An approach to suppress high-frequency resonance using model predictive and selective harmonic elimination combined strategy

SITONG CHEN, XIAOQIANG CHEN, YING WANG, YE XIONG

School of Automation and Electrical Engineering, Lanzhou Jiaotong University
Lanzhou, China
e-mail: sitong.chen@163.com
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Abstract: High-frequency resonance is a prominent phenomenon which affects the normal operation of the high-speed railway in China. Aiming at this problem, the resonance mechanism is analyzed first. Then, model predictive control and selective harmonic elimination pulse-width modulation (MPC-SHEPWM) combined control strategy is proposed, where the harmonics which cause the resonance can be eliminated at the harmonic source. Besides, the MPC is combined to make the current track the reference in transients. The proposed control has the ability to suppress the resonance while has a faster dynamic performance comparing with SHEPWM. Finally, the proposed MPC-SHEPWM is tested in a simulation model of CRH5 (Chinese Railway High-speed), EMUs (electric multiple units) and a traction power supply coupled system, which shows that the proposed MPC-SHEPWM approach can achieve the resonance suppression and shows a better dynamic performance.

Keywords: CRH5 EMUs and traction power supply coupled system, high-frequency oscillation, high speed railway, model predictive control (MPC), selective harmonic elimination pulse-width modulation (SHEPWM)

1. Introduction

High-frequency resonance has been frequently reported worldwide, which has a serious impact on the safety of the electric railway operation [1–4]. Sinusoidal pulse-width modulation (PWM) technology has been applied in electric vehicles for more than 20 years. There are high-frequency harmonics produced by PWM in these vehicles, which are injected into the traction network. When the high frequency harmonic injected into the traction network is the resonance frequency of the...
traction power supply system, the resonance occurs [5–7]. In recent years, high-speed railways have rapidly developed in China, large quantities of AC–DC–AC drive multiple units (EMUs) using PWM converter technology have been put into operation, and the resonance phenomenon occurs more and more often. In 2007, the Beijing-Harbin railway line was put into operation, there have been four resonance overvoltage accidents which caused failures of the converter in EMUs. In 2011, the resonance overvoltage accident happened to the Beijing-Shanghai railway line. In 2017, the Beijing-Guangdong railway line suffered voltage fluctuation, as shown in Fig. 1, which caused the tripping of the main breaker in a traction substation. These phenomena are caused by the harmonics resonance which makes aware that resonance suppression scheme should be valued.

Aiming at high-frequency resonance issues, some researchers have studied the influencing factors caused by the factors of a traction network, the positions, the running conditions, and the power of the vehicles [8–11]. In [12, 13], high-pass passive filters are installed in a traction power supply system (TPSS), where most of the high-frequency harmonics in the traction network can be filtered. Active power filters can be used on board vehicles and on the traction network to compensate for harmonics [14]. The resonance suppression is achieved in these studies by introducing filtering devices. Some scholars realize the suppression of the resonance without an extra filtering device, the adapted delta modulation strategy is adopted in the converter control to reduce the resonance harmonics injected to the TPSS, as documented in [15]. Some researchers brought forward the method of avoiding the resonant frequency harmonic generation, the selec-

![Traction power supply system](image-url)
### 2. Resonance mechanism traction power supply system

The TPSS in China is an all-parallel AT-fed power supply system with single-phase AC power and electrical parameters of $2 \times 27.5$ kV 50 Hz, there is a coupling relationship of mutual impedance and a stray capacitor between conductors of each line. In order to analyze the harmonic characteristics of the traction power supply system, the T-type equivalent circuit is established.
respectively for the transmission line on both sides of the vehicle, as it is shown in Fig. 3. \( Z_o \) is the equivalent impedance of the power supply facilities and the transmission line on the other side of the facilities, respectively. \( Z_{L1}, Z_{L2} \) and \( Y_{L1}, Y_{L2} \) are the equivalent impedance and admittance of the left-side and right-side transmission line seen from the harmonic source, respectively.

When the equivalent impedance of the left side and the right side of the vehicle is capacitive and inductive respectively, there is resonant frequency causing the parallel resonance to occur. In order to analyze the parallel resonance, the CRH5 EMUs can be equivalent to a current harmonic source \( I_N \) [23], which generates a number of odd number higher harmonics, literature [22] has analyzed the harmonic characteristics of the two-level four quarant pulse rectifier (4QPR). When the harmonic of \( I_N \) injected into the traction network is equal to the resonant frequency, the resonance would occur.

In Fig. 2, the input impedance \( Z_i \) of the traction network seen from EMUs can be calculated as:

\[
Z_i = \frac{Z_c \cosh \gamma l \times (Z_o \cosh \gamma l_1 + Z_c \sinh \gamma l_1)}{Z_o \sinh \gamma l + Z_c \cosh \gamma l},
\]

where: \( l = l_1 + l_2 \) is the total length of the transmission line, \( l_1, l_2 \) are the left-side length and the right-side length, respectively. \( Z_i \) is the characteristic impedance of the transmission line, \( \gamma = \sqrt{ZY} \) is the propagation constant, \( Z \) and \( Y \) are the equivalent impedance and admittance per unit length of the transmission line, respectively.

![Fig. 2. Vehicle-traction power supply coupled system](image)

When the resonance occurs, \( Z_i \) would attain the maximum value, so that the denominator in (1) would approach zero.

\[
Z_o \sinh \gamma l + Z_c \cosh \gamma l = 0,
\]

where: \( \gamma l < 1 \), so that \( \tan \gamma l \approx \gamma l, \ Z_o = \omega L_o = -Z_c/\gamma l). \)

At this point, the resonant frequency can be expressed as:

\[
f_{rp} = 1/(2\pi \sqrt{L_o C}),
\]

where \( C \) represents the equivalent inductance and the capacitance of \( Y \).
3. Proposed model of predictive selective and harmonic elimination combined strategy

The proposed control schematic is shown in Fig. 3, the control under $dq$ frame of the dual four-quadrant pulse two-level rectifier is adopted in CRH5 EMUs [24]. Because the rectifier is a single-phase one in EMUs, the voltage and current of the AC side are regarded as components in the $\alpha$ frame, and the imaginary voltage and current in the $\beta$ frame should be fabricated. Then the voltage and current are transferred to the $dq$ coordinate system through the Park transformation, so that the AC signal can be transformed into the DC signal. The DC voltage can be achieved by controlling the voltage in an outer loop and zero reactive power can be achieved by controlling the relationship of variables in the $dq$ system. The control under $dq$ frame is continued to be adopted in the proposed MPC-SHEPWM.

![Fig. 3. Control schematic of MPC-SHEPWM for a single-phase CRH5 EMUs rectifier](image)

To realize the suppression of the high-frequency resonance, SHEPWM is designed first and the switching signals $S^*_\text{SHE}$ of PWM is obtained in 3.1. However, there is a weakness of the dynamic performance in SHEPWM, aiming at this issue, the FCS–MPC is introduced in 3.2 to make up for the weakness, where there is an advantage of fast dynamic performance. The switching signals $S^*_\text{SHE}$ in SHEPWM is added as constraints in the cost function in the FCS–MPC in 3.3 to make it combined, and the performance of resonance suppressing and fast dynamic can be realized.

3.1. Selective harmonic elimination PWM

Reference [23] made theoretical analysis and introduction of the SHEPWM technology, which can control the fundamental and harmonic waveform by adjusting the positions of the switching angle.

The two-level rectifier is adopted in CRH5 EMUs, as shown in Fig. 4, there are $N$ switching angles in a quarter period, SHEPWM requires the waveform to have the characteristics of a quarter-period symmetry waveform, in this way, the even-order harmonic component and zero-order
component are naturally eliminated as well as the amplitude of the fundamental component and harmonic components.

![Two-level pulse waveform](image)

Fig. 4. Two-level pulse waveform

$b_n$ is the amplitude of the $n$–th order, the $n$–th harmonic can be eliminated by setting $b_n$ to zero. For the two-level rectifier, the expression of $b_n$ can be expressed as follows:

$$b_n = \sum_{i=1}^{N} (-1)^{i-1} \cos(n\alpha_i).$$  \hfill (4)

The switching frequency of CRH5 EMUs is 250 Hz, so that there are five controllable switching angles in a quarter period, as for the dual rectifier, there are 10 controllable switching angles. The control of the fundamental wave of rectifier 1 and rectifier 2 is independent, while the control of the harmonic is achieved by the control of a synthesized PWM waveform. The fundamental frequency can be written as:

$$\left( \sum_{i=1}^{5} (-1)^{i-1} \cos(\alpha_i) - M \right)^2 + \left( \sum_{i=6}^{10} (-1)^{i-1} \cos(\alpha_i) - M \right)^2 = 0.$$  \hfill (5)

$M \in (0.5, 1.0)$ is the modulation index which is governed by:

$$M = \frac{\pi}{4u_{dc}}b_1,$$  \hfill (6)

where: $b_1$ is the fundamental component, $u_{dc}$ is the DC voltage of the rectifier, and the constraint of the switching angles $\alpha_i$ is:

$$\begin{cases} 0 < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \frac{\pi}{2} \\ 0 < \alpha_6 < \alpha_7 < \alpha_8 < \alpha_9 < \alpha_{10} < \frac{\pi}{2} \end{cases}.$$  \hfill (7)

The harmonic elimination control can be realized by:

$$\left( \sum_{i=1}^{5} (-1)^{i-1} \cos(n\alpha_i) + \sum_{i=6}^{10} (-1)^{i-1} \cos(n\alpha_i) \right) = 0.$$  \hfill (8)
Among 10 controllable switching angles, two are used to control the fundamental components of rectifiers, so that up to 8 harmonics can be controlled. In this paper, the resonance frequency of the traction network supply system is 1550 Hz, since the even harmonics are not contained in the symmetrical pulse waveform, the higher harmonics $n$ are selected as 27-th, 29-th, 31-th, 33-th, and the remaining harmonics are lower harmonics, they are 3-th, 5-th, 7-th, 23-th. For transcendental equations with many dimensions and constraints, there may be no solutions under these conditions, so that (7) and (8) can be described as nonlinear optimization functions.

$$F_{\text{min}} = \left( \sum_{i=1}^{5} (-1)^{i-1} \cos(\alpha_i) - M \right)^2 + \left( \sum_{i=6}^{10} (-1)^{i-1} \cos(\alpha_i) - M \right)^2 + \left( \sum_{i=1}^{5} (-1)^{i-1} \cos(n\alpha_i) + \sum_{i=6}^{10} (-1)^{i-1} \cos(n\alpha_i) \right).$$

Due to the complexity of the calculations, it can be performed offline with different algorithms such as Newton’s iteration, the Walsh function method, particle swarm optimization and biogeographical based optimization [24–27]. The Newton iteration method is adopted in this paper to calculate ten sets of switching angles as showed in Fig. 5, so that the requiring angles can be obtained according to $M$ with a look-up table, then the switching angles can be converted into the switching driving signals $S_{\text{HE}}$ of traction rectifier bridge arms.

![Fig. 5. Sets of solutions of SHEPWM switching angles $\alpha_1$–$\alpha_5$ (a); SHEPWM switching angles $\alpha_6$–$\alpha_{10}$ (b)](image)

### 3.2. FCS–MPC applied in PWM

Fig. 6 depicts the four-quadrant pulse rectifier used in CRH5 EMUs. $R_d$ and $C_d$ stand for the equivalent impedance supporting capacitor in the DC link, respectively, $u_{dc}$ is the voltage in the DC link, $u_s$ and $i_s$ are the voltage and current of the rectifier of the traction network side, respectively, $R_s$ represents the equivalent resistance, $L_s$ is the inductance of the rectifier. In Fig. 6, the mathematical state equation of each part of the dual rectifier can be expressed as:

$$L \frac{di_s}{dt} = u_s - R_s i_s - u_{ab}.$$

(10)
The value of $R_s$ is small and can be ignored, the mathematical model of the rectifier in the rotating $dq$ coordinate can be derived as in (11).

$$
\begin{align*}
    u_{abd} &= u_{sd} - L \frac{di_{sd}}{dt} + \omega L i_{sq} \\
    u_{abq} &= u_{sq} - L \frac{di_{sq}}{dt} - \omega L i_{sd}
\end{align*}
$$

(11)

where: $i_{sd}$ and $i_{sq}$ represent the current $i_s$ decoupled in the $dq$ system, $u_{abd}$ and $u_{abq}$ are the voltage $u_s$ decoupled in the $dq$ system.

In this paper, the FCS–MPC is operated in a discrete time with a sampling time and one step prediction horizon, the schematic of the FCS–MPC is shown as Fig. 7, where variables $x(k)$ represent the instantaneous value at the current hour $kT_s$, $x(k+1)$ is the predictive value at $(k+1)T_s$, $S_{ab}$ is the switching function which is defined as:

$$
S_{ab} \in \{1, 0, -1\}.
$$

(12)
At the beginning of the $k$-th sampling cycle, the current $i_{sd}$ and $i_{sq}$ are sampled firstly, then the prediction model is used to calculate the $(k+1)$-th prediction values under the three switching states $S_{ab}$, and the switching state that minimizes the cost function $J$ is selected as the optimal state in (13).

$$S_{ab}(k) = \arg \{ \min J(k) \}.$$  

In order to calculate the prediction value, the discretization model of $i_s$ in the $dq$ frame can be expressed as:

$$\begin{align*}
\frac{di_{sd}}{dt} |_{r=k} &= \frac{i_{sd}(k+1) - i_{sd}(k)}{T_s} = \frac{u_{sd}(k) - S_{abd}(k)u_{dc}(k)}{L} + \omega i_{sq}(k) \\
\frac{di_{sq}}{dt} |_{r=k} &= \frac{i_{sq}(k+1) - i_{sq}(k)}{T_s} = \frac{u_{sq}(k) - S_{abd}(k)u_{dc}(k)}{L} - \omega i_{sd}(k).
\end{align*}$$  

According to (14), the predictive current $i(k+1)$ can be expressed as:

$$\begin{align*}
i_{sd}(k+1) &= i_{sd}(k) + \frac{T_s}{L} [u_{sd}(k) - S_{abd}(k)u_{dc}(k)] + \omega T_i i_{sq}(k) \\
i_{sq}(k+1) &= i_{sq}(k) + \frac{T_s}{L} [u_{sq}(k) - S_{abd}(k)u_{dc}(k)] - \omega T_i i_{sd}(k).
\end{align*}$$  

Formula (15) can be transformed into the $\alpha/\beta$ static coordinate as:

$$\begin{align*}
i_{sd}(k+1) &= i_{sd}(k) + \omega T_i i_{sq}(k) + \frac{T_s}{u_{sd}(k)L} \left\{ u_{sd}^2(k) - \left[ u_{sr}(k)S_{abra}(k) + u_{qf}(k)S_{abf}(k) \right] u_{dc}(k) \right\} \\
i_{sq}(k+1) &= i_{sq}(k) - \omega T_i i_{sd}(k) - \frac{T_s}{u_{sq}(k)L} \left\{ u_{sf}(k)S_{abra}(k) - u_{qf}(k)S_{abf}(k) \right\}.
\end{align*}$$  

In the control system of a single-phase PWM rectifier, only the voltage $u_{abra}(k)$ is needed to achieve the system control, the obtained voltage vector $u_{abf}(k)$ does not exist in the actual system, so that this virtual quantity can be discarded. Therefore, only the switching function $S_{abra}(k)$ is needed.

The control objective is to get accurate modulation wave of the rectifier $S_{abra}(k)$ by minimizing the error of the AC side current, so that the cost function $J$ can be expressed as:

$$J = \left[ i_{sd}^* - i_{sd}(k+1) \right]^2 + \left[ i_{sq}^* - i_{sq}(k+1) \right]^2,$$  

where $i_{sd}^*$ and $i_{sq}^*$ are the reference values of $i_{sd}$ and $i_{sq}$. By substituting the cost function $J$ and three switching states $S_{ab}$ into (13), the optimal switching state can be obtained.

### 3.3. FCS–MPC integrated into SHEPWM

To achieve better performance of the rectifier control, extra items are normally added to the cost function [28], therefore, SHEPWM is added in MPC in this section, which is shown in Fig. 8.

The cost function can be designed as in (18),

$$J = \left[ i_{sd}^* - i_{sd}(k+1) \right]^2 + \left[ i_{sq}^* - i_{sq}(k+1) \right]^2 + \sigma(k) \left[ S_{ab}(k) - S_{SHE}(k) \right]^2.$$  

$S_{SHE}(k)$ is selected as the optimal state in (13) in SHEPWM, $\sigma(k)$ is a weight coefficient.
In (18), the first item is the basic control objective, which tracks the current of the AC side. The second item is a constraint item, and the weighting factor $\sigma$ is introduced in, which constrains the switching action of the rectifier to fix the frequency spectrum of the PWM rectifier injected into the traction network. According to the SHEPWM algorithm proposed in Section 3.1, the corresponding voltage switching state $S_{\text{SHE}}^*$ is used as the reference, because MPC is operated in a discrete time, $S_{\text{SHE}}(t)$ should be discretized into $S_{\text{SHE}}^*(k)$, where:

$$S_{\text{SHE}}^*(k) = S_{\text{SHE}}^*(M(k), \delta(k)).$$  \hspace{1cm} (19)

The modulation index $M^*$ should be determined to obtain the optimal angles $\alpha_i$, and the switching states $S_{\text{SHE}}^*$ can be obtained by comparing $\alpha_i$ with the phase angle $\delta^*$ of the voltage $u_s$. The parameters $M^*$ and $\delta^*$ under the dq frame are defined as:

$$M^* = \frac{\pi}{4u_{dc}^*} \sqrt{(u_{sbd}^*)^2 + (u_{sabq}^*)^2}, \quad \delta^* = \theta - \arctan \left( \frac{u_{sabq}^*}{u_{sbd}^*} \right).$$  \hspace{1cm} (20)

Weighting factor $\sigma$ is introduced in the second item, when a low value is chosen as $\sigma$, the control system tends to be a standard FCS–MPC current control system, where the dynamic response will be fast, but the switching frequency and the harmonic spectrum are difficult to guarantee. On the contrary, there will be better harmonic spectrum characteristics in the waveform of PWM if $\sigma$ is high, but the dynamic response of the system will be slower. Considering the
control performance of both dynamic and steady performance, the weighting factor $\sigma$ can be designed as:

$$
\sigma(k) = \begin{cases} 
\sigma_{\text{max}} - \Delta i(k), & \sigma(k) \geq \sigma_{\text{min}} \\
\sigma_{\text{max}}, & \sigma(k) < \sigma_{\text{min}} 
\end{cases},
$$

(21)

where

$$
\Delta i(k) = \frac{\left[i_{\text{sd}}(k) - i_{\text{sd}}^*(k)\right]^2 + \left[i_{\text{sq}}(k) - i_{\text{sq}}^*(k)\right]^2}{\left[i_{\text{sd}}^*(k)\right]^2 + \left[i_{\text{sq}}^*(k)\right]^2}.
$$

(22)

$\sigma_{\text{max}}$ and $\sigma_{\text{min}}$ represent the upper limit and lower limit of the weighting factor $\sigma$, respectively.

In the steady state, the restriction of output spectrum characteristics of PWM is designed as the main function of the controller because of less current deviation. In the dynamic state, the current deviation increases, the controller mainly realizes the current tracking.

After the entire possible voltage level is evaluated, the optimal converter switching function $S_{ab}$ given by the proposed MPC-SHE can be carried out by minimizing the cost function.

### 4. Simulation and verification test

#### 4.1. Dynamic performance test

A simulation model of dual four-quadrant pulse EMU rectifiers is built in Matlab/Simulink, the parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary side voltage of the transformer</td>
<td>$U_s$</td>
<td>27.5</td>
<td>kV</td>
</tr>
<tr>
<td>Equivalent resistance of the rectifier</td>
<td>$R_n$</td>
<td>0.145</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Equivalent inductance of the rectifier</td>
<td>$L_n$</td>
<td>5.4</td>
<td>mH</td>
</tr>
<tr>
<td>DC-link voltage</td>
<td>$u_{dc}$</td>
<td>3600</td>
<td>V</td>
</tr>
<tr>
<td>DC-side equivalent resistance capacitance</td>
<td>$C_d$</td>
<td>10</td>
<td>mF</td>
</tr>
<tr>
<td>DC-side equivalent resistance</td>
<td>$R_d$</td>
<td>9</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Sampling frequency of the simulation</td>
<td>$f_s$</td>
<td>10</td>
<td>kHZ</td>
</tr>
<tr>
<td>Weighting factor</td>
<td>$\sigma_{\text{min}}$</td>
<td>0.01</td>
<td>/</td>
</tr>
<tr>
<td>Weighting factor</td>
<td>$\sigma_{\text{max}}$</td>
<td>8.5</td>
<td>/</td>
</tr>
</tbody>
</table>

The SHEPWM pattern prevails in MPC-SHEPWM control, when a step-change in the current is applied at 1.0 s, as shown in Fig. 9(a), which produces an increment in the current relative deviation $\Delta i(k)$, the MPC pattern prevails, currents $i_{\text{sd}}$ and $i_{\text{sq}}$ quickly reach a new stable working state (about 0.018 s), and there is no overshoot in the adjusting process. In order to make a comparison, the same step-change is also used in PI-SHEPWM. There is no longer adjustment
time of the rectifier with PI-SHEPWM as shown in Fig. 9(b), which shows that MPC-SHEPWM performed a better dynamic performance.

![Graph showing dynamic performance comparison between PI-SHEPWM and MPC-SHEPWM](image)

**Fig. 9.** Dynamic simulation results based on two control methods

### 4.2. Resonance suppressing test

The simulation model of a CRH5 EMUs-TPSS is built in MATLAB/Simulink, as shown in Fig. 10, the encapsulation module represents 15 km traction networks, one with subscript “1” is the upgoing line, and “2” is the downgoing line; subscripts “in” and “out” represent the input and output, respectively; T is the contact line, R is the rail, G is the connecting wire, F is the feeder line, and P represents the protection line.

![Simulation model of CRH5 EMUs-TPSS](image)

**Fig. 10.** Simulation model of CRH5 EMUs-TPSS

The impedance measurement under different frequencies is made for the traction power network model shown in Fig. 11, which shows that the impedance value is highest at a frequency of 1550 Hz nearby, so when a vehicle’s current of about 1550 Hz enters the traction power network, it causes the resonance phenomenon of the traction power system.
It can be seen from Fig. 12(c) that the harmonics of the pantograph current of CRH5 EMUs injected into the traction power network are concentrated around 31-th, which coincides with the resonant frequency stimulating the EMUs-TPSS resonance, the harmonics around 1550 Hz in the
voltage of the traction power network becomes rich, as shown in Fig. 12(b), and the the voltage of the traction network is amplified to 40 kV, which is shown in Fig. 12(a).

The resonance is suppressed by adopting the proposed MPC-SHEPWM, as shown in Fig. 13, the voltage in Fig. 13(a) is distorted less significantly than that of the traditional controller in Fig. 12(a), the harmonics from 1500 to 2000 are nearly eliminated. The THD of the pantograph current of CRH5 EMU voltage decreases to only 0.86%, and the voltage of the traction network decreases to only 0.62% as shown in Figs. 13(b) and (c).

5. Conclusion

In this paper, aiming at the phenomenon of high-frequency resonance, the resonance mechanism is analyzed first. Based on the characteristic of a single-phase rectifier, the MPC-SHEPWM method is applied in the multiple 4QPR of CRH5 EMUs in China. Some conclusions can be drawn:

1) an MPC-SHEPWM controller has a better dynamic performance than a PI-SHEPWM controller,
2) the MPC-SHEPWM can suppress the characteristic harmonics of 3-th, 5-th, 7-th, 23-th, 27-th, 29-th, 31-th, 33-th,
3) the MPC-SHEPWM control can reduce the characteristic harmonics and THD of pantograph current and successfully suppress the high frequency resonance of the traction network voltage.

References


