



## Research paper

# Fine optimization of rigid frame bridge parameters based on the genetic algorithm

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**Abstract:** The primary aim of this paper is to study the optimization of rigid frame bridge parameters. With a three-span continuous rigid frame bridge as the engineering background, finite element models were established. Then an index about bridge force condition was proposed to calculate the optimal side-to-mid span ratio with different side-to-mid span ratio parameters. Based on the ratio, the values of the girder depth at the pier and the bottom curve degree of the box-girder were taken as parameters in their common ranges for further optimization. A comprehensive multi-objective evaluation index correlated with the mid-span section stress, the mid-span deflection, and the concrete consumption was proposed to do fine optimization through the genetic algorithm method. The result of this study shows that the genetic algorithm is an effective method for bridge optimization and could provide better girder design parameter combinations for the comprehensive performance, and the optimal result could be obtained in the continuous parameter definition domains. It also shows that a larger girder depth at the pier to span ratio and a smaller curve degree in their common ranges should be taken for the bridge's comprehensive performance.

**Keywords:** rigid frame bridge, side-to-mid span ratio, girder parameters, genetic algorithm, fine optimization

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## 1. Introduction

Continuous rigid frame bridges (CRFB) have good mechanical properties and experienced construction technologies. With its good structural stiffness, strong span ability, comfortable driving feeling, and light and beautiful appearance, CRFB are widely used in the world. For the design of CRFB, girder parameters are determined preliminarily by experiences and existing cases [1, 2]. Among these design parameters, the side-to-mid span ratio, the girder depth at the pier to span ratio, and the curve degree of the box-girder are the more critical parameters. Existing research also mainly focused on these parameters [3, 4], such as the studies on better girder performance with the optimal form according to different combinations of parameters [5, 6].

Optimization procedures were used to achieve the goals of making bridges safer and more economical [7]. The optimum design was defined by Wilde [8], and the purpose of structural optimization is to find the most efficient structure that matches the criteria [9]. As these parameters have significant impacts on the force condition of CRFB, it is necessary to carry out a refined optimization analysis to verify whether the optimized value has reached a good level in the typical range. Researchers have developed many methods of optimization, such as the ant colony optimization [10], the gradient-based approach [11], the simulated annealing [12], the genetic algorithm (GA) and so on. Since optimizations usually have multiple objectives, comprehensive evaluation indexes were applied to some form of these algorithms [13, 14].

The genetic algorithm was first proposed by John Holland in the 1970s [15] and has been widely used in many engineering fields. It was generally used to find an optimal solution for its simplicity and capability [16, 17]. In many branches of civil engineering, the genetic algorithm is used for design optimization: such as tunnel profiles under different geological conditions [18], the optimal cross-sectional design of steel beams [19], the cable optimization of cable-stayed bridges [20, 21], the temperature gradient of reinforced concrete bridges [22], and the optimal design by the hybrid genetic algorithm for steel arch bridges and concrete arch bridges [23, 24]. Because structural designs often have several variable design parameters, which would lead to high computational works for multi-objective optimization, a relatively efficient and straightforward calculation process is also critical [25, 26]. To achieve this goal, the selection of objective functions for optimization is of great significance.

By taking a continuous rigid frame bridge as the research background, this paper studied the multi-objective optimization of design parameters as the side-to-main span ratio, the girder depth to span ratio, and the curve's degree of the box-girder. Firstly, through the finite element simulation and regression analysis, the optimal value of the side-to-mid span ratio parameter was obtained. With this result, the mid-span section stress, the mid-span deflection, and the total concrete consumption were selected as evaluating indicators. Then, the influences of design parameters on the above indicators were analyzed to facilitate the application. In order to get more accurate results and provide an algorithm case for the refined optimization design of CRFB, the genetic algorithm was applied to obtain and verify the comprehensive evaluation index lastly.

## 2. Optimization of the side-to-mid span ratio

### 2.1. Engineering background

A three-span concrete continuous rigid frame bridge was used as the engineering background, and the span distribution is 103 + 190 + 103 m. The main span is the pre-stressed reinforced concrete box-girder. The bridge's side-to-mid span ratio is 0.54, the girder depth at the pier is 12 meters (the depth-to-span ratio is 0.063), and the degree of the girder bottom curve is 1.6. The elevation drawing of the bridge is shown in Figure 1, and the cross-section of the girder at the pier is shown in Figure 2.

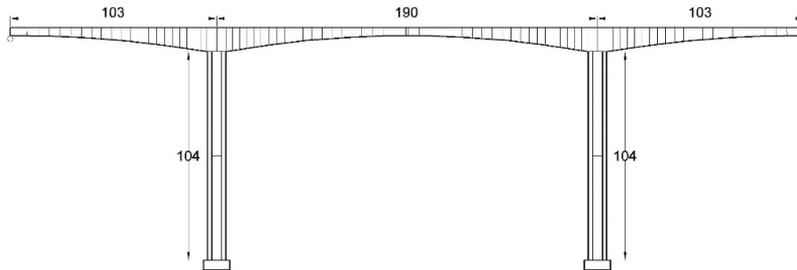


Fig. 1. The elevation drawing of the bridge (unit: m)

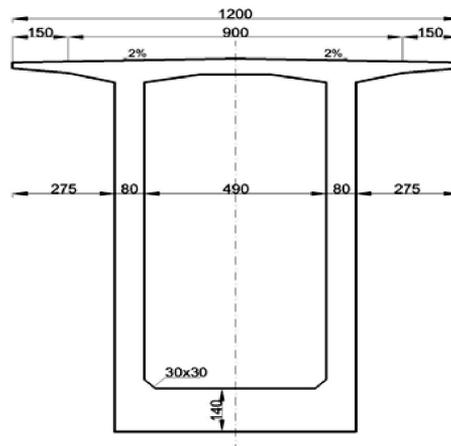


Fig. 2. The cross-section of the girder at the pier (unit: cm)

### 2.2. The side-to-mid span ratio analysis

The side-to-mid span ratio of CRFB is dependent on the overall layout of bridges and requires coordination with natural conditions. Therefore, it is usually determined first. Simultaneously, the ratio is also based on girder force distributions and the convenience of construction. It is commonly distributed between 0.52 and 0.57. In this paper, the values of ratio parameter  $i$  were

selected in the common range as 0.521, 0.526, 0.532, 0.537, 0.547, 0.553, 0.558, 0.563 and 0.568, respectively. A series of finite-element models were established for numerical simulation. The results of the mid-span bending moment, the bending moment at the pier, and the mid-span deflection were taken for evaluation. It should be noted that due to the large impact of the pre-stressed reinforcement, it is necessary to study it separately when optimization. Therefore, in order to better investigate the impact of the main girder parameters above, the pre-stressed reinforcements basically remained unchanged.

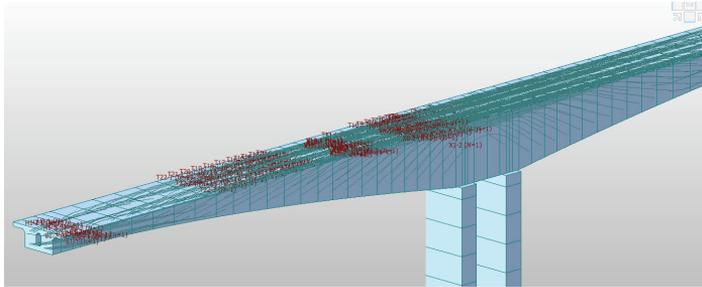


Fig. 3. The finite element model of the bridge

The results are shown in Figure 4-6.

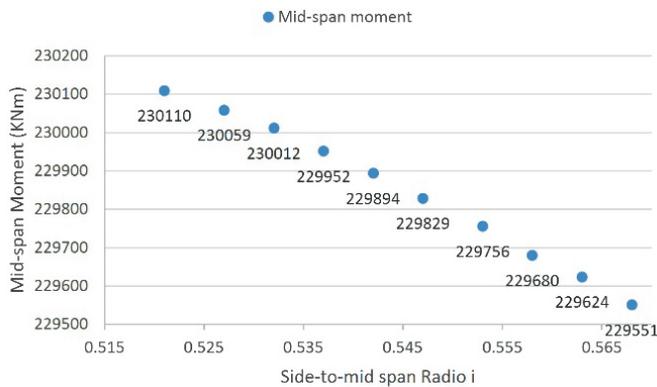


Fig. 4. The relation between the mid-span moment and the side-to-mid span ratio (unit: kN · m)

The above figures demonstrate that as the ratio increases, the mid-span bending moment, the mid-span deflection decrease, and the bending moment at the pier increases gradually.

To evaluate the side-to-mid span ratio parameter more directly, the above three results were used to propose a comprehensive evaluation index for the multi-objective optimization. Obtain the ratios of the three results (the mid-span bending moment, the bending moment at the pier, and the mid-span deflection) of different parameters to the original values of background engineering, respectively. Then take the sum of these ratios as the evaluation index  $K_1$ .

$$(2.1) \quad K_1 = \frac{M_m}{[M_m]} + \frac{M_p}{[M_p]} + \frac{f}{[f]}$$

where:  $M_m$  is the mid-span bending moment,  $M_p$  is the bending moment at the pier, and  $f$  is the mid-span deflection.  $[M_m]$ ,  $[M_p]$  and  $[f]$  are the corresponding values of the background engineering, respectively.

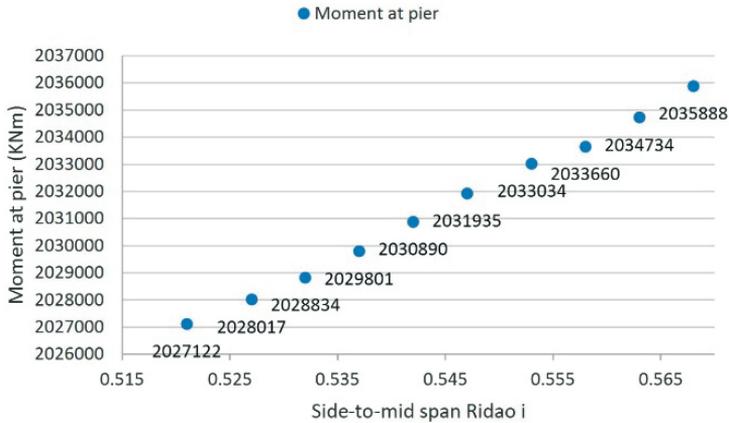


Fig. 5. The relation between the moment at the pier and the side-to-mid span ratio (unit:  $\text{kN} \cdot \text{m}$ )

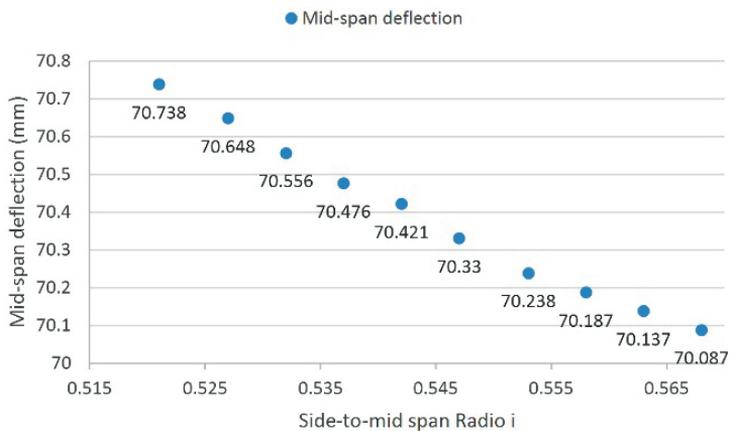


Fig. 6. The relation between the mid-span deflections and the side-to-mid span ratio (unit: mm)

The relationship between the side-to-mid span ratio value  $i$  and the index  $K_1$  is shown in Figure 7.

In order to find the optimal value, different function curves were fitted and compared, then obtained the better fitting result of 6 degrees polynomials function. The function was with ratio  $i$  as an independent variable and index  $K_1$  as a dependent variable, and its shape is shown in Figure 8. From the fitting equation, the minimum value of  $K_1$  was obtained nearby  $i = 0.55732$ , and the index  $K_1$  correlated with the mid-span bending moment, the bending moment at the pier, and the mid-span deflection under the most unfavorable load achieved the best.

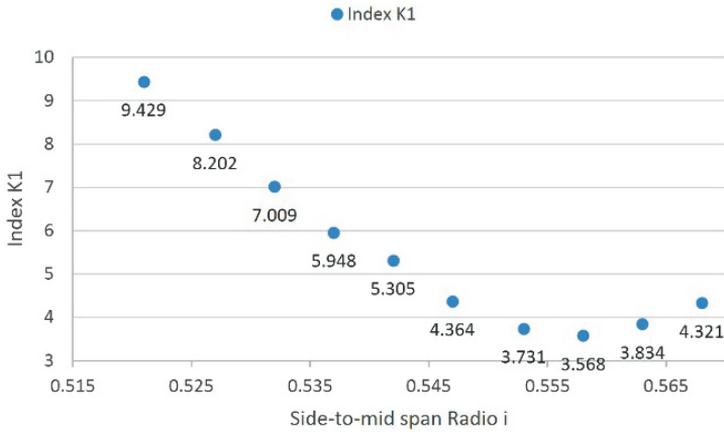
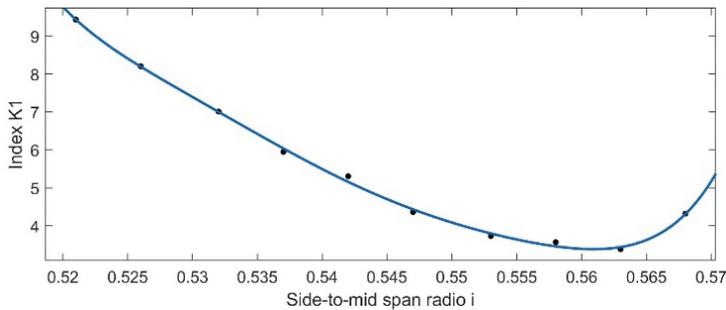
Fig. 7. The relation between  $i$  and  $K_1$ 

Fig. 8. The fitting equation graph

### 3. Fine optimizations based on the genetic algorithm

#### 3.1. Selection and calculation of evaluation indicators

The optimization analysis of the girder depth at the pier to span ratio and the degree of the girder bottom curve was based on the previous section's optimal side-to-mid span ratio. The ratio of girder depth at pier  $h$  to span  $L$  ranges commonly between 0.055 and 0.065 (for the convenience of study, the depth  $h$  determined by the depth-to-span ratio was taken as a variable parameter), and the degree of girder bottom curve range commonly distributes from 1.5 to 2.0. According to background engineering and other studies, the value of height  $h$  was selected in the above range as 11.2 m (the depth-to-span ratio 0.059), 11.6 m (0.061), 12 m (0.063) 12.4 m (0.065), respectively. The degree of curve  $\alpha$  was as 1.5, 1.67, 1.83, 2.0, respectively. A set of bridge models with the full permutation of  $h$  and  $\alpha$  were established for numerical simulation.

In order to effectively evaluate the bridge's comprehensive capacity, the maximum stress and the deflection of the mid-span section are selected as indicators. Moreover, considering the economic and environmental protection performance, the concrete material consumption

of the whole bridge is also taken as an indicator. Then, the following evaluation index  $K_2$  was used for the multi-objective optimization.

$$(3.1) \quad K_2 = \frac{\sigma}{f_{tk}} + \frac{f}{f_l} + \frac{F}{G}$$

where,  $\sigma$  and  $f$  are the maximum stress and the deflection at the mid-span section;  $f_{tk}$  is the specification strength of C55 concrete, valued as  $2.74 \text{ N/mm}^2$ ;  $f_l$  is the mid-long term deflection limits,  $f_l = L/1600$ ;  $F$  is the total concrete consumption of the model bridges;  $G$  is the concrete consumption of the actual bridge.

The full permutations and corresponding results are shown in Figure 9–12.

Figures 9–12 illustrate the relationship between the parameters and the results of the stress, the deflection, and the amount of concrete. It also shows that the index  $K_2$  increases as the increase of the curve degree  $\alpha$  in common range, while it decreases when the girder depth  $h$

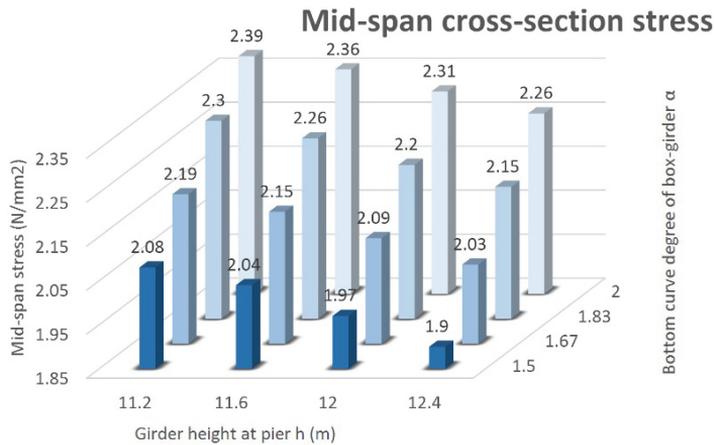


Fig. 9. Results of the mid-span section stress (unit:  $\text{N/mm}^2$ )

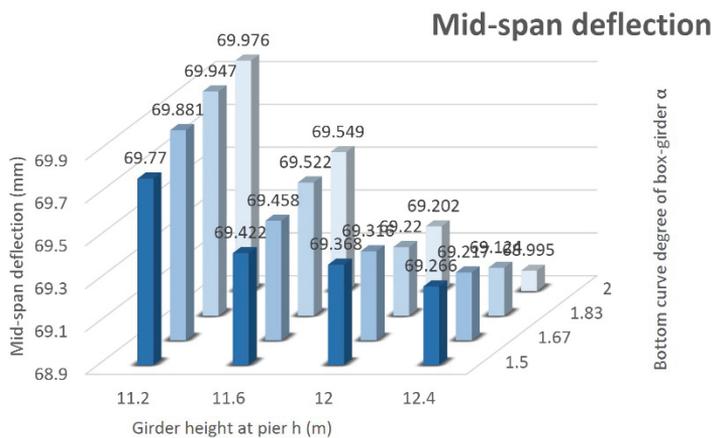


Fig. 10. Results of the mid-span deflection (unit: mm)

increases. The combination of the girder depth valued 12.4 m (the depth-to-span ratio 0.065) and the degree of curve valued 1.5 makes the best result for the multi-objective optimization correlated with the mid-span section stress, the mid-span deflection, and the amount of concrete consumption.

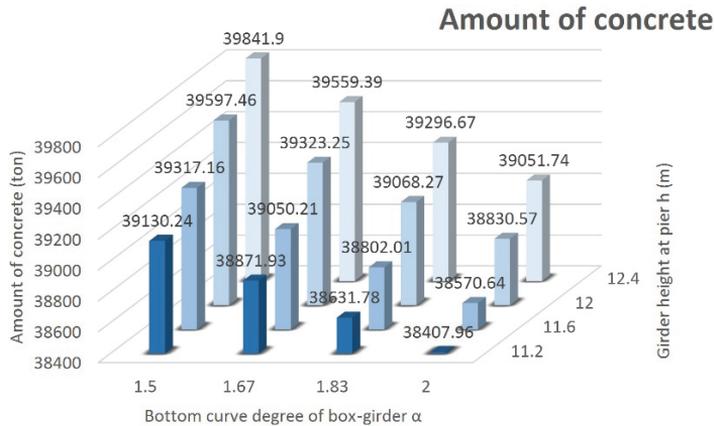


Fig. 11. Results of the total consumption of concrete material (unit: ton)

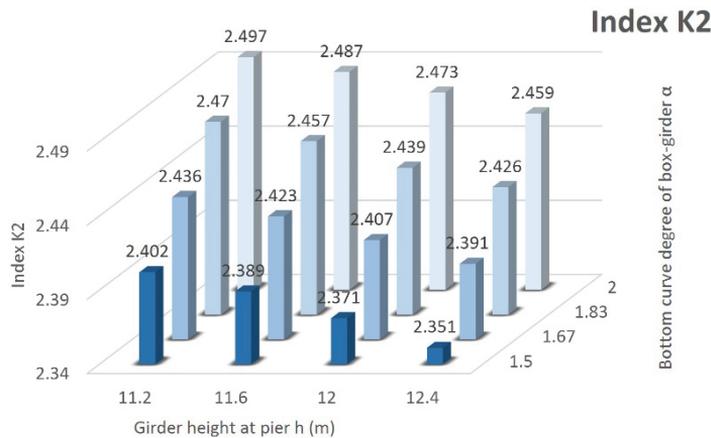


Fig. 12. Results of the index  $K_2$

In order to facilitate the targeted optimization of each indicator in engineering, a simple binary regression was carried out on the above results. With the coefficient of determination R-squared as the standard ( $R^2 > 99.0\%$ ) for fitting, the fitted equations were obtained as:

$$(3.2) \quad y_1 = 2.546 - 0.1313x_1 + 0.6685x_2$$

$$(3.3) \quad y_2 = 119.5 - 9.396x_1 + 10.26x_2 + 0.4371x_1^2 - 0.8744x_1x_2$$

$$(3.4) \quad y_3 = 34950 + 575.2x_1 - 1518x_2$$

where,  $y_1$ ,  $y_2$ ,  $y_3$  are the mid-span section stress, the mid-span deflection, and the amount of concrete consumption, respectively.  $x_1$ ,  $x_2$  are the girder depth and the degree of curve.  $R^2$  are 0.995, 0.991, 0.995, respectively.

### 3.2. The genetic algorithm optimization

Because of the discreteness of parameters, there is a possibility that the optimal solution was between the selected parameter values in the common ranges. In addition, with the higher requirements of bridge structure optimizations, the number of design parameters may also increase. The accuracy and reliability of the multiple regression way need to be improved, and it is necessary to propose a method that can effectively solve the multi-parameter optimization. Therefore, with the background bridge as the example, to obtain a more precise combination of parameters and provide a case algorithm for the optimization design of CRFB, the genetic algorithm was applied for the further fine optimization of the index  $K_2$ .

The genetic algorithm is used to search for the optimal global solution by imitating the natural evolution process. According to the genetic algorithm, a target function proposing to distinguish the good or bad individuals in the group should be determined first. Then a fitness function is obtained based on the target function, which drives to simulate the natural evolution process. In this study, the target function was fitted based on the girder depth  $h$  and the degree of curve  $\alpha$  as independent variables and the multi-objective index  $K_2$  as the dependent variable. A binary cubic polynomial was taken as it matched well by comparing the functions of different fitting forms. Then, the function with two independent variables and one dependent variable,  $y = f(x_1, x_2)$ , was obtained:

$$(3.5) \quad y = -38.17 + 9.905x_1 + 2.997x_2 - 0.8223x_1^2 - 0.295x_1x_2 - 0.5975x_2^2 + 0.02279x_1^3 + 0.007812x_1^2x_2 + 0.04375x_1x_2^2$$

where:  $y$  is the index  $K_2$ ,  $x_1$  is the girder depth at the pier  $h$ ,  $x_2$  is the degree of curve  $\alpha$ , and  $x_1 \in [11.2, 12.4]$ ,  $x_2 \in [1.5, 2.0]$ .

The target function is to obtain the minimum value of  $f(x_1, x_2)$  in domains of  $x_1$  and  $x_2$ . According to the actual meaning of the target function, the domain ranges, and the range of initial value of the objective function, the target function could meet the requirements of the fitness function, and then set the fitness function  $g(x) = f(x_1, x_2)$ .

The GA toolbox of MATLAB was used for the optimization, and the basic setting is as, the ranges of the control variables  $x_1$ : [11.2, 12.4],  $x_2$ : [1.5, 2.0]; the group size was 50; the initial value range of  $y$ : [2.351, 2.497]; the number of individuals per generation was 50. Binary coding was used, and the coding form was random uniform distribution for reproduction, one-point cross for crossover, and constraint for mutation.

The results showed that the genetic algorithm calculation terminated in the 50th generation and meets the conditions; the average value in population was 2.3537. The fitness function  $g(x)$  achieves the best value of 2.3536 at the point  $(x_1, x_2) = (12.3999, 1.5)$ . The iterative calculation process is shown in Figure 13.

The results by the genetic algorithm further show that when the fitness function obtains the minimum value, the best parameter combination is  $h = 12.4$  m (the ratio of the girder depth at

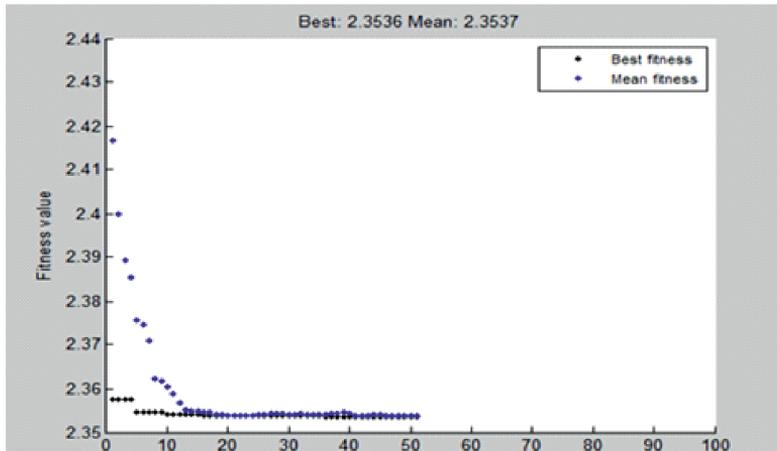


Fig. 13. Process and result of the fitness function within 50 iterations and optimization calculation

the pier to span is 0.065) and  $\alpha = 1.5$ . The function obtains the best result at the boundary of parameters' continuous domains, too.

Compared with the background engineering, the optimal mid-span stress and the mid-span deflection decreased by 9.09% and 0.72%, respectively, and the amount of concrete consumption increased by 1.32%.

## 4. Conclusions

In this paper, the optimization of CRFB design parameters was investigated. Through the finite element simulation and regression analysis, the optimal side-to-mid span ratio parameter was obtained. Then based on this ratio, the mid-span section stress, the mid-span deflection, and the total concrete consumption were taken as evaluating indicators. The multi-objective optimization of design parameters as the girder depth to span ratio and the curve's degree of the box-girder was carried out by the genetic algorithm. The main results are as follows.

1. The continuous rigid bridge achieves a better comprehensive performance correlated with the side-to-mid span ratio parameter when the ratio is 0.557.
2. The mid-span bending moment and the mid-span deflection under the most unfavorable load decrease when the girder depth parameter increases, while increase when the degree of curve increases. The bridge's total concrete consumption increases when the girder depth increases, while it decreases when the curve degree increases.
3. When taking the multi-objective performance correlated with the mid-span stress, the mid-span deflection, and the concrete consumption as the comprehensive evaluation index, a relatively more significant girder depth and a minor degree of box-girder bottom curve should be taken within their common ranges.
4. The genetic algorithm is an effective method for rigid frame bridge optimizations. The fine optimization by the genetic algorithm shows that the optimal value is obtained at the

point on the boundary in the continuous domain of the variable parameters. Moreover, the best combination is 0.065 for the girder depth at the pier to span ratio and 1.5 for the degree of the box-girder bottom curve.

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