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Terrain-aliasing effects on gravimetric geoid determination

Sujan Bajracharya, Michael G. Sideris

Department of Geomatics Engineering, The University of Calgary,
2500 University Drive N.W., Calgary, Alberta, T2N 1N4, Canada
e-mail: bajrachs@ucalgary.ca

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Abstract: This paper investigates the terrain-aliasing effects on geoid determination using different gravimetric reduction schemes. The high resolution of digital terrain model (DTM), if available, should be used for every gravimetric reduction scheme since it can precisely map the details of the terrain. The reduction methods used in this study are the Rudzki inversion method, Helmert's second method of condensation, the residual terrain model (RTM) method, and the Pratt-Hayford (PH) topographic-isostatic reduction technique. The effect of using different DTM grid resolutions of 6", 15", 30", 45", 1' and 2' on gravity anomalies and absolute geoid undulations is studied for each of these reduction schemes. A rugged area in the Canadian Rockies bounded by latitude between 49°N and 54°N and longitude between 236°E and 246°E is selected to conduct numerical tests. Our results suggest that a DTM grid resolution of 6" or higher is required for precise geoid determination with an accuracy of a decimetre or higher for any gravimetric reduction method chosen to treat the topographical masses above the geoid in rugged areas. The most precise geoid models obtained in this test are the ones obtained using Rudzki, Helmert, and RTM methods with 6" DTM resolution.

Keywords: Aliasing, gravimetric reduction, digital terrain model, geoid

1. Introduction

The challenge for centimetre geoid determination these days is a main reason to study its various theoretical as well as practical aspects. The gravimetric terrain reduction is one of the main components of geoid computational process that plays a prominent role in precise geoid determination especially in mountainous areas. The high-resolution digital terrain models (DTM) are available these days and are being used in various engineering and scientific applications. There have been some recent studies on terrain-aliasing effects for Helmert's second method of condensation (Bajracharya et al., 2002; Featherstone and Kirby, 2000). The study on precise geoid determination in rugged areas requires the exploration of various gravimetric reduction methods, among other issues, to treat topographical masses in addition to most commonly used Helmert's second method of condensation and RTM method. A high resolution DTM should be incorporated in every gravimetric reduction method for precise geoid determination.

The purpose of this paper is to investigate the importance of using various DTM grid resolutions for different mass reduction schemes within the context of precise geoid determination. The terrain aliasing effects are studied for the RTM topographic reduction, PH topographic isostatic reduction, Helmert's second method of condensation, and the Rudzki inversion method (Rudzki, 1905). The results obtained from using densest DTM (6" resolution) in every reduction method are taken as the control values in this paper. The difference between control values and the results obtained using other coarser DTMs is regarded as the terrain-aliasing effects. The resolution of DTM affects both the computation of complete removal of topography (complete Bouguer reduction) and the compensated, condensed, or inverted masses depending on the gravimetric reduction scheme chosen.

2. Computational formulas

Gravimetric geoid solution is carried out using the remove-compute-restore technique (RCR) in this investigation for all gravity reduction methods. The reduced gravity anomalies in RCR procedure can be expressed as:

$$\Delta g = \Delta g_F - \Delta g_T - \Delta g_{GM} + \delta g \tag{1}$$

where Δg_F is the free-air (FA) anomaly, Δg_T is the direct topographical effect on gravity, and Δg_{GM} is the reference gravity anomaly. δg is the indirect effect on gravity, which reduces gravity anomaly from the co-geoid to the geoid and can be expressed using the simple free-air gradient (Heiskanen and Moritz, 1967):

$$\delta g = 0.3086\, N_{ind} \,\mathrm{mGal} \tag{2}$$

The direct topographical effect on gravity Δg_T in equation (1) for each mass reduction scheme can be expressed as:

$$\Delta g_T = A - A_{(Inv,\ Comp,\ Ref)} \tag{3}$$

where A is the attraction of all topographic masses above the geoid and $A_{(Inv,\ Comp,\ Ref)}$ represents the attraction of either the inverted topographical masses, or the compensated masses, or the reference topographic masses for the Rudzki, PH, and RTM reduction schemes, respectively.

The two components on the right hand side of equation (3) can be expressed for each reduction scheme as follows:

$$A = G\rho \iint_E \int_0^h \frac{(h_p - z)}{s^3(x_p - x, y_p - y, h_p - z)}\, dxdydz$$

$$A_{Inv} = G\rho' \iint_E \int_{-h'}^0 \frac{(h_p - z)}{s^3(x_p - x, y_p - y, h_p - z)}\, dxdydz$$

$$A_{Comp\,(\mathrm{Pratt})} = G\Delta\rho \iint_{E} \int_{-D}^{0} \frac{(h_p - z)}{s^3(x_p - x, y_p - y, h_p - z)} dxdydz$$

$$A_{Ref} = G\rho \iint_{E} \int_{0}^{h_{Ref}} \frac{(h_p - z)}{s^3(x_p - x, y_p - y, h_p - z)} dxdydz \tag{4}$$

where h and h_{Ref} are the height and the reference height respectively. The reference height is defined by low pass filtering of local terrain heights (Forsberg, 1984). E is the area and s is the distance between the computation point and mass elements. ρ' and h' are the density and depth of inverted masses that are equal to the density and height of the topographical masses in planar approximation (Heiskanen and Moritz, 1967; Bajracharya and Sideris, 2004), respectively. $\Delta\rho$, the difference between standard crust density and the actual density in PH model, can be given as (Heiskanen and Moritz, 1967)

$$\Delta\rho_{(\mathrm{Pratt})} = \rho - \rho_r = \frac{h}{D + h}\rho \tag{5}$$

where ρ_r is the actual crust density and D is the depth of compensation for the PH model.

The direct topographical effect on gravity in Helmert's second method of condensation, Δg_T in equation (3), is equal to the negative of terrain correction. The terrain correction can be expressed as

$$c = G\rho \iint_{E} \int_{h}^{h_p} \frac{(h_p - z)}{s^3(x_p - x, y_p - y, h_p - z)} dxdydz \tag{6}$$

where h_p is the height of computation point. The integrals in equation (4) and (6) can be numerically integrated using rectangular prisms with the computation point coinciding with the origin of the coordinate system (Nagy, 1966):

$$A_T = G\rho \;|||\; x\ln(y + r) + y\ln(x + r) - z\arctan\frac{xy}{zr} \Big|_{x_1}^{x_2} \Big|_{y_1}^{y_2} \Big|_{z_1}^{z_2} \tag{7}$$

where the coordinates x_1, x_2, y_1, y_2, z_1 and z_2 represent the corners of a prism.

The total geoid obtained from the restore step in RCR procedure can be expressed as:

$$N = N_{\Delta g} + N_{ind} + N_{GM} \tag{8}$$

where N_{GM} denotes the long wavelength part of the geoid obtained from a geopotential model, $N_{\Delta g}$ represents residual geoid obtained by using Δg from equation (1) in Stokes's formula, and N_{ind} is the indirect effect on the geoid, which depends on the mass reduction scheme used. Stokes's integral formula with the rigorous spherical kernel by the one-dimensional fast Fourier transform algorithm is applied in this paper (Haagmans et al., 1993).

The formulas for the computation of Δg_{GM} and N_{GM} are given in Heiskanen and Moritz (1967). The indirect effect on geoid, in equation (8), can be computed from Bruns's formula, as follows:

$$N_{ind} = \frac{\Delta T}{\gamma} \tag{9}$$

where ΔT is the change in the potential at the geoid, which depends on the reduction method used and can be expressed as follows:

$$\Delta T = T - T_{(Inv,\ Cond,\ Comp)} \tag{10}$$

where T is the gravitational potential of the actual topographical masses and $T_{(Inv,\ Cond,\ Comp)}$ represents the potential of the inverted, condensed, or compensated masses for the Rudzki, Helmert, and PH reduction schemes, respectively. ΔT in the equation (10) is zero for the Rudzki inversion scheme since the potential of the topography is equal to that of the inverted topography.

The potentials of the topographical masses and the compensating masses for the PH models can be expressed as:

$$T = G\rho \iint_E \int_0^{h(x,y)} \frac{1}{s\,(x_p - x, y_p - y, h_p - z)}\, dxdydz$$

$$T_{Comp(\mathrm{Pratt})} = G\Delta\rho \iint_E \int_{-D}^{0} \frac{1}{s\,(x_p - x, y_p - y, h_p - z)}\, dxdydz \tag{11}$$

The integrals in equation (11) can also be numerically integrated using rectangular prisms with the computation point coinciding with the origin of the coordinate system (Nagy, 1966):

$$T = G\rho\ |||\ xy\ \ln(z + r) + xz\ \ln(y + r) + yz\ \ln(x + r)$$

$$-\frac{x^2}{2}\tan^{-1}\left(\frac{yz}{xr}\right) - \frac{y^2}{2}\tan^{-1}\left(\frac{xz}{yr}\right) - \frac{z^2}{2}\tan^{-1}\left(\frac{xy}{zr}\right) \Big|_{x_1}^{x_2} \Big|_{y_1}^{y_2} \Big|_{z_1}^{z_2} \tag{12}$$

The indirect effect on the geoid for Helmert's second method of condensation can be obtained in planar approximation as (Wichiencharoen, 1982)

$$N_{ind} = -\frac{\pi G\rho}{\gamma} h_p^2 - \frac{G\rho}{6\gamma} \iint_E \frac{h^3 - h_p^3}{s_o^3}\, dxdy \tag{13}$$

where γ is the normal gravity and $s_o = [(x - x_p)^2 + (y - y_p)^2]^{1/2}$. The RTM reduction method gives the quasigeoid, and the restored effect on the quasigeoid due to the removal of topography according to this model is given by (Forsberg, 1984)

$$N_{res-eff} = \frac{G\rho}{\gamma} \iint_E \int_{h_{Ref}}^{h} \frac{1}{s} dxdydz = \frac{G\rho (h - h_{Ref})}{\gamma} \iint_E \frac{1}{s_o} dxdy \tag{14}$$

The separation between quasigeoid and geoid can be computed from (Heiskanen and Moritz, 1967)

$$\zeta - N = -\frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} h \approx -\frac{\Delta g_B}{\bar{\gamma}} h \tag{15}$$

where $\bar{g}$, $\bar{\gamma}$, and Δg_B represent the mean gravity along the plumb line between geoid and ground, mean normal gravity along the normal plumb line between ellipsoid and telluroid, and the Bouguer anomalies, respectively.

3. Numerical tests and discussions

A numerical test is carried out in the most rugged area of Canadian Rockies bounded by latitude between 49°N and 54°N and longitude between 236°E and 246°E. Table 1 presents the statistics of the DTMs used in this test for different grid resolutions. Figure 1 shows the topography model of Canadian Rockies. The original grid resolution available for this test is 3". Other grid files of resolutions 6", 15", 30", 45", 1' and 2' are created by selecting the point height values from the 3" grid for the corresponding grid levels. There were 9477 gravity measurements available for this test. The distribution of gravity points is presented in Fig. 2. The constant density of topographical masses is assumed to be 2.67 g/cm^3.

The program TC developed by Forsberg (1984) is modified to compute direct topographical effects on gravity using different gravimetric reduction schemes used in this test. A radius of 300 km is used around the computation point to compute the gravitational

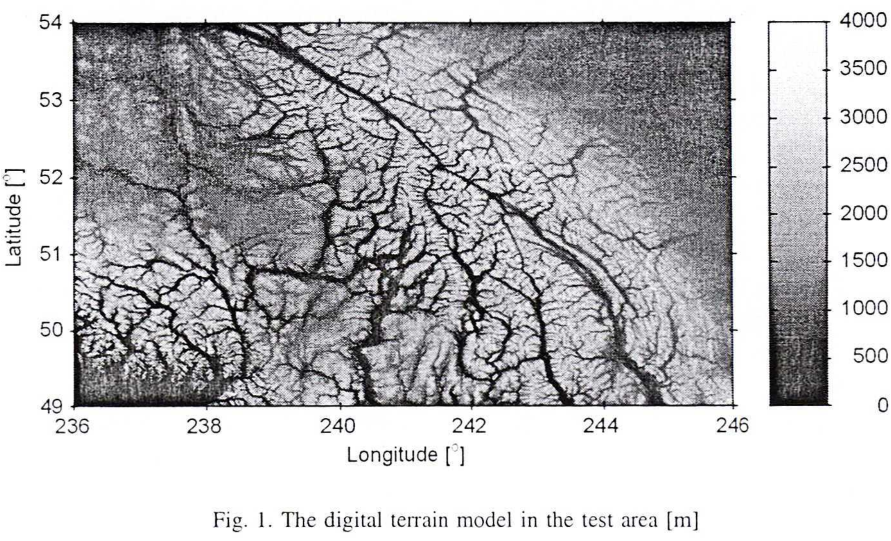

Fig. 1. The digital terrain model in the test area [m]

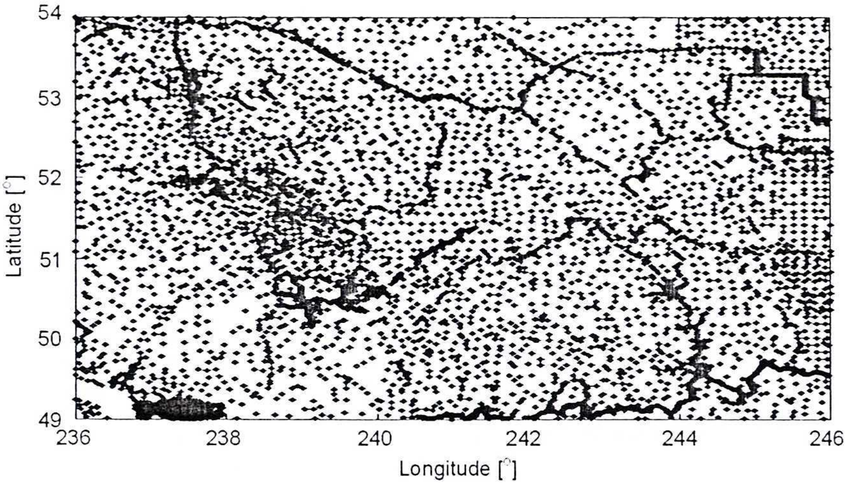

Fig. 2. The distribution of gravity points in the test area

Table 1. Statistical characteristics of DTMs [m]

Grid resolution	Max	Min	Mean	RMS	STD
6"×6"	3937	0	1396	1460	543
15"×15"	3840	0	1355	1460	543
30"×30"	3785	0	1355	1460	543
45"×45"	3656	0	1354	1459	543
1'×1'	3429	0	1354	1459	543
2'×2'	3275	0	1353	1458	544

attraction of the topography and the attraction of the compensated or inverted masses. The height of the smooth reference surface, h_{Ref}, for RTM is computed with the resolution of 100 km. The EGM96 is used as the reference global field for all schemes. The GPS levelling data set is used to assess the precision of Rudzki, RTM, Helmert, and PH gravimetric geoid solutions. 258 GPS benchmarks were available for this test. The distribution of GPS benchmarks is given in Fig. 3.

The geoid is computed from Stokes's integral formula with the rigorous spherical kernel by the one-dimensional fast Fourier transform algorithm. The 5'×5' grid of gravity anomalies is used in Stokes's computation. The aliasing effects on gravity anomalies and on the geoid are not only due to the different DTM resolutions used in this test, but also to the use of 5'×5' grid of gravity anomalies. Thus the results shown in this study should not be considered as absolute aliasing effects.

First, the gravity anomalies are computed using different DTM grid resolutions for each mass reduction technique. The gravity anomalies obtained by using the highest DTM resolution are regarded as the control gravity anomalies for each reduction scheme. Figures 4 and 5 show the difference in maximum value and standard deviation, respectively, between control gravity anomalies and gravity anomalies obtained by using coarser DTM grid resolutions.

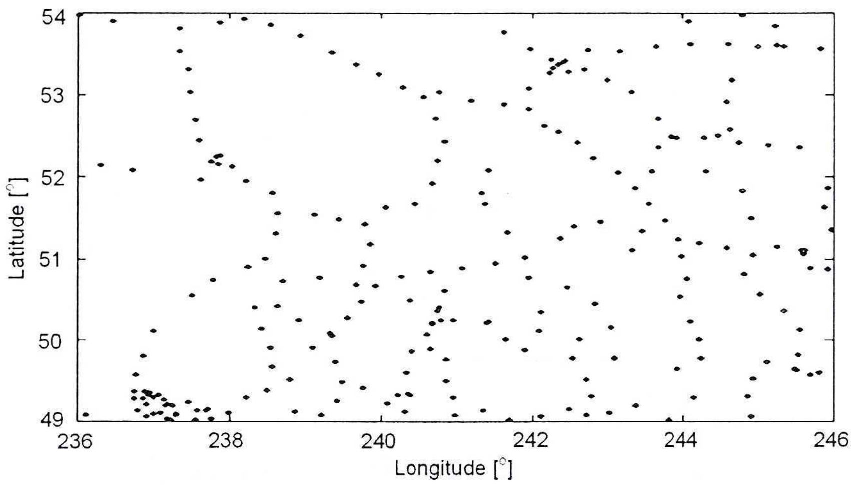

Fig. 3. Distribution of GPS/levelling points in the test area

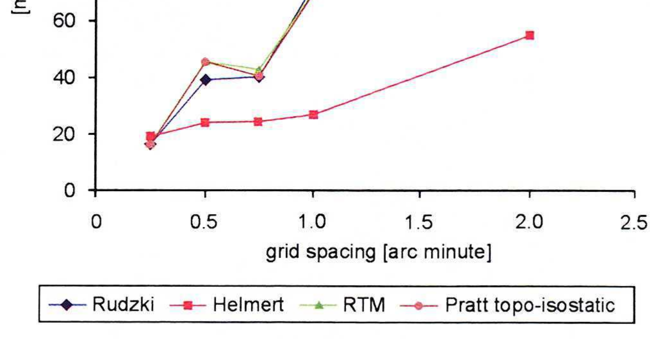

Fig. 4. The difference in maximum value between control gravity anomalies and anomalies obtained using different DTM resolutions

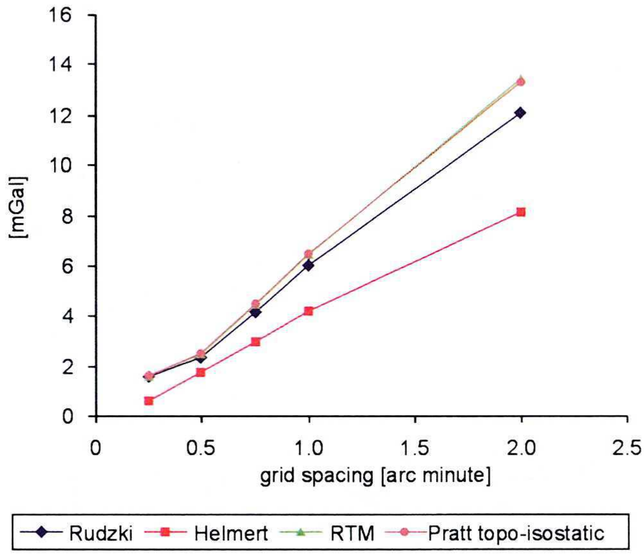

Fig. 5. The difference in standard deviation between control gravity anomalies and anomalies obtained using different DTM grid resolutions

The use of a 2' DTM grid resolution instead of a 6" one changes the gravity anomalies from 55 mGal to 123 mGal in maximum value, depending on the type of the gravimetric reduction technique chosen. Similarly, the difference in standard deviation changes from 8 mGal to 13 mGal. As the grid spacing decreases, we can note for every reduction scheme that the resultant map of gravity anomalies tends to get smoother. This can be easily seen by comparing two solutions for Rudzki gravity anomalies shown in Fig. 6 and Fig. 7.

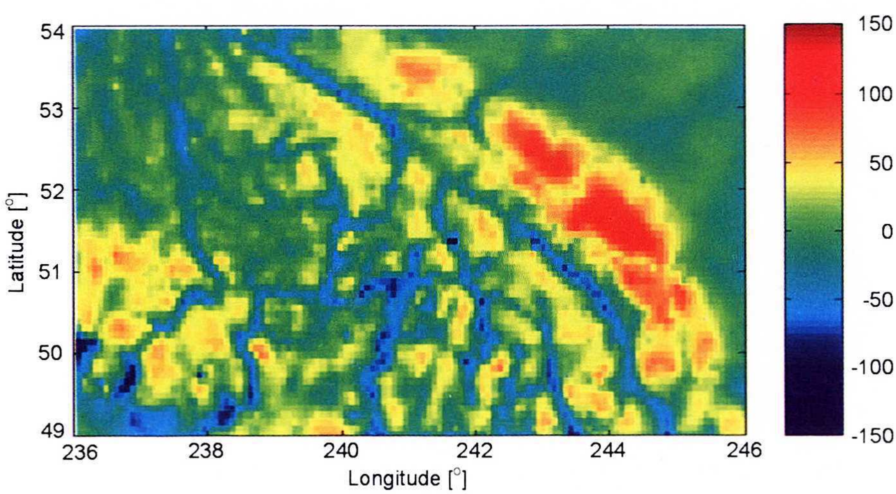

Fig. 6. 6"×6" Rudzki anomalies using 6" DTM grid resolution [mGal]

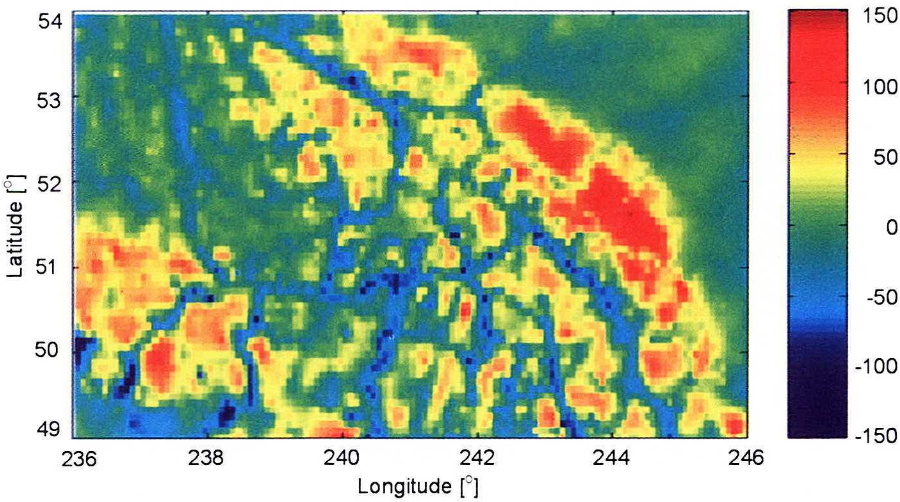

Fig. 7. 5'×5' Rudzki anomalies using 2' DTM grid resolution [mGal]

Second in this investigation, the gravimetric geoid solution is carried out using each reduction scheme with different DTM resolutions. The geoid solution obtained by using the highest DTM resolution is regarded as the control geoid. Figure 8 shows the difference in maximum value between the control geoid and geoid models obtained using different DTM resolutions. These differences can reach 2.05 m to 2.75 m in maximum value depending on the mass reduction scheme selected for geoid determination. These results in the Canadian Rockies suggest that a DTM grid resolution of 6" or higher is required for precise geoid

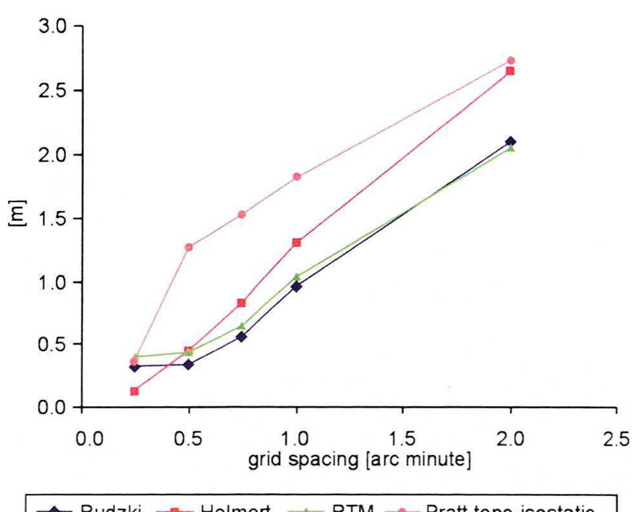

Fig. 8. The difference in maximum value between the control geoid and geoid models obtained using different DTM resolutions

determination with an accuracy of a decimetre or higher for any gravimetric reduction method in rugged areas. A DTM resolution not coarser than 45" is required for the geoid determination with accuracy of a metre using Rudzki, Helmert, and RTM gravimetric reductions, whereas a DTM resolution not coarser than 15" is required for geoid determination using PH topographic-isostatic reduction to gain the accuracy of a metre.

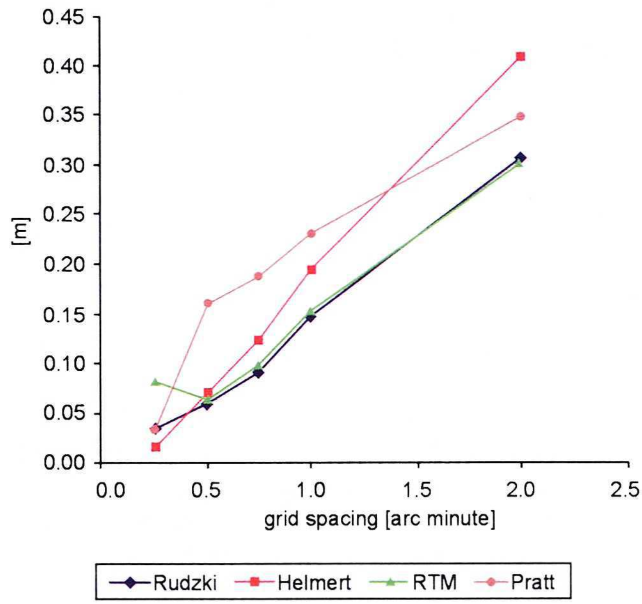

Fig. 9. The difference in standard deviation between the control geoid and the geoid models obtained using different DTM resolutions

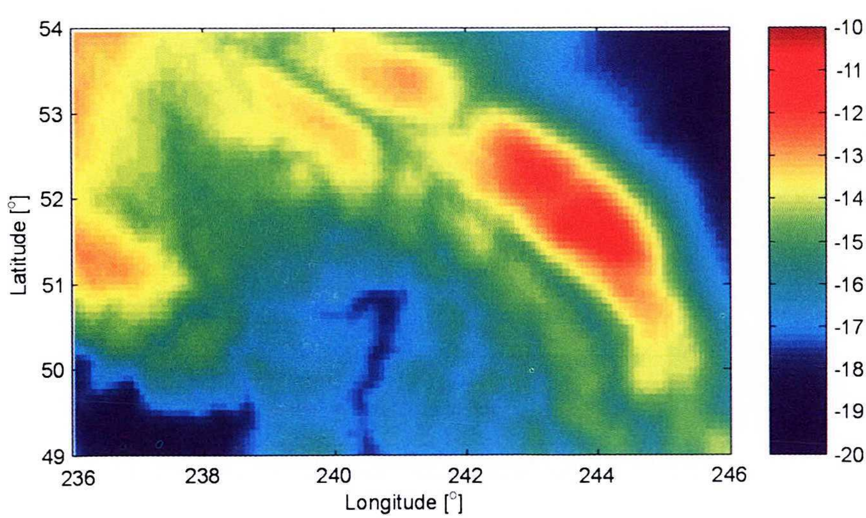

Fig. 10. 6"×6" Rudzki geoid using 6" DTM grid resolution [m]

Figure 9 shows the trend of increasing the standard deviation with increasing DTM resolution, which is similar for all gravimetric reduction schemes. The difference in standard deviation between the control geoid model and the geoid models obtained using different DTM resolutions is between 3 and 4 decimetres depending on the reduction method selected. Comparing the Rudzki geoid using 6" and 2' DTM resolution shown in Fig. 10 and Fig. 11, the geoid using 6" DTM resolution is smoother as well as less correlated with the topography. This is true for all reduction methods used in this test.

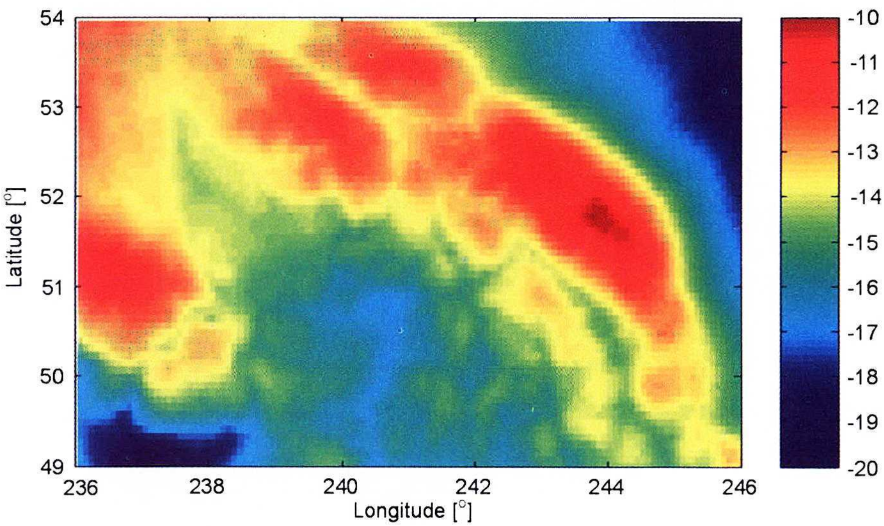

Fig. 11. 5'×5' Rudzki geoid using 2' DTM grid resolution [m]

Finally, the gravimetric geoid solutions obtained from using different DTM resolutions for every reduction method are compared with the GPS-levelling geoid solution. Figures 12

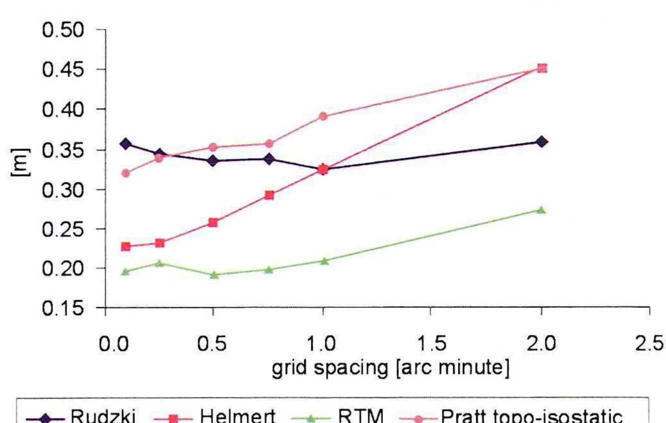

Fig. 12. Standard deviation of the differences between different geoid undulations with GPS/levelling geoid before fit

and 13 present the graphs of standard deviation of the differences between different gravimetric geoid solutions with the GPS-levelling geoid before and after fit. The increasing trend of the standard deviation of the differences between gravimetric geoid solutions using different DTM grid resolutions with the GPS-levelling geoid looks similar for every reduction technique after fit. There is an increase of nearly a decimetre in standard deviation when using a 2' DTM grid resolution instead of a 6" one.

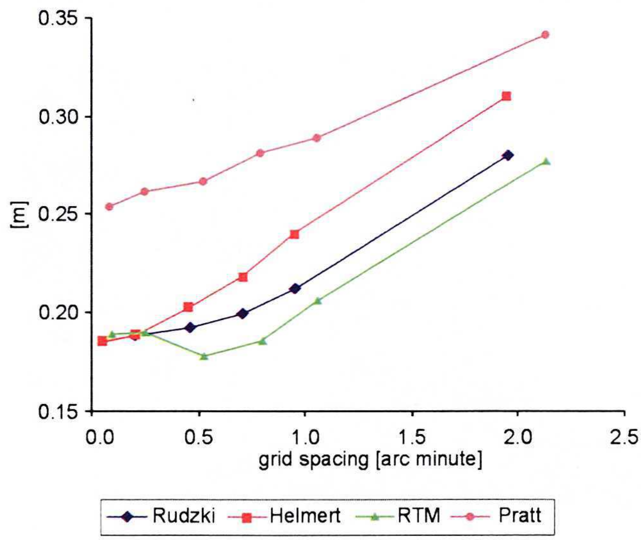

Fig. 13. Standard deviation of the differences between different geoid undulations with the GPS/levelling geoid after fit

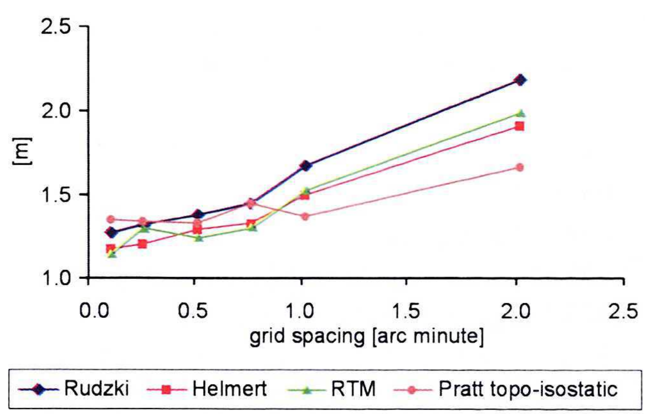

Fig. 14. The range of differences between different geoid undulations with the GPS/levelling geoid after fit

The most precise geoid models obtained from this test are the ones obtained using Rudzki, Helmert, and RTM method with 6" DTM resolution. The use of 2' DTM grid instead of 1' brings the change of nearly 7 cm in standard deviation for every reduction method. The finer the DTM grid resolution, the smaller are the standard deviation and the range (as shown in Fig. 14) of the differences between the gravimetric and GPS/levelling geoid.

4. Conclusions

This paper investigated the terrain aliasing effects on geoid determination using different gravimetric mass reduction schemes within the context of precise geoid determination. As earlier stated, the aliasing effects on gravity anomalies and on the geoid are not absolute since the 5' × 5' grid of gravity anomalies is used in Stokes's computation. A DTM grid resolution of 6" or denser should be used to achieve an accuracy of a decimetre or higher for any gravimetric reduction method chosen to treat the topographical masses above the geoid in mountainous areas. A DTM spacing of 45" or higher should be applied to gain an accuracy of a metre using the Rudzki, Helmert, or RTM gravimetric reductions. A DTM not sparser than 15" should be used to possess metre accuracy when using PH topographic-isostatic reduction. The choice between using a 6" and a 2' DTM can alter the geoid from 2.05 m to 2.75 m in maximum value depending on the mass reduction scheme selected for geoid determination.

The Rudzki, Helmert, and RTM gravimetric geoid solutions obtained using 6" DTM resolution presented the best results in this study. The standard deviation and the range of the difference between the gravimetric geoid and GPS-levelling geoid become smaller as the DTM resolution goes higher. More studies using higher resolution of DTM (if and where available) should be carried out to study this effect in different parts of the world especially in mountainous areas for various gravimetric reduction schemes.

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Wpływ rozdzielczości modelu terenu na wyznaczenie grawimetrycznej quasigeoidy

Sujan Bajracharya, Michael G. Sideris

Wydział Inżynierii Geomatycznej, Uniwersytet w Calgary,
2500 University Drive N.W., Calgary, Alberta, T2N 1N4, Kanada
e-mail: bajrachs@ucalgary.ca

Streszczenie

W pracy badany jest wpływ rozdzielczości modelu terenu na wyznaczenie geoidy przy użyciu różnych metod redukcji grawimetrycznych. Numeryczny model terenu umożliwia precyzyjne mapowanie szczegółów terenowych, toteż, w miarę dostępności, powinien być on wykorzystywany w każdej redukcji grawimetrycznej. W badaniach użyto następujących metod redukcji grawimetrycznych: metoda inwersji Rudzkiego, druga metoda kondensacji Helmerta, metoda residualnego modelu terenu (RTM) i metoda topograficzno-izostatycznej redukcji Pratta-Hayforda (PH). Dla każdej z wybranych metod redukcji grawimetrycznej badano wpływ rozdzielczości 6", 15", 30", 45", 1' i 2' modelu terenu na obliczone anomalie grawimetryczne oraz na absolutne undulacje geoidy. Do przeprowadzenia testów numerycznych wybrano silnie pofałdowany obszar w kanadyjskiej części Gór Skalistych pomiędzy równoleżnikami 49°N i 54°N i pomiędzy południkami 236°E i 246°E. Uzyskane wyniki wskazują na to, że do wyznaczenia w terenie górzystym geoidy z dokładnością co najmniej decymetrową konieczne jest, niezależnie od wybranej metody redukcji grawimetrycznej, użycie modelu terenu o rozdzielczości co najmniej 6" w celu uwzględnienia efektów mas topograficznych ponad geoidą. W wyniku przeprowadzonych testów numerycznych najdokładniejsze modele geoidy uzyskano przy zastosowaniu metod Rudzkiego, Helmerta i RTM z użyciem DTM o rozdzielczości 6".