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METHODS OF GEOMETRIC PARAMETERS ESTIMATION FOR A MECHANISM BASED ON POINT COORDINATES MEASUREMENTS

The paper presents selected methods used for estimation the unknown geometrical parameters of a spatial mechanism model, used to describe the position and orientation of the end-effector, for example the coordinates of the center points of spherical joints, link dimensions etc. These data are necessary when dealing with computer simulation of any mechanism. The parameters are estimated based on coordinate measurements of selected points, located on the real mechanism links, using a portable manipulator with serial structure composed of 6 revolute joints and a spherical probe, but other techniques of acquiring point coordinates are applicable as well. The described methods can be used in the cases of a disassembled link, an assembled mechanism and redundant data sets. The methods are characterized by accuracy and robustness in the presence of different levels of noise, stability with respect to degenerate data sets, and low computation time. Special attention is paid to the case when the wanted parameters are hard to measure directly. Numerical examples are presented dealing with 5-link mechanism used to guide front wheels of a car.

NOTATIONS

R – orthogonal rotation matrix,
p – position vector of any point,
o – position vector of reference system origin,
t – translation vector.

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1. Introduction

The problem of determining the location of an object with respect to another one or to a given coordinate system is an important step in mechanism modeling, robot calibration, or other applications. Several methods have been developed to solve the position problem from information on the homologous features of the body in considered position. The most widely used are based on the description of rigid body pose by a position vector of selected point, as an origin of local reference frame, and an orientation matrix of this frame with respect to a global frame. This problem can be solved using at least three noncollinear points. Nevertheless, this approach leads to an inaccurate estimation of the body pose in real experimental arrangements where the measurements of coordinates have some degree of error. The determination of an orthogonal orientation matrix from set of n points ($n \geq 3$) is a nonlinear problem. Its solution involves numerical optimization procedures. These method are computationally efficient, but lead to purely numerical solutions that complicate the subsequent kinematic analysis as well as the sensitivity analysis to estimate the model parameters.

This paper presents some procedures for estimation of unknown geometrical parameters of a mechanism model, upon having acquired noisy data from the sensory system. The methods are based on the coordinate measurements of selected points located on the considered object, assumed to be rigid. The point coordinates can be determined by contact or optical methods. The presented methods are intended to be general, no matter of what kind of mechanism is dealt with.

The application area of the presented methods could be described by accuracy and robustness in the presence of different levels of noise, stability with respect to degenerate data sets, and low computation time. The goal is to minimize the effects of measurement errors. In many applications, accurate determination of the pose (i.e. position and orientation) of an object is often required. Some methods are difficult to apply to car suspensions because of limited allowable workspace where the suspension is jointed to the car body. An immediate way to accomplish this task is to measure the properties of each component of the mechanism separately. But in some cases it is not possible to disassemble the mechanism, and the model has to be defined form observations made on the operating mechanism.

Simulation analysis of the considered mechanism usually is performed by using its kinematical model properly dimensioned and described by using constrained equations. Experimental study on a real mechanism can deal with coordinates of selected points which can be measured by using serial manipulator, for example Portable Coordinate Measuring Machine (CMM) is one of the most suitable [6, 7]. In practical applications, the measurements of coordinates for some points are particularly difficult because their positions and limited reach ability, for example the coordinates of joint centers. The useful dimensions of mechanism model can be read from technical drawings, however, this way is often restricted.

This paper presents selected estimation methods for the unknown geometrical parameters of a spatial mechanism, used to describe the position and orientation of the end-effector, for example the coordinates of the centre points of spherical joints, link dimensions etc. These data are necessary when dealing with computer simulation of any mechanism. The parameters are estimated based on coordinate measurements of selected points, located on the real mechanism links. This problem can be solved by using optimization method to find minimum of the goal function describing the deviation from desired value of the selected parameter describing the trajectory of the end-effector.

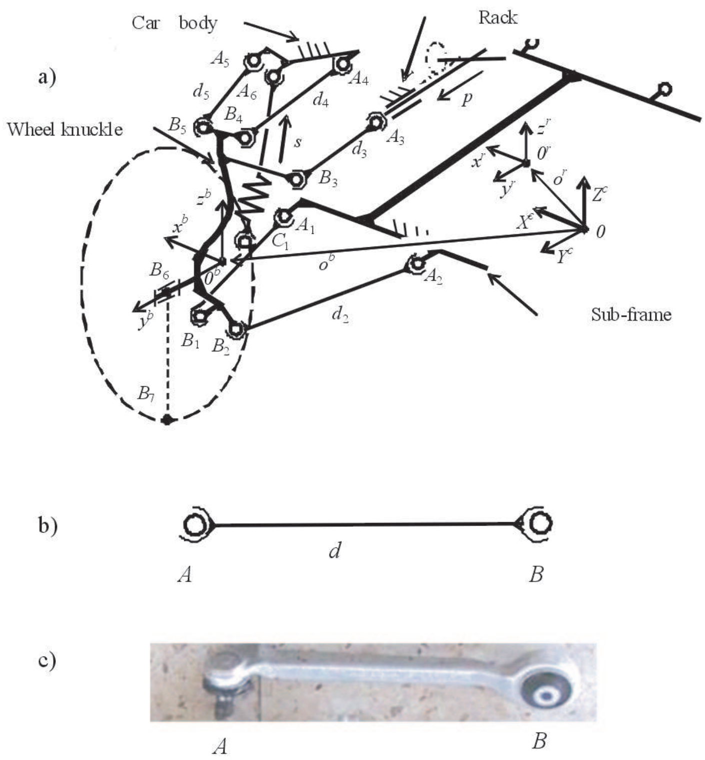


Fig. 1. a) Kinematical model of the wheel guide mechanism; b) Kinematic model of a separate link; c) View of real link of mechanism

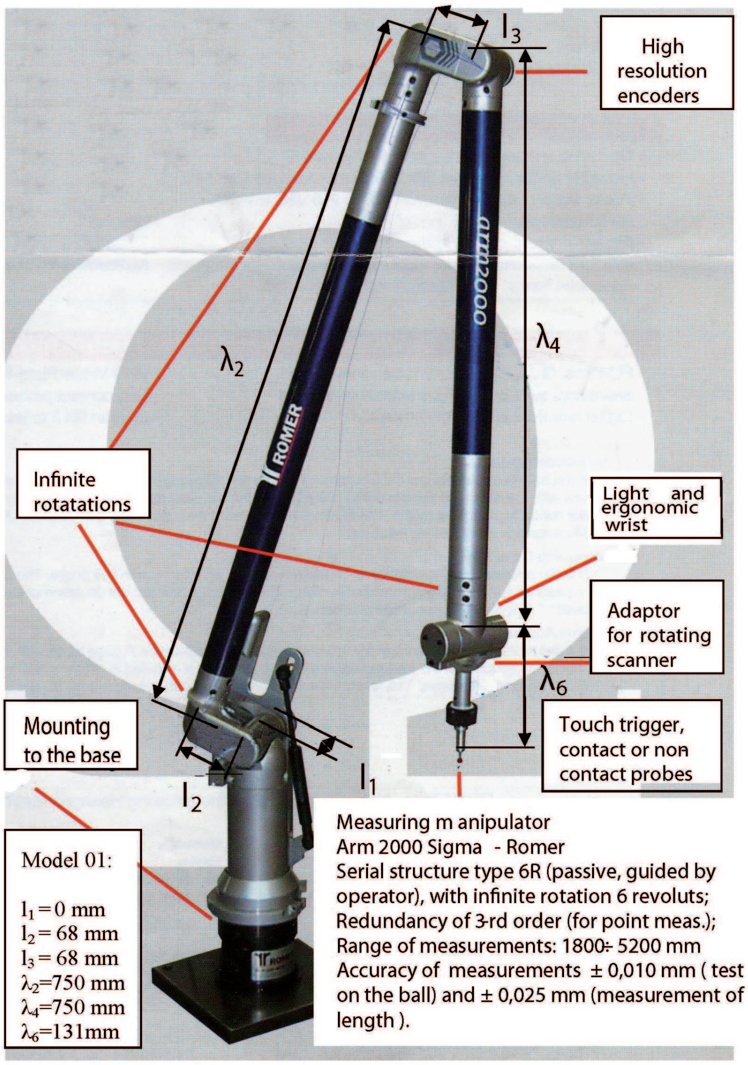


Fig. 2. View of measurement manipulator and some specifications [10]

The simulation results obtained by using kinematic model of the multi-link mechanism, for example 5-link wheel guide mechanism of passenger car [3, 4], presented in Fig. 1a, can be used in practice, if the model parameters were previously estimated, because they are not measurable directly. Two kind of estimation problem are solved by using the presented method: (I) estimation of geometrical parameters for the considered link; (II) estimation of link position and orientation with respect to the mechanism base frame.

2. Estimation of the rigid body pose and displacement

In order to describe the pose and displacement of the considered link (Fig. 1), one performs the coordinate measurements of selected points located on the link b (wheel knuckle) with respect to the base c (car body frame). These measurements are followed for successive positions of considered mechanism.

The position vector $\mathbf{b}_i^b$ of point B_i belonging to link b and described in the reference frame of this link, can be determined in the reference frame of the base c, by using transformation [1]:

$$\mathbf{b}_i^{b,c} = \mathbf{R}^{b,c}\mathbf{b}_i^b + \mathbf{o}^{b,c} \tag{1}$$

where:

$\mathbf{R}^{b,c} = [\hat{\mathbf{x}}^{b,c}\hat{\mathbf{y}}^{b,c}\hat{\mathbf{z}}^{b,c}]$ – orthogonal orientation matrix of the reference frame of link b with respect to reference frame of base c;

$\mathbf{o}^{b,c} = [o_x^{b,c} o_y^{b,c} o_z^{b,c}]^T$ – position vector of origin of the reference frame of link b with respect to origin of frame c.

The coordinate measurements of reference points, realized by using Portable Coordinate Measuring Machine [6, 7, 10], can be suitable to solve the problem of estimation of position and orientation of link b with respect to the base c, by minimizing the goal function [1]:

$$\varepsilon = \sum_{i=1}^{n} \left[\mathbf{b}_{ip}^{b,c} - \mathbf{R}^{b,c}\mathbf{b}_i^b - \mathbf{o}^{b,c}\right]^2 \tag{2}$$

where: $\mathbf{b}_{ip}^{b,c}$ – measured position vector of point B_i with respect to frame c.

This problem can be solved by using the coordinate measurements of three (at least) points respectively situated on the studied link. Assuming that the local reference frame is based on the measured points, the position vector of its origin is measured directly (for example point B_1), but the elements of orientation matrix $\mathbf{R}^{b,c}$ (for example described by three angles) are determined by using equation (2). In order to diminish the sensitivity of estimates to the deviations or errors of measurement results, a greater number of points ($n > 3$) is recommended [1].

3. Estimation of kinematical pair parameters

Assembled and disassembled mechanism

Two joints are connected with one rod of the considered mechanism (Fig. 1c), the spherical joint ((A) ball-type) situated at one side, and the cylindrical joint (B) with elastomeric bushing at the other side. Kinematical model of this link (shown on Fig. 1) is formed by two spherical joints, connected by a link of length d. The compliant bushing (B) is substituted by a spherical joint, what is reasonable in the case where radial deformation of the bushing has small influence on the characteristics of mechanism [4, 5].

Geometrical parameters for kinematical model of the considered mechanism are determined by using methods of analytical geometry and coordinate measurements for position of ball center of the end-effector of the measuring manipulator, placed at respective points of the measured profile [2]. The minimum number (n) of measured points used to determine the considered element is equal to the number of parameters to describe its characteristics. A greater number of points is recommended to estimate the deviations of measurements and to diminish the errors.

The distance (r_i) form any measured point to the assumed profile is analytically determined. In the case of ideal profile, the distances are equal to zero ($r_i = 0$). In the case of real profile, the parameters describing the sphere or cylinder are determined based on the minimum of all distances. This problem can be described as minimization of the functional E based on the method of least squares [2]:

$$E = \sum_{i=1}^{n} r_i^2 \min \tag{3}$$

The solution can be obtained by using an algorithm of nonlinear programming [6], because the components of functional (1) frequently are nonlinear functions of unknown parameters.

In the case of link modeling (Fig. 1b), the problem is to find the coordinate for the sphere center of joint (A) and the coordinates of the cylinder axis of joint (B). The sphere center (A) can be determined by using two approaches: (I) by measuring the point coordinates of the ball surface when the spherical joint is disassembled; or (II) by measuring the cylindrical surface of the link in different angular positions and by determination the intersection point of cylindrical axes [2, 5].

The coordinates of cylindrical surface (B) can be determined by measuring the points lying on the outer and inner bushing of the joint (Fig. 1) and on the surfaces corresponding to the base planes of the cylinder.

Geometrical parameters are estimated for the 5-link suspension mechanism used for front wheels of VW Passat and shown in Fig. 3 (front left wheel) as joined to the frame of the base of the rig.

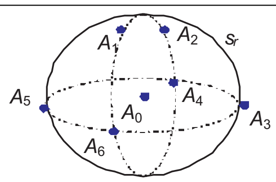
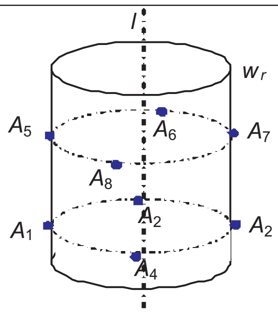
The position of the considered mechanism (Fig. 1a) is described by two independent variables: s – deformation of spring element (ride motion) and p – displacement of the steering rack (steering motion).

The link lengths are determined by using the described method for the link disassembled from mechanism. In order to estimate the position and orientation of the wheel knuckle corresponding to current mechanism positions, it is useful to measure the coordinates of three points located on the additional plate fixed to the wheel knuckle at car body side (see Fig. 3). The plate points are accurately dimensioned and positioned with respect to the wheel axis. The coordinates of points located on the wheel knuckle can be also measured, but the shape deviations of the surface can cause greater errors.

The successive positions of the measured plate are determined for the five following mechanism positions: (I) $s = 0$, $p = 0$; (II) $s = \max$, $p = 0$; (III) $s = \min$, $p = 0$; (IV) $s = 0$, $p = \min$; (V) $s = 0$, $p = \max$ and are show in Fig. 4. By using minimization of function (3), the parameters of position and orientation of the wheel knuckle are calculated, and collected in Tab. 2. One calculates the convention of Euler angles, which are called the Roll-Pitch-Yaw (δ, γ, ϕ) and used in vehicle dynamics [4].

Table 1.

Determination of parameters of the substitute geometrical elements on the basis of coordinate measurements of points

Geometrical element	Number of measured points (A_i, $i = 1, 2 \ldots n$)	Scheme of real profile with measured points	Coordinates describing profile
Sphere s	$n \geq 4$		Sphere center: $A_0(x_0, y_0, z_0)$, Sphere radius: r
Cylinder w	$n \geq 5$		Cylinder axis: l Cylinder center: $A_0(x_0, y_0, z_0) \in l$ Radius: r High h

The position of the considered mechanism (Fig. 1a) is described by two independent variables: s – deformation of spring element (ride motion) and p – displacement of the steering rack (steering motion).

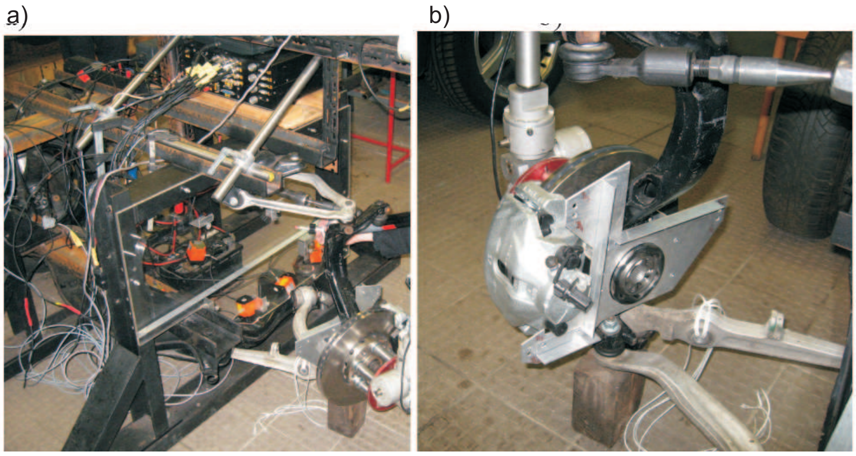


Fig. 3. a) View of 5-link suspension mechanism assembled with the base frame of the rig; b) View of the additional platform fixed to the wheel knuckle with measured points

The link lengths are determined by using the described method for the link disassembled from mechanism. In order to estimate the position and orientation of the wheel knuckle corresponding to the current mechanism positions it is useful to measure the coordinates of three points located on the additional plate fixed to the wheel knuckle at car body side (see Fig. 3). The plate points are accurately dimensioned and positioned with respect to the wheel axis. The coordinates of points located on the wheel knuckle can be also measured, but the shape deviations of the surface can caused greater errors.

The successive positions of the measured plate are determined for the five following mechanism positions: (I) $s = 0$, $p = 0$; (II) $s = \max$, $p = 0$; (III) $s = \min$, $p = 0$; (IV) $s = 0$, $p = \min$; (V) $s = 0$, $p = \max$ and are show in Fig. 4. By using minimization of function (3), the parameters of position and orientation of the wheel knuckle are calculated, and collected in Tab. 2. One calculates the convention of Euler angles, which are called the Roll-Pitch-Yaw $(\delta, \gamma, \varphi)$ and used in vehicle dynamics [4].

In order to determine the same scale of measurement accuracy, one calculates the constant distances between the points (B_1, B_2, B_3) assumed on the measured plate. The differences between the obtained distances in successive positions of the mechanism, given in Table 3, can describe the inaccuracy of the measurement system. The calculated confidence interval (fidelity level 99,7%) are given in the last column of Table 3. The mean relative is at the level of ±0.3%, what is satisfying value.

Table 2.
Coordinates for position and orientation of the wheel knuckle corresponding to successive positions of mechanism (position nr I is assumed as reference position)

Poitions	I	II	III	IV	V
x [mm]	0.00	–6.23	3.89	–15.48	11.03
y [mm]	0.00	–12.92	–5.82	–11.78	–3.74
z [mm]	0.00	–73.73	75.26	2.05	1.91
δ [deg]	0.00	0.77	1.26	23.16	-21.96
γ [deg]	0.00	0.97	0.10	-0.95	0.99
φ [deg]	0.00	0.61	-0.14	-0.62	0.56

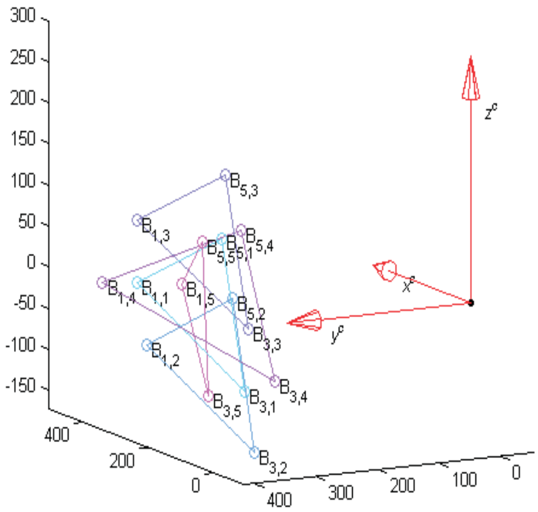


Fig. 4. The successive positions of the measured plate determined on the base of coordinate measurements with respect to the basic frame

Table 3.
Distances between reference points selected on the measured plate in successive positions of the mechanism, determined on the basis of measurements

Number of successive position	I	II	III	IV	V	Confidence interval (99%) [mm]
$\|\|\mathbf{b}_1 - \mathbf{b}_2\|\|$ [mm]	323.67	323.58	323.41	323.44	324.03	±0.75
$\|\|\mathbf{b}_1 - \mathbf{b}_3\|\|$ [mm]	271.47	271.26	271.27	271.36	271.92	±0.81
$\|\|\mathbf{b}_2 - \mathbf{b}_3\|\|$ [mm]	187.85	187.49	187.77	187.15	187.83	±0.89

4. Conclusions

Preparation of the input data for a simulation model is an important and work-consuming stage of simulation study. Inaccuracies in determination of model parameters directly influence the agreement between model and reality.

Parameters of the model can be estimated in the case when it is possible to measure real object by using suitable sensors.

Estimation accuracy of geometrical parameters of mechanism model is dependent on the accuracy of coordinate measurements of selected points with respect to the local or base reference frame, and on the number or distribution of the measured points influencing the conditioning of measurements. The numerical procedure used to transform experimental data into final results cannot be neglected.

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Metody estymacji parametrów mechanizmu na podstawie pomiarów współrzędnych punktów

S t r e s z c z e n i e

W pracy przedstawiono metody estymacji położenia i przemieszczeń mechanizmu przestrzennego na podstawie znanych współrzędnych wybranych punktów mechanizmu, które zmierzono metodą stykową za pomocą szeregowego manipulatora pomiarowego o 6 parach obrotowych. Opisano kolejne etapy pomiarów i estymacji, np.: pomiary punktów, osi, cylindrów i środków sfer, transformacje układów współrzędnych, miary rozrzutu pomiarów. Procedurę tę wykorzystano do wyznaczenia nieznanych parametrów geometrycznych mechanizmu oraz do weryfikacji jego modelu symulacyjnego. Przykład numeryczny dotyczy wielo-wahaczowego mechanizmu prowadzenia koła samochodu.