steps m, n (first of all, by increasing the latter ones). If, however, we need to improve of accuracy of the approximate solution (to reduce the closure error ε_*), we increase the parameters of the partition m and n and solve the difference problem (6) – (9) again. The optimality of the relationship between m and n is achieved analogously to BOMBA *et al.* [2007b; 2008] by optimising the functional analogues of Riemann integral. The argumentation of the created algorithm for „step-by-step recording of the process and environment (medium) characteristics, conformance parameter, internal and boundary joint connections of the curvilinear area" shall be carried out analogously to BOMBA *et al.* [2007b; 2008] using the ideas of block iterative methods.

RESULTS AND DISCUSSION

After doing calculations according to the described algorithm with the following numerical values: total filter thickness l = 3 m, depth of drainage laying h_1 = 0.675 m, drain radius r = 0.025 m, distance from drainage to impenetrable basis C_*C^* h_2 = 1.3 m, thickness of one layer of filter t_f = 0.01 m, thickness of two layers of filter b_{tp} = 0.05 m, partition mn = 30×16 of the G_z area, accuracy of approximation $\varepsilon = 10^{-5}$, conductivity coefficients of the medium k_1 = 50 m·day^{-1}, k_2 = 10 m·day^{-1}, k_3 = 1 m·day^{-1} for k = 2847 steps, we shall obtain a hydrodynamic motion grid (see Fig. 1a), velocity field, we shall also find the total seepage discharge Q = 1.065 m·day^{-1} and other process characteristics.

Particularly, Table 1 demonstrates the results of numerical calculations of the seepage discharge depending on the change of k_1, k_2, k_3, t_f, b_{tp} parameters.

Table 1. Results of numerical calculations of the seepage discharge Q depending on the change of k_1, k_2, k_3, t_f, b_{tp} parameters

k_1	k_2	k_3	t_f	b_{tp}	Q
50	10	1	0.01	0.05	0.920320260082948
50	12	1	0.01	0.05	0.960873665785776
50	14	1	0.01	0.05	0.987617056663534
50	16	1	0.01	0.05	1.00835392085078
50	18	1	0.01	0.05	1.03851097519713
50	20	1	0.01	0.05	1.06577163362688
50	22	1	0.01	0.05	1.10114973682152
40	10	1	0.01	0.05	0.896314526530931
45	10	1	0.01	0.05	0.902018405768276
50	10	1	0.01	0.05	0.920320260082948
55	10	1	0.01	0.05	0.927833093747881
60	10	1	0.01	0.05	0.943876562915457
65	10	1	0.01	0.05	0.948026524377088
70	10	1	0.01	0.05	0.950105136285527
50	20	1	0.01	0.05	1.06577163362688
50	20	1	0.015	0.05	1.02366903997431
50	20	1	0.02	0.05	0.952081581579486
50	20	1	0.025	0.05	0.946986240371109
50	20	1	0.01	0.035	1.04081701488922
50	20	1	0.01	0.04	1.04539368741735
50	20	1	0.01	0.045	1.05341165619218
50	20	1	0.01	0.05	1.06577163362688

Explanations: k_1, k_2, k_3 = conductivity coefficients of the medium, t_f = thickness of one layer of filter, b_{tp} = thickness of two layers of filter.
Source: own study.

CONCLUSIONS

Therefore, the paper suggests a practical methodology for solving boundary value problems on conformal mappings for the calculation of the filtration process in the horizontal symmetrical drainage.

During the argumentation (as well as construction) of the corresponding algorithm which is based on the alternating „freeze" of the anticipated conformance parameter, the internal and boundary connections of the curvilinear area etc., we have used the ideas of block iterative methods [MARCHUK 1980].

The results of the conducted numerical calculations confirmed the effectiveness of the suggested problem formulations and algorithms of their numerical solution and the possibility of their use in the modelling of nonlinear filtration processes occurring in horizontal drainage systems, as well as in the design of drainage facilities and optimising other hydrotechnical systems, and, hence, the paper results are hugely valuable.

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Matematyczne modelowanie procesów filtracji w systemach drenarskich z zastosowaniem odwzorowania konforemnego

STRESZCZENIE

W przypadku podwyższenia poziomu wody gruntowej znacząco ponad „normalną" wysokość pobór wody, obniżanie jej zwierciadła i inne działania wymagają urządzeń drenarskich. Obecnie najbardziej efektywnymi technikami numerycznych badań odpowiednich problemów brzegowych są metody odwrotnej analizy wartości brzegowych (odwzorowania konforemne i quasi-konforemne). Jako podstawę prowadzonych badań przyjęto przykład połączenia metod fikcyjnej domeny z quasi-konforemnym odwzorowaniem rozwiązania problemu nieliniowych wartości brzegowych do obliczenia reżimu filtracji w środowisku o wolnej powierzchni brzegowej (krzywe depresji) i w strefach „górskich".

W pracy dokonano przeglądu rozwiązań pionowej stacjonarnej filtracji płynów do horyzontalnego symetrycznego drenażu. Zaproponowano praktyczną metodologię rozwiązywania problemów wartości brzegowych w przekształceniu konforemnym do obliczania procesu filtracji w horyzontalnym symetrycznym drenażu.

Wykorzystano zasadę metod iteracyjnych podczas tworzenia odpowiednich algorytmów opartą na naprzemiennym „zamrażaniu" przewidywanego parametru odwzorowania z wewnętrznymi i brzegowymi powiązaniami krzywoliniowego obszaru.

Wyniki przeprowadzonych numerycznych obliczeń potwierdzają efektywność formułowania sugerowanego problemu i algorytmów ich numerycznego rozwiązania oraz możliwości ich zastosowania w modelowaniu nieliniowych procesów filtracji występujących w horyzontalnym systemie odwadniania, w projektowaniu urządzeń drenarskich i w optymalizacji innych systemów wodnych. Z tego powodu uzyskane wyniki mają ogromne znaczenie.

Słowa kluczowe: *modelowanie matematyczne, ochrona przeciwpowodziowa, odwzorowanie konforemne, procesy filtracji, systemy drenarskie*