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DRIVING TECHNIQUES FOR MINIMIZING FUEL CONSUMPTION DURING RECORD VEHICLE COMPETITION

The problem of optimal driving techniques during fuel economy competition is considered. The kinetic model of the record wheeled vehicle is proposed. It is regarded as a particle moving on a trace with variable slope angle. Engine characteristics are taken into account. The fuel consumption is minimized as the vehicle goes over a given distance. The problem is formulated in optimal control. The direct pseudospectral Chebyshev's method is employed. The motion of student's vehicle representing the Faculty of Power and Aeronautical Engineering during Shell Eco-marathon in Nogaro, France, in 2006 , is used as an example.

NOMENCLATURE

A	–	initial point,
B	–	final point,
F	–	rolling resistance [N],
f	–	rolling resistance per weight unit,
G_p	–	total fuel consumption [kg],
C_x	–	drag coefficient,
g	–	gravitational acceleration [m/s^2],
g_p	–	specific fuel consumption [kg/(W·s)],
h(x)	–	shape function of the trace [m],
L	–	total length of the trace [m],

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m – mass of the vehicle with the driver [kg],
P – instantaneous power of the engine [W],
P_x – aerodynamic drag [N],
R – normal reaction of the ground [N],
S – reference area [m^2],
T – forward propulsive force [N],
t – time [s],
v – velocity of the vehicle [m/s],
x – horizontal coordinate (approximately covered distance) [m],
α – local slope of the trace,
η – power setting,
ρ – air density [kg/m^3].

1. Introduction

One of the most important challenge of the latest decades is minimization of fuel consumption of the vehicles (aircraft, cars, ships, trains). It is enforced by increasing prices of crude oil and shortage of natural resources on the one hand, and increasing emission of carbon dioxide on the other hand. Two ways are possible to meet this challenge: by improvement of vehicle performances via reduction of masses, resistances of motion, using more efficient engines, and by using optimal driving techniques. Minimization of fuel consumption (operating costs) has a long tradition in aircraft engineering (Maroński and Łucjanek, 1979; Maroński, 1988; Panasz and Maroński, 2005), where variations of the aircraft altitude and velocity are possible. Such techniques are of less importance for car drivers due to limitations imposed, for example, by traffic regulations and traffic jams. Such constraints do not appear during the competitions to achieve highest fuel economy. An annual event known as Shell Eco-marathon is organized in Nogaro, France.

The aim of this study is formulation and solution of the minimum-fuel problem for the record vehicle participating in such an event. The paper is not devoted to theoretical difficulties due to the existence of optimal solutions in piece-wise continuous control functions, when "chattering-controls" give best results, and to the so-called singular arcs, where classic necessary conditions of optimality do not hold. The discussion of such cases one can find, for example, in Maroński (1988).

2. Problem formulation

Fundamental assumptions of the model are as follows:

1. The record vehicle is regarded as a particle. It is assumed that the dimensions of the car's body are small in comparison to the distance to be covered.
2. The velocity is not reduced in turnings, therefore the motion is considered in vertical plane. The shape of the trace is described by a given smooth function h(x) of the horizontal coordinate x.
3. The transmission ratio is constant. The vehicle is not equipped with any gear box.
4. The brakes are not used, because braking causes irreversible losses of energy.
5. The velocity of the vehicle is controlled only via regulation of power setting (forward propulsive force). The engine is running over a whole distance and the clutch is thrown in.
6. The wind is not considered.
7. The mass of the vehicle is constant. That means that the mass of the fuel consumed during the event is negligible.
8. The local slope of the trace α is small, therefore sinα may be approximated by tgα. The curvature of the trace is neglected.

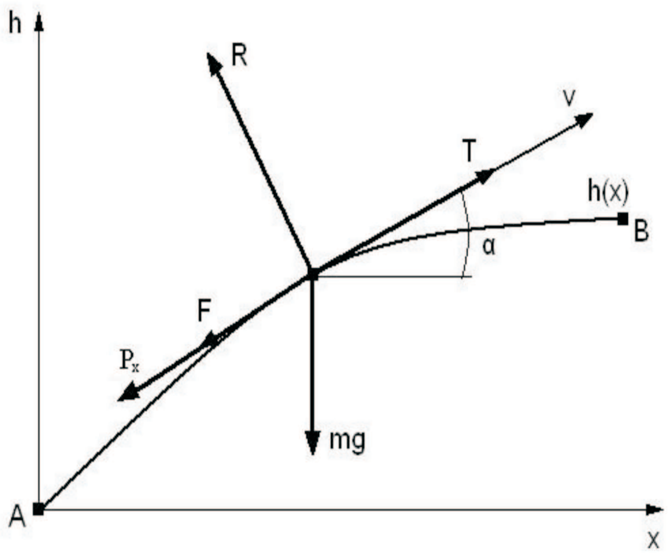

Fig. 1. Particle model of the vehicle

The equation of vehicle motion resulting from the second Newton law is as follows (cf. Arczyński, 1994; Maroński, 1999)

$$m\,v\,\frac{dv}{dx} = \frac{P(v,\eta)}{v} - 0.5\,\rho\,v^2\,S\,C_x - F - m\,g\,\frac{dh}{dx}\,. \tag{1}$$

It should be considered with given boundary conditions representing vehicle velocities at the beginning and at the end of the race

$$v(x_A) = v_A\,, \qquad v(x_B) = v_B\,. \tag{2}$$

Minimized is total fuel consumption necessary to cover the given distance from A to B

$$G_p = \int_A^B \frac{g_p(v,\eta)\;P(v,\eta)}{v}\,dx\,. \tag{3}$$

The throttle setting η(x) to be found is unknown function of the distance x, and it belongs to the range

$$0 \leq \eta(x) \leq 1\,. \tag{4}$$

The problem is formulated in the following manner. Find the control function η(x) satisfying (4) that minimizes the fuel consumption (3). Equation of motion (1) with boundary conditions (2) should be satisfied. This is the typical problem of optimal control. In the paper, it has been solved applying direct pseudospectral Chebyshev's method. It employs N-th degree Lagrange polynomial approximations for state and control variables. The values of these variables at the Chebyshev-Gauss-Lobatto points are expansion coefficients. Such an approach converts optimal control problem to a nonlinear programming problem, where unknown parameters are state and control variables at these points (Fahroo and Ross, 2000). The method is implemented in Matlab using sequential quadratic programming algorithm (MATLAB Optimization Toolbox, 2000).

3. Numerical analysis

All computations refer to the student's record vehicle PAC-Car E/2006 with RYOBI 26CC engine and the trace in Nogaro, France. The data are as follow (cf. Staniszewski, 2006): mass of the vehicle with the driver m = 100 kg, drag coefficient C_x = 0.26, ratio of the chain transmission i = 13.2, reference area S = 0.5164 m^2, rolling resistance per weight unit f = 0.006 [F=mg·f], air density ρ = 1.225 kg/m^3, radius of car wheel is 0.23 m, gravitational acceleration g = 9.81 m/s^2, distance to be covered (x_B-x_A) = 3636 m, initial and final velocities v_A = v_B = 31 km/h. Preliminary computations have shown that the number of node points N = 200 is a rational compromise between the desired accuracy and the computing time. The performances of RYOBI 26CC engine are given in Fig. 2 and Fig. 3.

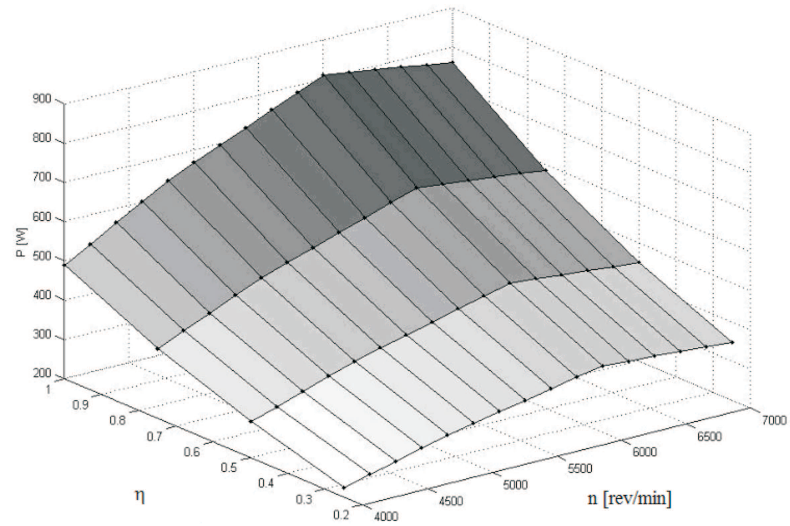

Fig. 2. Power of the engine versus revolutions and power setting

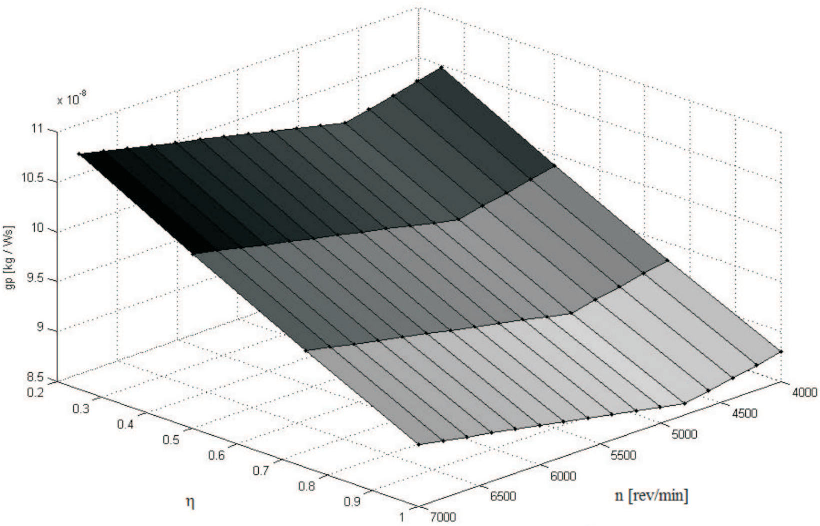

Fig. 3. Specific fuel consumption versus revolutions and power setting

The shape of the trace in Nogaro, France (elevation above horizontal versus distance) is depicted in Fig. 4.

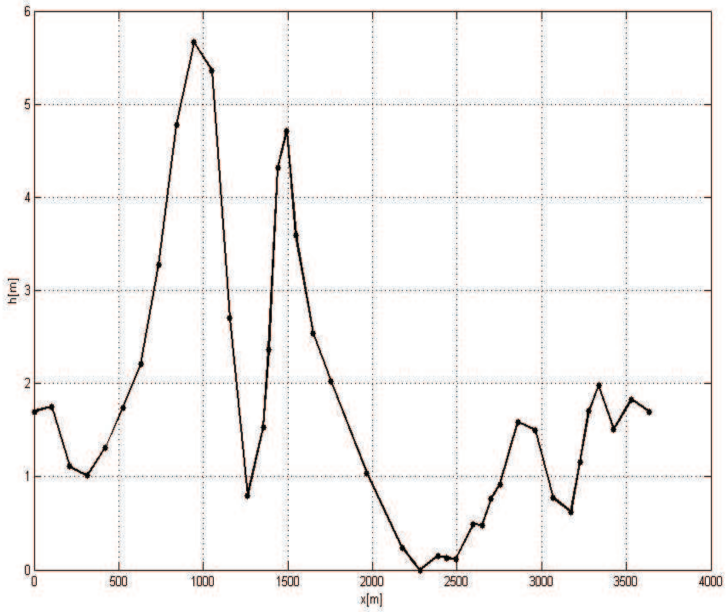

Fig. 4. Shape of the trace

As the final result of computations, one obtains optimum profiles of the throttle setting and the velocity (Fig. 5 and Fig. 6).

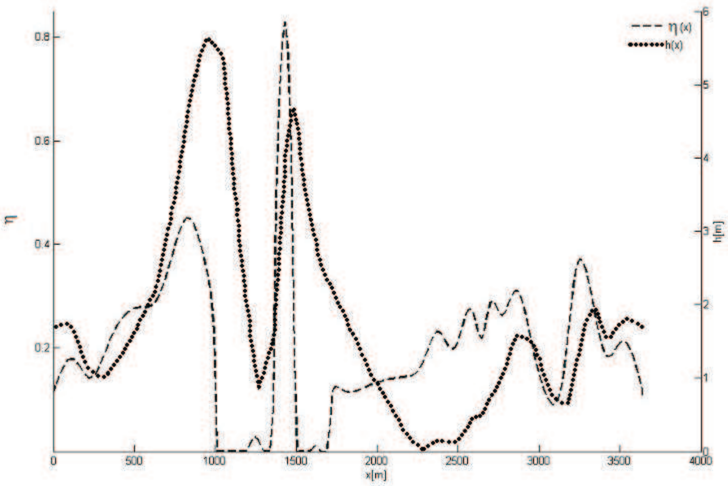

Fig. 5. Optimum throttle setting versus covered distance. The dotted line is the profile of the trace (cf. right-hand scale)

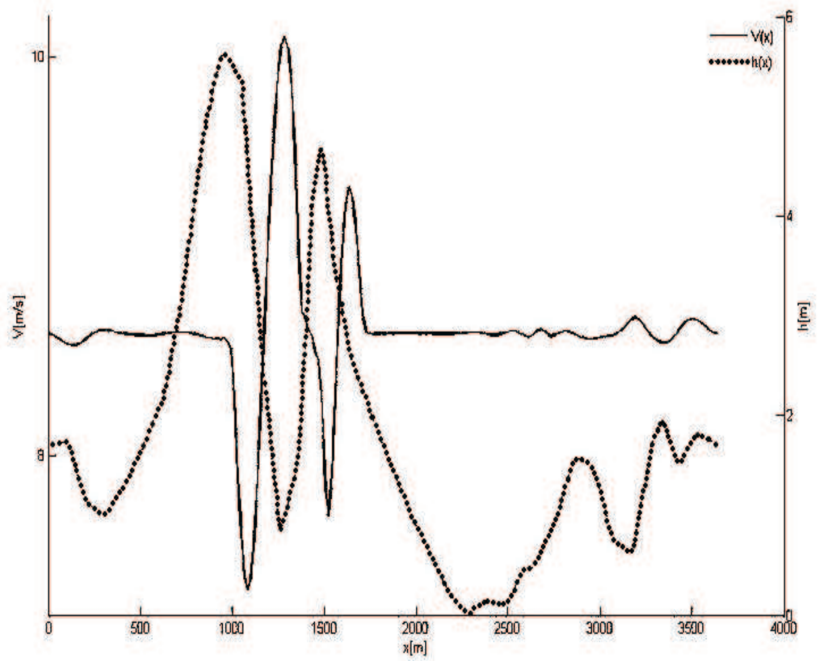

Fig. 6. Optimum velocity versus covered distance (dotted line – the profile of the trace)

The optimum velocity graph may be divided into three segments. Two of them, where the variations of velocity are negligible, appear at the beginning and at the end of the trace. The velocity does not depend on the local trace slope. This is contradictory to intuitive speculations. However, it is consistent with optimum results obtained for bicycle riding where the optimal control problem is formulated as an isoperimetric one (Maroński, 2002). For greater slope angles (the middle segment), the variations of velocity are significant. The vehicle should move faster during descent and slower during climbing.

4. Conclusions

In this study, the authors consider the problem of minimum-fuel consumption of the record vehicle during the motion on the track having variable slope angle. The problem is formulated in optimal control and solved applying direct pseudospectral Chebyshev's method. The numerical approach is efficient, and the algorithm is acceptably stable. One may also study the influence of model parameter perturbations onto the value of the minimum-fuel consumption. Computations show that the increase of total mass of the vehicle form m = 100 kg to m = 200 kg causes 8.2% decrease of the distance to be covered using 1 liter of fuel. The increase of the coefficient of rolling resistance from f = 0.006 to f = 0.012 causes 5.7% decrease of the distance. On the other hand, if the drag coefficient is reduced from $C_x = 0.26$ to $C_x = 0.13$, the distance to be covered using 1 liter of the fuel is 5.1% longer. Further details one can find in Rogowski (2008). Computations show that for small slopes the constant vehicle velocity should be recommended. This conclusion is in accordance with the results obtained for optimal cycling strategies and rowing techniques (cf. Maroński, 2002; Sanderson and Martindale, 1986). For steeper slopes, optimum velocity varies, and greater values of power setting are necessary as the vehicle moves up, and lower values as it moves down.

The presented study has a weakness however. The maximum distance to be covered using the strategy obtained in this paper is 307,4 km on 1 liter of fuel. It is much smaller than the result 487 km obtained by the student's vehicle during the event in 2006. In the opinion of the authors of this paper it is due to the fact that assumption 5 of the model of the process is not satisfied. During real competitions, the clutch is thrown in and out, and the engine is running only onto some sections of the trace. On the other ones the vehicle coasts down and the propulsive force of the engine is not employed. Such a strategy is much more sophisticated because the instants of time of the engine/clutch junctions should be selected in optimal manner, and it is out of the scope of the present study. The strategies during active segments of the trace are opened for further discussion and such strategies can be found using the method presented in this paper. The unexploited reserves of the strategies are sufficient and further studies of the problem are necessary.

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Strategie minimalizujące zużycie paliwa w czasie zawodów pojazdów rekordowych

S t r e s z c z e n i e

Rozważono zagadnienie wyznaczenia optymalnej strategii sterowania pojazdem w czasie zawodów minimalizacji zużycia paliwa. Zaproponowano model dynamiki pojazdu rekordowego. Pojazd potraktowano jak punkt materialny poruszający się po torze o zmiennym kącie pochylenia. Uwzględniono charakterystyki silnika. Minimalizowano zużycie paliwa potrzebne do pokonania zadanego dystansu. Zagadnienie rozwiązano metodami sterowania optymalnego. Zastosowano bezpośrednią pseudospektralną metodę Czebyszewa. W charakterze przykładu rozważono przejazd pojazdu studentów Wydziału Mechanicznego Energetyki i Lotnictwa Politechniki Warszawskiej w Nogaro, Francja, w 2006 roku.