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Reliability aspect of incomplete trigonometric networks

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Abstract: The study presents results of the internal reliability analysis of structural modules used for the determination of horizontal displacement in incomplete trigonometric network. The influence of such elements as: number of control points, sight line length and arrangement of control points around the instrument station on reliability was analysed. Furthermore the analysis of the influence of diversification of reliability indices calculated for individual observations on the detection efficiency of non-dislocated control points was performed. The presented numerical example illustrates the possibility of incorrect valuation of control point stability because of a large diversification of reliability indices. The summary contains recommendations from the point of view of internal reliability for optimal designing of structural modules in incomplete trigonometric networks.

Keywords: Displacement measurements, incomplete trigonometric network, network reliability, identification of reference base

1. Introduction

In studies on displacement and deformation in engineering objects, different geodetic and non-geodetic measurement methods are being used. The geodetic methods are often used because of their advantages (Lazzarini, 1977a). Among those methods there is also the determination of horizontal displacements in analysed objects by the use of the so called incomplete trigonometric network. This method known already for many years is eagerly used under difficult field conditions because of its elasticity thanks to a specific structure of control networks. It is worth to notice that in case of short sights, the angle measurements have an absolute advantage over distance measurements.

Despite a long functioning of this method in practice of displacement measurements, it is difficult to find publications within surveying literature that would treat the aspect of reliability of this specific network. This elaboration presents results of studies related to internal reliability of the network structural module constituted by the instrument station and related to it control points. In such module the displacement of the instrument station is obtained by solving multiple angular resection. Shown will be also the influence of internal reliability indices calculated for individual observations on identification efficiency of non-dislocated control points. The summary contains recommendations concerning the design of optimal configuration of structural modules in incomplete trigonometric networks.

2. Reliability analysis for a single module of incomplete trigonometric network

The incomplete trigonometric network is a commonly known measurement construction. We will lead the reliability analysis for its structural module (distinguished by bold line in Figure 1) with exclusion of sight lines to object points. The observation to the control points (in the form of direction differences) will be treated as equally precise, with the standard deviation estimated during the stage of station adjustment.

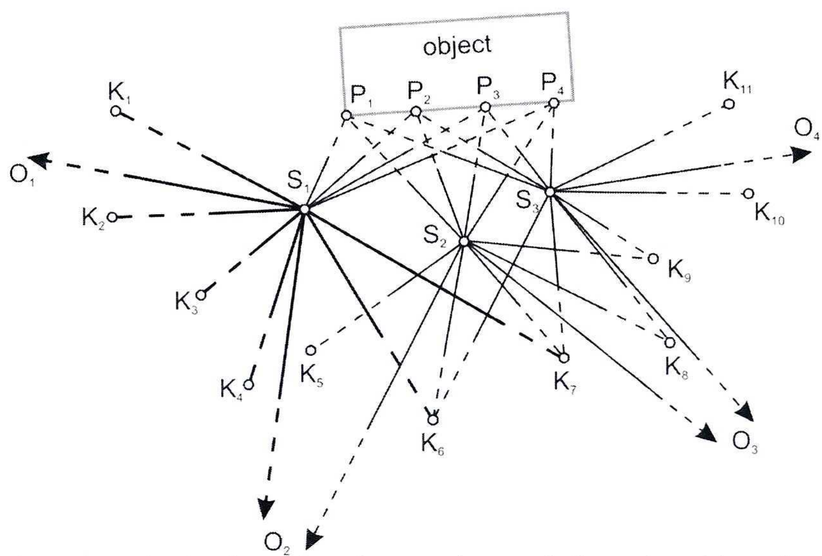

Fig. 1. Example of an incomplete trigonometric network: $P_1, \ldots, P_4$ – object points; S_1, S_2, S_3 – instrument stations; $K_1, \ldots, K_{11}$ – control points; $O_1, \ldots, O_4$ – orientation points

Formulae known from literature (Caspary, 1988; Prószyński and Kwaśniak, 2002) for the calculation of the global index $\bar{f}$ of internal reliability and the index f_i calculated for the specific observation in a network are being recalled

$$\bar{f} = \frac{\sum f_i}{n} = \frac{n-u}{n} \tag{1}$$

$$f_i = \{\mathbf{R}\}_{ii} \quad \text{whereby} \quad \mathbf{R} = \mathbf{I} - \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}} \tag{2}$$

where n – number of observations in the network;

u – number of unknowns within the observation system;

$\mathbf{A}(n \times u)$ – matrix of coefficients in the standardized system of observation equations (rank $\mathbf{A} = u$).

In practice also an equivalent form of the index f_i is being used as

$$\sigma_{V_i} = \sqrt{\{\mathbf{R}\}_{ii}} \tag{3}$$

that is the standard deviation estimator of correction to the i-th observation within the standardized system (Prószyński, 1994).

A reliability level of the structural module will be analysed taking into account the following factors:

- number of control points;
- sight line length;
- sight line arrangement around the instrument station.

2.1. Influence of the number of control points on internal reliability of structural module

In a single module we shall determine three unknowns: Δx_S, Δy_S being components of the station displacement and the orientation constant z_S. Thus, with n measured directions, basing on (1) the global index of internal reliability amounts to

$$\bar{f} = \frac{n-3}{n} \tag{4}$$

whereby, according to (1), it depends only on the number of those directions. Table 1 shows variability of the index $\bar{f}$ according to the number of measured directions.

Table 1. Global index of internal reliability for different number of lines to points

n	3	4	5	6	7	8	9	10
$\bar{f}$	0.00	0.25	0.40	0.50	0.57	0.63	0.67	0.70

To have the global index of internal reliability fulfil the reliability criterion, as given in (Prószyński, 1994), the reliability index has to be

$$(\sigma_V)_{av} = \sqrt{\bar{f}} \geq 0.71 \quad \text{and hence} \quad \bar{f} \geq 0.50 \tag{5}$$

Thus, in conformity with Table 1, the structural module shall contain at least 6 lines to points.

It's worth to give attention to the following fact. If out of n equally precise direction differences we pass over to horizontal angle differences, that results in reducing the orientation constant of the station z_S, we will receive $n-1$ independent angle differences constituting also the system with equally precise observations. The determination of the station displacement requires now the evaluation of two unknowns Δx_S, Δy_S. In this situation the value of the global index of internal reliability $\bar{f}_\beta$ amounts to

$$\bar{f}_\beta = \frac{(n-1)-2}{n-1} = \frac{n-3}{n-1}$$

Comparing the value $\bar{f}_\beta$ with the value of the index $\bar{f}$ we see that

$$\bar{f}_\beta - \bar{f} = \frac{n-3}{n-1} - \frac{n-3}{n} = \frac{n-3}{n(n-1)} \tag{6}$$

Table 2 shows values of the difference $\bar{f}_\beta - \bar{f}$ according to the number of direction differences determining the station displacement.

Table 2. Difference $\bar{f}_\beta - \bar{f}$ for different number of directions

n	3	4	5	6	7	8	9	10
$\bar{f}_\beta - \bar{f}$	0.00	0.08	0.10	0.10	0.10	0.09	0.08	0.08

It results from Eq. (6) and Table 2 that from the point of view of reliability theory the observation system composed of directions is weaker in terms of reliability by about 0.1 than the system composed of angles calculated basing on those directions.

2.2. Influence of sight lengths on internal reliability of the structural module

Assuming stability of control point, the observation equation for direction difference (Janusz, 1964; Lazzarini, 1977b) is explicitly expressed in terms of the sight line lengths

$$-z_S + \frac{1}{d_i}(\Delta x_S \cos\varphi_i - \Delta y_S \sin\varphi_i) = l_i + v_i \tag{7}$$

with z_S – orientation constant of the station;
Δx_S, Δy_S – station displacement components;
$l_i = k_i' - k_i$ – difference of observed directions;
k_i, k_i' – observed horizontal direction up to the i-th control point accordingly in the initial and the actual measurement;
φ_i – sight line azimuth up to the i-th control point.

Assuming the following matrix notations:

$$\mathbf{A} = \begin{bmatrix} -1 \\ -1 \\ \cdots \\ -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \cos\varphi_2 & -\sin\varphi_2 \\ \cdots & \cdots \\ \cos\varphi_n & -\sin\varphi_n \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} d_1^{-1} & 0 & \cdots & 0 \\ 0 & d_2^{-1} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & d_n^{-1} \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \Delta x_s \\ \Delta y_s \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} l_1 \\ l_2 \\ \cdots \\ l_n \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix}$$

the equation (7) can be written in the form

$$\begin{bmatrix} \mathbf{A} & \mathbf{DB} \end{bmatrix} \begin{bmatrix} z_S \\ \mathbf{x} \end{bmatrix} = \mathbf{w} + \mathbf{v} \tag{8}$$

and the system of normal equations has the following form

$$\begin{bmatrix} \mathbf{A}^T\mathbf{A} & \mathbf{A}^T\mathbf{DB} \\ \mathbf{B}^T\mathbf{DA} & \mathbf{B}^T\mathbf{D}^2\mathbf{B} \end{bmatrix} \begin{bmatrix} z_S \\ \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^T\mathbf{w} \\ \mathbf{B}^T\mathbf{Dw} \end{bmatrix} \tag{9}$$

Basing on (9) one can state that the sight line lengths have here an analogous function as observation weights. Thus, their variability implies the change of internal reliability indices calculated for individual observations. This conclusion is confirmed by numerical studies.

For a structural module with 6 control points, distributed uniformly in the horizon (Fig. 2), a study of the influence of sight line lengths on the value of internal reliability indices (in form of σ_V) has been carried out.

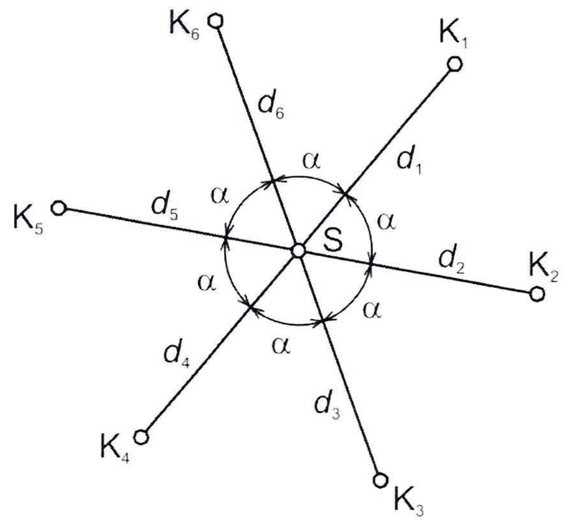

Fig. 2. Structural module with 6 control points

Figure 3a shows variability diagrams of reliability indices σ_V when changing the sight line length to point K_3 (range: 50÷250 m) and keeping constant the remaining lengths of 150 m each, while Figure 3b shows the variability of σ_V when changing the sight line length to point K_6 (range: 50÷250 m) and keeping constant the remaining lengths of 150 m each.

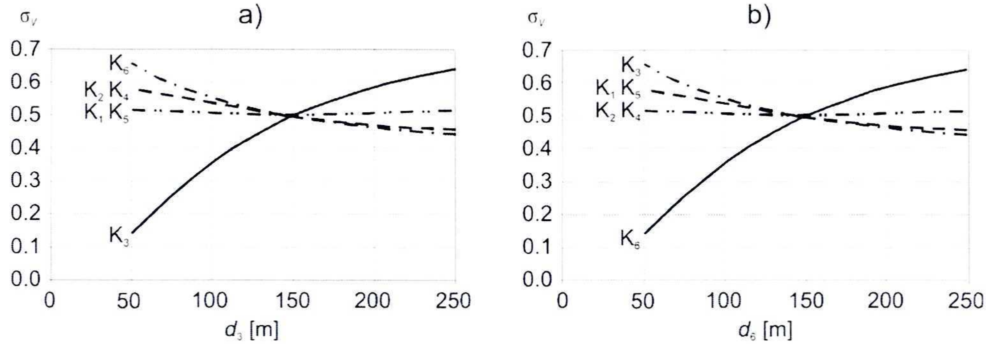

Fig. 3. Variability graph of reliability indices σ_{V_i} according to the sight line length: a) to point K_3, b) to point K_6

From Figure 3 it follows that the change in length of one of the sight lines causes the changes of values of reliability indices for all directions. The reliability for this direction which sight line length changes, varies the most. With the increase of sight line length, its reliability index σ_V also increases. Simultaneously for other directions, the reliability index values diminish. The magnitude of those changes depends on the location of given direction against that direction, which sight line length changes.

If though the lengths of all sight lines are transformed in the following way:

$$\frac{d_1'}{d_1} = \frac{d_2'}{d_2} = \cdots = \frac{d_n'}{d_n} = r \tag{10}$$

the system of equations (9) will change into

$$\begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{A} & r\mathbf{A}^{\mathrm{T}}\mathbf{B} \\ r\mathbf{B}^{\mathrm{T}}\mathbf{A} & r^2\mathbf{B}^{\mathrm{T}}\mathbf{B} \end{bmatrix} \begin{bmatrix} z_s \\ \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{w} \\ r\mathbf{B}^{\mathrm{T}}\mathbf{w} \end{bmatrix} \tag{11}$$

Using the formula for the inverse of block matrix it can be proven that

$$\text{if } \begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{A} & \mathbf{A}^{\mathrm{T}}\mathbf{B} \\ \mathbf{B}^{\mathrm{T}}\mathbf{A} & \mathbf{B}^{\mathrm{T}}\mathbf{B} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{Q} & \mathbf{U} \\ \mathbf{U}^{\mathrm{T}} & \mathbf{W} \end{bmatrix} \text{ then } \begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{A} & r\mathbf{A}^{\mathrm{T}}\mathbf{B} \\ r\mathbf{B}^{\mathrm{T}}\mathbf{A} & r^2\mathbf{B}^{\mathrm{T}}\mathbf{B} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{Q} & r^{-1}\mathbf{U} \\ r^{-1}\mathbf{U}^{\mathrm{T}} & r^{-2}\mathbf{W} \end{bmatrix}$$

Thus, if

$$\begin{aligned} \mathbf{R} = \mathbf{I} - \begin{bmatrix} \mathbf{A} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{Q} & \mathbf{U} \\ \mathbf{U}^{\mathrm{T}} & \mathbf{W} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{\mathrm{T}} \\ \mathbf{B}^{\mathrm{T}} \end{bmatrix} &= \mathbf{I} - \begin{bmatrix} \mathbf{AQ} + \mathbf{BU}^{\mathrm{T}} & \mathbf{AU} + \mathbf{BW} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{\mathrm{T}} \\ \mathbf{B}^{\mathrm{T}} \end{bmatrix} = \\ &= \mathbf{I} - \left(\mathbf{AQA}^{\mathrm{T}} + \mathbf{BU}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} + \mathbf{AUA}^{\mathrm{T}} + \mathbf{BWB}^{\mathrm{T}}\right) \end{aligned} \tag{12}$$

than after change of the sight line lengths

$$\mathbf{R}' = \mathbf{I} - \begin{bmatrix} \mathbf{A} & r\mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{Q} & r^{-1}\mathbf{U} \\ r^{-1}\mathbf{U}^{\mathrm{T}} & r^{-2}\mathbf{W} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{\mathrm{T}} \\ r\mathbf{B}^{\mathrm{T}} \end{bmatrix} =$$
$$= \mathbf{I} - \begin{bmatrix} \mathbf{AQ} + \mathbf{BU}^{\mathrm{T}} & r^{-1}(\mathbf{AU} + \mathbf{BW}) \end{bmatrix} \begin{bmatrix} \mathbf{A}^{\mathrm{T}} \\ r\mathbf{B}^{\mathrm{T}} \end{bmatrix} =$$
$$= \mathbf{I} - \left(\mathbf{AQA}^{\mathrm{T}} + \mathbf{BU}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} + \mathbf{AUA}^{\mathrm{T}} + \mathbf{BWB}^{\mathrm{T}}\right) = \mathbf{R}$$

It results from here that r time rescaling of all sight line lengths does not influence reliability indices of this construction calculated for the individual observations.

2.3. Influence of sight line arrangement around the instrument station

In the functional model (8) the arrangement of the lines to control points in the horizon of instrument station is represented by the matrix **B**. From the analysis of the influence of the matrix **B** on the values of internal reliability indices (Eq. (12)) it follows that this dependence is very complex and the change of element values even in one line of the matrix **B** (change in azimuth of one of the sight lines) causes the change of values of internal reliability indices for all observations. In this situation, to capture specific reliability characteristics of the structural module, numerical studies have been carried out.

Figure 4 shows two variants of a module with 6 control points and equal sight line lengths. In variant 1 (Fig. 4a), the angles formed by pairs of neighbouring lines (with exclusion of a pair K_1, K_6) are equal, while in variant 2 (Fig. 4b) those angles are different.

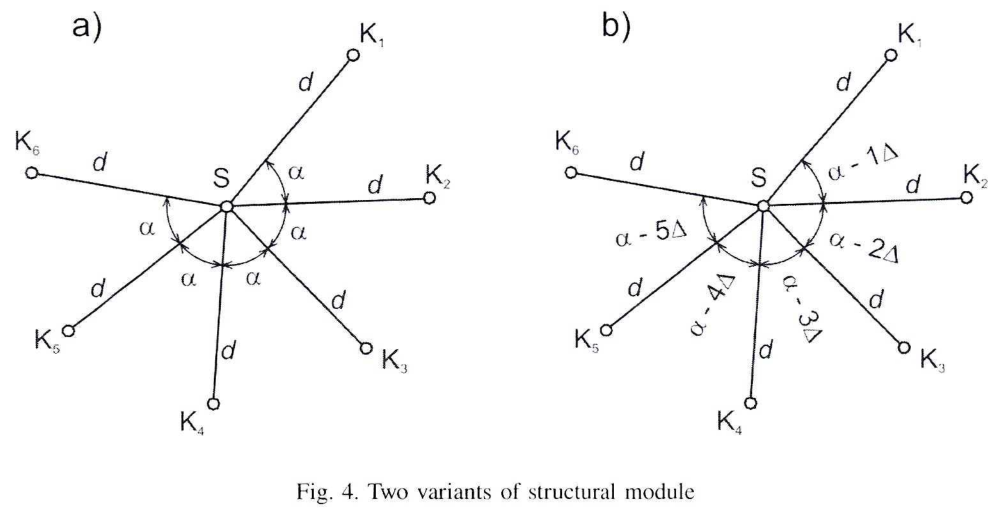

Fig. 4. Two variants of structural module

When introducing variability in variant 1 for the value of angle α, and in variant 2 for the value of Δ, one calculates the internal reliability indices σ_V for individual directions. Those index variations are shown in the form of graphs in Figure 5a (for variant 1) and Figure 5b (for variant 2).

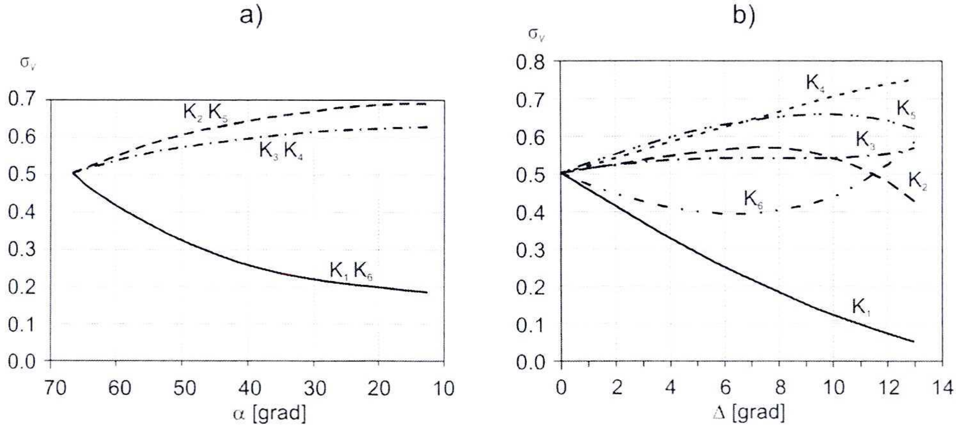

Fig. 5. Graphic illustration of the variability of internal reliability indices σ_V depending on: a) value of angle α, b) value of Δ

Analysing Figure 5a one can see that the size of angle α has an essential influence on the values of internal reliability σ_V for those directions. As the angle α gets smaller, the values of the indices σ_V for boundary directions (here: K_1 and K_6) significantly decrease, whereas for remaining directions increase. This conforms to (4), because the average (global) internal reliability index is constant for the given construction and depends only on the number of observed directions. The symmetry of indices σ_V, i.e. $\sigma_{V_1} = \sigma_{V_6}$, $\sigma_{V_2} = \sigma_{V_5}$, $\sigma_{V_3} = \sigma_{V_4}$ can also be notified.

In case of non-identical values of angles between neighbouring lines (Fig. 4b), there is a lack of symmetry of the σ_V – index values (Fig. 5b). As the angles get smaller (that is when increasing the value Δ) the value of the index σ_{V_1} decreases the most, while σ_{V_4} and σ_{V_5} increase. For some directions the indices σ_V change in a non-monotonous way.

The conclusions resulting from this chapter shall be considered when designing the shape of the structural module. To obtain a shape being optimal from the point of view of reliability theory, limitations resulting from field conditions shall also be surmounted. For that reason the following chapter presents the analysis of the influence of an essential diversification of reliability indices on the results of reference base identification for displacements being determined.

3. Detectability of dislocated control points versus internal reliability indices of structural module

In geodetic practice, an essential diversification of the values of σ_V indices for observations in a module takes place. Such situation does not remain without influence on the efficiency of the determination of the non-dislocated control points, forming the reference base for the station. Already at the first step of reference base identification (testing of zero hypothesis, assuming the stability of all control points) the diversification of reliability indices for individual directions may cause that a control point being actually dislocated is considered as constant. This results from the fact that during the global test if the assumed zero hypothesis is true, the σ_0' is being tested, which is calculated on the basis of adjustment corrections v for direction differences

$$\sigma_l' \leq \sigma_{l,crit}' \quad \text{whereby}: \quad \sigma_l' = \frac{\hat{\sigma}_l}{\sigma_l} = \frac{1}{\sigma_l}\sqrt{\frac{[vv]}{n-3}} \quad \text{and;} \quad \sigma_{l,crit}' = \sqrt{\frac{\chi^2_{n-3,\alpha}}{n-3}} \tag{13}$$

where σ_l – standard deviation of single direction difference determined basing on station adjustments of the initial and actual measurement;

$\hat{\sigma}_l$ – standard deviation estimator of a single direction difference determined basing on adjustment corrections v for a module;

$\sigma_{l,crit}'$ – the critical value for $\hat{\sigma}_l$;

$[vv]$ – sum of squares of adjustment corrections;

$\chi^2_{n-3,\alpha}$ – quantile of χ^2 – distribution for n – 3 degrees of freedom and significance level α;

n – number of control points.

For the global test to be fulfilled, the following condition should be met

$$\frac{[vv]}{\sigma_l^2} \leq \chi^2_{n-3,\alpha} \tag{14}$$

And since $\mathbf{v} = -\mathbf{Rw}$ then $\mathbf{v}^T\mathbf{v} = [vv] = \mathbf{w}^T\mathbf{RRw} = \mathbf{w}^T\mathbf{Rw}$ and hence $\mathbf{w}^T\mathbf{Rw} \leq \sigma_l^2 \cdot \chi^2_{n-3,\alpha}$ where $\mathbf{R}$ is the reliability matrix and $\mathbf{w}$ is the vector of direction differences.

The contribution of the direction difference l_i to the value of $[vv]$ amounts to $[vv]_i = l_i^2\{\mathbf{R}\}_{ii} = l_i^2\sigma_{V_i}^2$. Assuming that $[vv]_{\max} = \sigma_l^2\chi^2_{n-3,\alpha}$ was caused by a single direction difference, one obtains

$$l_i^2\sigma_{V_i}^2 = \sigma_l^2\chi_{n-3,\alpha}$$

and on this basis

$$l_i^{\max} = \frac{\sigma_l}{\sigma_{V_i}}\sqrt{\chi^2_{n-3,\alpha}} \tag{15}$$

Analysing the above dependence it can be seen that the lower internal reliability of a given direction difference, the higher value l_i is tolerated for test (13) fulfilment. Thus, if the direction difference for the line to dislocated control point has a low reliability index, it is possible to include this point into the reference base of the station. Figure 6 illustrates the dependence of the value $l_i^{\max}$, expressed in multiplicity of σ_l, on the values of internal reliability σ_{V_i} for structural modules with 4 and 8 control points.

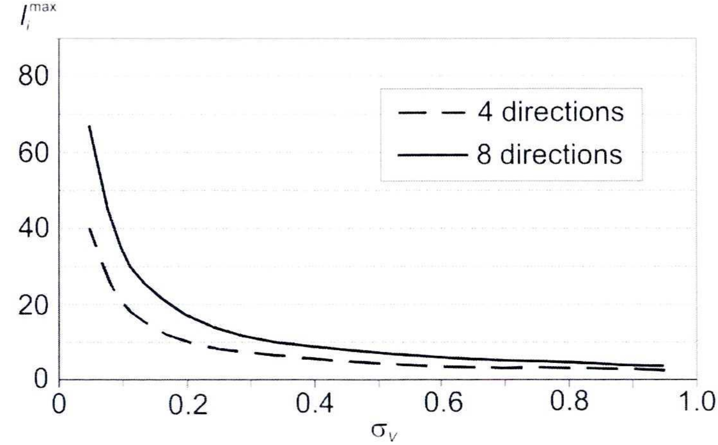

Fig. 6. Dependence of value l_i^{max} on the value of internal reliability index σ_{V_i}

In the analysed method the displacement of the i-th control point influences the observed direction difference l_i in the same way as the gross error occurring in the direction measurement (initial k_i or actual k_i'). Thus, to detect displaced control points, various gross error detection methods developed in geodesy can also be used, i.e. the method of unified correction (Prószyński and Kwaśniak, 2006).

Let's analyse the effect of the reliability of a structural module on the efficiency of the detection of dislocated control points using the unified correction method.

Because of $\mathbf{v} = -\mathbf{Rw}$, in case of disturbance g_i in the observation for the i-th direction (caused by the control point displacement) one gets

$$\Delta\mathbf{v} = -\mathbf{R}_{\bullet i} g_i \quad \text{hence} \quad \Delta v_i = -\{\mathbf{R}\}_{ii}\, g_i \quad \textit{and} \quad \Delta v_j = -\{\mathbf{R}\}_{ij}\, g_i \tag{16}$$

If the correlation of adjustment corrections v_i, v_j is k_{ij} ($0 < |k_{ij}| < 1$) then the value of the element $\{\mathbf{R}\}_{ij}$ is $\{\mathbf{R}\}_{ij} = k_{ij}\sqrt{\{\mathbf{R}\}_{ii}\{\mathbf{R}\}_{jj}}$ and hence $\Delta v_j = -k_{ij}\sqrt{\{\mathbf{R}\}_{ii}\{\mathbf{R}\}_{jj}}g_i$. Let's assume that the unified corrections for observation i and j within the non-disturbed observation system are correspondingly u_i^o and u_i^o. Than after occurrence of disturbance g_i

$$\begin{aligned} u_i &= u_i^o - \{\mathbf{R}\}_{ii}\, g_i \Big/ \sqrt{\{\mathbf{R}\}_{ii}} = u_i^o - \sqrt{\{\mathbf{R}\}_{ii}}g_i = u_i^o - \sigma_{v_i} g_i \\ u_j &= u_j^o - k_{ij}\sqrt{\{\mathbf{R}\}_{ii}\{\mathbf{R}\}_{jj}}g_i \Big/ \sqrt{\{\mathbf{R}\}_{jj}} = u_j^o - k_{ij}\sqrt{\{\mathbf{R}\}_{ii}}g_i = u_j^o - k_{ij}\sigma_{v_i} g_i \end{aligned} \tag{17}$$

The incorrect indication of disturbed observation will occur if $|u_j| > |u_i|$. Solving this inequality and taking under consideration (17), one obtains

$$\frac{u_i^o + u_j^o}{\left(1 + k_{ij}\right)\sigma_{V_i}} < g_i < \frac{u_i^o - u_j^o}{\left(1 - k_{ij}\right)\sigma_{V_i}} \tag{18}$$

From Eq. (18) it follows that an incorrect indication of disturbed observation is possible and the occurrence of such situation depends on the correlation level k_{ij} of adjustment corrections and on the values and signs of u_i^o, u_i^o and g_i (see numerical example).

The problem of non-detection of the displaced point (or detection as displaced of an actually non-displaced point) could also occur in the phase of reference base identification of the given station by the use of the method $\delta\varepsilon = \Delta l$ being very popular in surveying practice. In this method, the affiliation criterion of the considered control point to a reference base constitutes an inequality (Prószyński and Kwaśniak, 2006)

$$q_{be} \leq k_\alpha \sigma_{q_{be}} \quad \text{or} \quad \frac{q_{be}}{\sigma_{q_{be}}} \leq k_\alpha \tag{19}$$

whereby

$$q_{be} = \Delta\varepsilon_{be} - \Delta l_{be} \tag{20}$$

where: b – point belonging to the working base;
e – point not belonging to the working base;
$\Delta l_{be} = l_e - l_b$ – difference of observed direction differences (between actual and initial measurements) corresponding to control points e and b;
$\Delta\varepsilon_{be} = \varepsilon_e - \varepsilon_b$ – difference of station displacement influence on direction differences corresponding to control points e and b;
$\sigma_{q_{be}}$ – standard deviation of q_{be};
k_α – critical value depending on applied confidence level α.

The quantity Δl_{be} is calculated basing on measured values and does not depend on the determination of accuracy of station displacement. However, value $\Delta\varepsilon_{be}$ is calculated basing on determined station displacement components and its accuracy; it is a function of geometry of the module. It will be shown that this method is in fact the method of unified correction, therefore it is also sensitive to high variability of internal reliability indices of the module.

The observation equation for the direction difference (assuming the stability of control point) can be presented in terms of angular quantities

$$-z_S + \varepsilon_i = l_i + v_i \tag{21}$$

whereby $\varepsilon_i = B_i \Delta x_s - A_i \Delta y_s$; $A_i = \dfrac{\cos\varphi_i}{d_i}$; $B_i = \dfrac{\sin\varphi_i}{d_i}$.

For point e not belonging to the working base and point b of the working base, Eq. (21) has a corresponding form

$$-z_S + \varepsilon_e = l_e + v_e$$

$$-z_S + \varepsilon_b = l_b + v_b$$

Subtracting the above equations by sides one obtains

$$\varepsilon_e - \varepsilon_b = l_e - l_b + v_e - v_b$$

what can be written as

$$\Delta\varepsilon_{be} = \Delta l_{be} + \Delta v_{be} \tag{22}$$

The obtained observation equation concerns the change of angle, with its vertex being the instrument station and the arms being the control points b and e.

From (20) and (22) one finally gets

$$q_{be} = v_{qbe} \tag{23}$$

Thus, q_{be} is an observation correction of the angle change and $q_{be}/\sigma_{q_{be}}$ is its unified correction.

As presented in Section 2.1, the global reliability index for structural module, as shown in Fig. 2, where horizontal directions are observations, is inferior by about 0.1 than in case where the observations are horizontal angles calculated on the basis of measured directions. For that reason one can expect that the identification of the station reference base using the $\Delta\varepsilon = \Delta l$ method will be more efficient than the method of unified correction applied for directions as observations.

4. Numerical example

On station S two measurements of horizontal directions to 6 control points (Fig. 7) have been carried out in a certain time interval. Table 3 contains the differences of measured directions and approximate coordinates of the control points and the station. The standard deviation of single direction difference is $\sigma_l = 6.5^{cc}$. The task is to perform the reference base identification for the station by the use of the unified correction method and the $\Delta\varepsilon = \Delta l$ method.

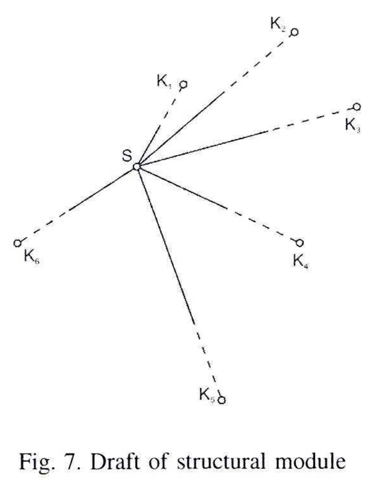

Fig. 7. Draft of structural module

Table 3. Observed direction differences and approximate coordinates of points

Point	$l = k' - k$ [cc]	x_0 [m]	y_0 [m]
K_1	–16.3	1048.289	1026.866
K_2	27.9	1080.032	1092.927
K_3	62.5	1036.242	1133.902
K_4	108.4	954.376	1094.525
K_5	58.8	855.441	1051.266
K_6	–32.7	953.072	927.444
S		1000.000	1000.000

Compiling the system of observation equations for direction differences and solving it using the least squares method one obtains

$$[z_S \;\; \Delta x_S \;\; \Delta y_S]^{\mathrm{T}} = [-8.974^{cc} \quad -13.302mm \quad -9.703mm]^{\mathrm{T}}$$

Table 4. Observation corrections v, internal reliability indices σ_V and unified corrections u for direction differences

Point	v [cc]	σ_V	u
K_1	2.10	0.47	0.68
K_2	0.53	0.88	0.09
K_3	–6.23	0.89	–1.08
K_4	–1.18	0.67	–0.27
K_5	6.58	0.82	1.23
K_6	–1.79	0.28	–0.98
$\sigma_0 = 5.52^{cc}$; $\sigma_0' = 0.85$; $\sigma_{0,crit} = 1.61$			

From Table 4 it follows that the global test $\sigma_0' \leq \sigma_{0,crit}$ as well as the local tests $|u| \leq k_\alpha$ for all observation differences are fulfilled at $\alpha = 0.05$ ($k_\alpha = 2.0$). This means that all the control points can constitute the reference base for the station.

Let's assume now that the control point K_4 got displaced and the value of direction difference up to this point changed as follows

$$l_4^* = l_4 + 6\sigma_l = 108.4^{cc} + 6 \cdot 6.5^{cc} = 147.4^{cc}$$

Solving the system of observation equations one obtains again

$$\left[\begin{array}{ccc} z_s & \Delta x_s & \Delta y_s \end{array}\right]^{\mathrm{T}} = \left[\begin{array}{ccc} -10.520^{cc} & -15.920\mathrm{mm} & -11.791\mathrm{mm} \end{array}\right]^{\mathrm{T}}$$

Table 5. Observation corrections v, internal reliability indices σ_V and unified corrections u for direction differences after introduction of disturbance in l_4

Point	v [cc]	σ_V	u
K_1	–2.71	0.47	–0.88
K_2	5.30	0.89	0.92
K_3	4.41	0.89	0.76
K_4	–18.83	0.67	–4.31
K_5	19.93	0.82	3.73
K_6	–8.09	0.28	–4.42
$\sigma_0 = 17.05^{cc}$; $\sigma_0' = 2.62$; $\sigma_{0,crit} = 1.61$			

As Table 5 shows, this time both the global test and the local tests for observation differences for points K_4, K_5 and K_6 are not fulfilled. According to the method of unified correction, the control point K_6 (having the highest absolute value of unified correction) shall be recognized as displaced which is not compatible with the assumption made. The reason for this divergence in stability valuation of control points is the high variability of reliability indices σ_V for individual direction differences, as well as the correlation level of adjustment corrections of those directions.

Let's take a closer look at the dependencies between the adjustment corrections. The matrix $\mathbf{K}$ of their correlations has the following form

$$\mathbf{K} = \begin{bmatrix} 1.000 & -0.793 & -0.400 & 0.388 & 0.363 & 0.088 \\ -0.793 & 1.000 & -0.232 & -0.205 & -0.120 & -0.241 \\ -0.400 & -0.232 & 1.000 & -0.457 & -0.246 & 0.059 \\ 0.388 & -0.205 & -0.457 & 1.000 & -0.619 & 0.853 \\ 0.363 & -0.120 & -0.246 & -0.619 & 1.000 & -0.896 \\ 0.088 & -0.241 & 0.059 & 0.853 & -0.896 & 1.000 \end{bmatrix}$$

It is obvious that the correction v_4 ($\sigma_V = 0.67$) is correlated the most with the correction v_6 ($k_{46} = 0.853$), of which the reliability index is the lowest one ($\sigma_V = 0.28$). Substituting appropriate data ($u_i^o = -0.27$, $u_{ji}^o = -0.98$, $\sigma_{V_4} = 0.67$, $k_{46} = 0.853$, $z_4 = 6$) in Eq. (17) one obtains the confirmation of the identification result

$$u_4 = u_4^o - \sigma_{V_4} g_4 = -4.31$$

$$u_6 = u_6^o - k_{46}\sigma_{V_4} g_4 = -4.42$$

whereas from the substitution of the data above to (18) it results that $|u_6| > |u_4|$ for $1.0\ \sigma_l < g_4 < 7.2\sigma_l$.

The identification of reference base using the method $\Delta\varepsilon = \Delta l$ will be carried out by assuming control points K_1, K_5 and K_6 as a working base. The calculation results for data without disturbance (Table 2) are presented in Table 6.

Table 6. Results of the reference base identification by use of the method $\Delta\varepsilon = \Delta l$ and working base K_1, K_5, K_6

Point	l_e [cc]	ε_e [cc]	Δl_{1e} [cc]	$\Delta\varepsilon_{1e}$ [cc]	q_{1e} [cc]	σ_q [cc]	$\lvert q_{1e}\rvert/\sigma_q$
K_2	27.9	17.30	44.20	40.10	–4.10	8.09	0.51
K_3	62.5	43.15	78.80	65.95	–12.85	8.89	1.45
K_4	108.4	90.43	124.70	113.23	–11.47	11.64	0.98

Those results confirm the stability of all control points at $k_\alpha = 2$.

Considering the disturbance at l_4 introduced above and calculating once again at the same working base leads to the results presented in Table 7.

Table 7. Results of identification of the reference base by use of the method $\Delta\varepsilon = \Delta l$ and working base K_1, K_5, K_6 after introducing of disturbance at l_4

Point	l_e [cc]	ε_e [cc]	Δl_{1e} [cc]	$\Delta\varepsilon_{1e}$ [cc]	q_{1e} [cc]	σ_q [cc]	$\lvert q_{1e}\rvert/\sigma_q$
K_2	27.9	17.30	44.20	40.10	–4.10	8.09	0.51
K_3	62.5	43.15	78.80	65.95	–12.85	8.89	1.45
K_4	147.4	90.43	163.70	113.23	–50.47	11.64	4.33

From Table 7 it results that the point K_4 shall be recognized as displaced which conforms to the assumption made, because the direction difference for just this point has been disturbed. If though the point K_4 will get included into the working base than the identification results for the reference base (Table 8) show only the point K_6 as displaced, not as one expected for all points not belonging to the working base.

Table 8. Identification results of the reference base by use of the method $\Delta\varepsilon = \Delta l$ and working base K_1, K_4, K_5 after introducing of disturbance at l_4

Point	l_e [cc]	ε_e [cc]	Δl_{1e} [cc]	$\Delta\varepsilon_{1e}$ [cc]	q_{1e} [cc]	σ_q [cc]	$\lvert q_{1e}\rvert/\sigma_q$
K_2	27.9	57.12	44.2	44.83	0.63	7.76	0.08
K_3	62.5	98.78	78.8	86.49	7.69	7.53	1.02
K_6	–32.7	–107.31	–16.4	–119.60	–103.20	23.81	4.33

This is the effect of the high correlation of the adjustment correction v_4 and v_6, as well as a low internal reliability index for the direction difference l_6.

5. Summary and final conclusions

From the presented analyses it results that despite the simplicity of the construction determining the station within the incomplete trigonometric network, its internal reliability has a high influence on a proper identification of the reference base, and consequently on the correctness of determined dislocations.

The level and variability of reliability indices of the structural module is influenced by: the number of control points, sight line lengths to those points as well as angle size between neighbouring control points.

If the sight lines to the control points take only a certain horizon sector of the station, the boundary directions will have the lowest values of internal reliability indices. An analogue statement related to the approximation task can be found in (Nowak and Nowak, 2006).

At the stage of network design one should aim to decrease of variability of reliability indices through proper choice of: number of control points, sight line lengths to those points as well as their arrangement within the measurement station horizon.

In practice, restrictions regarding the location of control points and measurement stations resulting from field conditions do often cause that the reliability indices calculated for individual directions are of significantly varying magnitudes.

At a high variability of reliability indices one should be conscious that identification errors can occur and consist in:

- conceding the control point being in fact fixed as dislocated, if the adjustment correction to observation to this point is strongly correlated with the correction to observation to the point being in fact dislocated and if the reliability index of observation to that dislocated point is low;
- conceding the control point being in fact dislocated as fixed, if the reliability index of observation to concerning point is low.

For specified structural module, on the stage of reference base identification it is more favourable to operate in angles as direction differences than directions alone. The obtained results of reliability indices are by about 0.1 higher, which improves the efficiency of reference base identification.

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Aspekt niezawodnościowy w sieciach trygonometrycznych niepełnych

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Streszczenie

W pracy przedstawione zostały wyniki analizy niezawodności wewnętrznej modułu sieci trygonometrycznej niepełnej służącego do wyznaczenia składowych przemieszczenia stanowiska. Przeanalizowano wpływ na niezawodność takich czynników jak: liczba punktów kontrolnych, długość celowych oraz rozmieszczenie punktów kontrolnych w horyzoncie stanowiska. Dokonano analizy wpływu zróżnicowania wskaźników niezawodności liczonych dla poszczególnych obserwacji na skuteczność wykrywania nie przemieszczonych punktów kontrolnych. Zamieszczony przykład numeryczny ilustruje możliwość błędnej oceny stałości punktu kontrolnego na skutek dużego zróżnicowania wskaźników niezawodności. W podsumowaniu podano zalecenia do optymalnego, z punktu widzenia niezawodności, projektowania modułów pomiarowych stosowanych w sieciach trygonometrycznych niepełnych.