THE ARCHIVES OF MECHANICAL ENGINEERING

VOL. LIII 2006 Number 4

Key words: sorting process, active fence, modelling

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ACTIVE FENCE WITH FLEXIBLE ELEMENT IN A DRIVE SYSTEM

The article presents a dynamic analysis of the sorting process of load units transported on conveyor belts. The process is realized by means of an active fence with one freedom degree. During analytic research the manipulated loads were treated as undeformable bodies, or in the case of load impact against the fence as the bodies with nonlinear springy-damping properties. A flexible element, described by Kelvin linear model, was used in the drive of the fence of a rotatably fixed beam. Appropriate choice of construction parameters makes it possible to mitigate the dynamic interactions of the sorted loads.

1. Introduction

Crank rocker arm mechanisms find their application in the drive of an active rotary fence used in the process of automatic sorting of load unit stream (e.g. mail packages). These mechanisms, which consist of the fence and the remaining part of the drive (worm gear), are structures characterized by very high stiffness. This property guarantees stability of the fence angular location in relation to the position of the drive crank (a guarantee of realization of the assumed fence motion course) regardless of the load forces. However, such a concept of a drive does not meet requirements of contemporary logistic processes, whose aim is to achieve higher and higher speed for the moving objects, as well as minimizing dynamic interactions generated during application of the manipulating activities on the object (in the presented work these activities are referred to as manipulation process). Application of manipulators of very stiff constructions diminishes the possibilities of absorbing and dissipating the impact energy released during the contact of the package

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with the manipulator. Thus, it is hardly possible to achieve high efficiency of the manipulation process without exceeding permitted overloads.

Mechanical system consisting of a belt conveyor and an arm fixed above it (active rotary fence or a set of passive fences) is also used in the process of positioning, which means assigning a precisely defined destination position to each load unit (rotary and translatory [11]). In the works pertaining to this application of fences (e.g. [1], [2], [3], [10], [18]), the problem of interaction between the manipulator and the load is reduced only to quasi-statistic events – inertia forces are neglected. In these works it is assumed that during the process of positioning the conveyor convection speed is relatively low, and therefore dynamic reaction forces are negligibly low compared to friction forces.

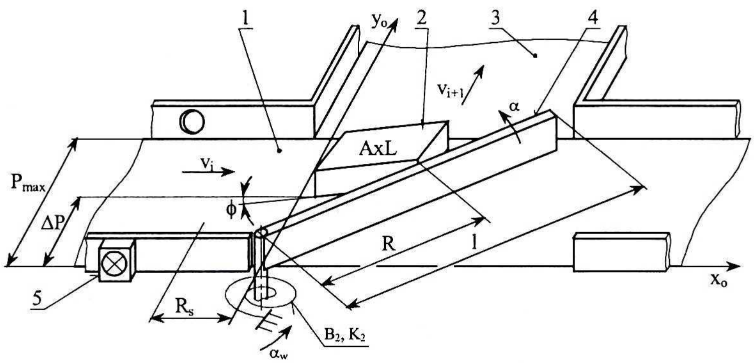

Fig. 1. Manipulator with an active rotary fence: 1 and 3 belt conveyors, 2 – the scraped load of the dimensions AxL, 4 – fence, 5 – light barrier, R_s – position of the load front at the moment of the fence activation, ΔP – distance of the load from the conveyor edge, R – distance of the load corner from the fence end at the moment of contact, P_{max} – the conveyor width, l – the fence length, α – the fence deflection angle, α_w – kinematic constraint of the fence, ϕ – load position angle, v_i – conveyor velocity of transportation, B_2, K_2 – damping and stiffness coefficient of the flexible element

The problem of mitigating reaction forces in the sorting process was considered in work [14], where a possibility of replacing the classic rectilinear fence by a curvilinear one was considered. In work [16], an application of a pneumatic system in the drive was discussed. These examples do not fully represent all the construction solutions aiming at reduction of dynamic reactions of the manipulated load units. Mitigation of the impact effects occurring in the sorting process can also be achieved by introducing elements with springy-damping properties into a classical rocker drive system of the fence. The present work discusses and analyses the concept of using a flexible element for connecting the stiff fence with a rocker drive system. Introduction of elements diminishing the impact effects into the manipulator 'releases' the fence enabling it to move around its equilibrium position partly independently on the drive system constraints. (Fig. 1).

In order to evaluate the fence and the manipulated object behaviour as a response to the sorting process parameters and the manipulator drive dynamic properties it is necessary to develop an appropriate theoretical model. One of the most important elements of such a model is the description of the impact phenomenon.

2. Model of the body impact process

The parameters of load motion after the load impacts against a fence with an infinitely large mass or against a fence of finite mass treated as a free body (not clutched with the manipulator drive system) can be determined basing on the classical theory of rough body impact. This theory uses a stiff body model, and the impact itself is treated as a temporary process (timeless [6], [13], [19]). In this way, such quantities as force or impact time can not be determined. However, in order to compare dynamic interactions to which the load is subjected in the process of sorting (in the case of using a fence characterised by the freedom of movement, as it was mentioned before) with permitted overloads, which are usually referred to as overloads appearing during a free fall of the load from the permitted height on an undeformable surface [21], it is enough to know the impulse of a force of the occurring impact – determined in the classical impact theory:

$$S_y = \frac{m_1 I_2 w_y(0)(1 + e)}{m_1 R^2 + I_2} \tag{1}$$

where:

R – the distance of the point of the load impact against the fence from the fence rotation axis,

$e = -w_y'/w_y$ – restitution coefficient,

w_y – relative velocity of the body before the impact in the normal direction,

w_y' – relative velocity of the body after the impact,

I_2 – mass inertia moment of the fence arm.

A problem arises when we want to examine the process of impact against the manipulator working element which exhibits flexibility properties – e.g. a fence connected with the drive system by a springy-damping element. The classical impact theory (regarding impact as a discrete process) is useless in this case. Treating the impact phenomenon as a continuous process, where we take into account the changes of interactions between the contacting bodies expressed in the function of time (i.e. describing the course of the impact force and displacement caused by it) provides a solution to this problem.

However, not every model treating the impact process as a continuous one can meet the requirements of the considered application. For example, the most popular one, well-known from the literature, classical Kelvin model is unreliable [15]. In the case of impact of an inelastic body (i.e. when the restitution coefficient e<l that is characteristic for loads in the analysed sorting process) the course of impact force, calculated from the classical linear Kelvin model described by the equation:

$$N = b_1\dot{x} + k_1 x \tag{2}$$

where:
x and $\dot{x}$ – displacement and displacement velocity of the impacting bodies,
b_1 and k_1 – damping and stiffness coefficient,
is not consistent with the physical nature of the process [6]. The impact force at the initial impact moment (t = 0) assumes a non-zero value (Fig. 2a), and its maximal value is not sensitive to the fence drive system changes – Fig. 2b. For this reason, it is not possible to assess how the springy-damping properties introduced to the fence drive system (or even total release of the fence, Fig. 2b(3)) can mitigate the dynamic interactions effects.

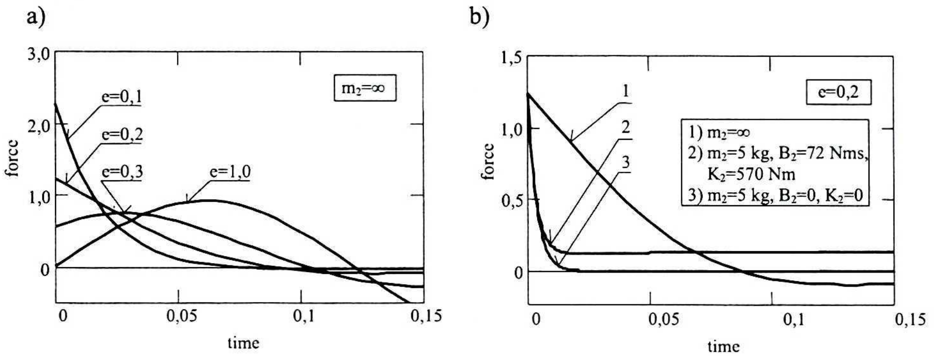

Fig. 2. Graphs of forces arising during the impact of the load against the fence (the impact phenomenon is treated as a continuous process described by Kelvin linear model – markings according to Fig. 1): a) loads with different restitution coefficients e and a fence with an infinitely large mass $m_2 = \infty$, b) load restitution coefficient e = 0,2; (1) – fence with an infinitely large mass, (2) fence as a stiff body clutched with the drive system by a springy-damping element with parameters B_2 and K_2, (3) – fence as a stiff free body

Irrespective of the flexible element properties (introduced to the drive system, and described by the parameters B_2 and K_2) the maximal value of the impact force is the same as for the case in which the fence mass is infinitely large – Fig. 2b(1). Only the course of impact force immediately after the initiation does change. However, the achieved effect is not great enough to decide about advantages resulting from the modification of the manipulator construction. Kelvin linear model limits the analysis of the impact process to the case, for which the displacement accompanying the impact does not result in extension of the contact area between the impacting bodies.

The result of taking into consideration the relations between the displacement of impacting bodies and the contact surface is the appearance of nonlinearity in the equations describing normal impact force. This property is accounted for by the theory of contact phenomena by Hertz. The application of this method is limited only to loads of bodies accompanying elastic strains. A model representing the load and the fence impact process should account for a high degree of the manipulated loads inelasticity – e.g. as for mail packages during free fall on undeformable surface, which reach restitution coefficient of about e = 0,2 [13].

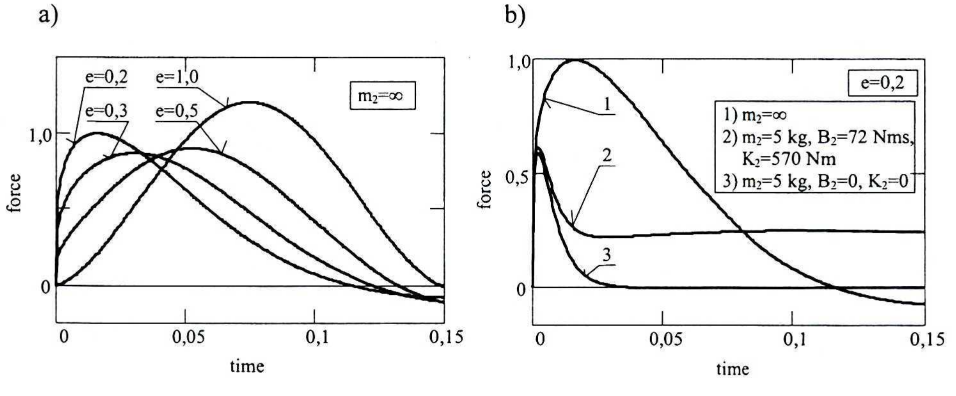

Fig. 3. Graphs of forces arising during load – fence collision (impact phenomenon is treated as a continuous process described by modified Kelvin nonlinear model): a) loads of different restitution coefficients e, and the fence with infinitely large mass $m_2 = \infty$, b) load restitution coefficient e = 0,2; (1) – fence with infinitely large mass, (2) – fence as a stiff body with a drive system clutched by a springy-damping element with parameters B_2 and K_2, (3) – fence as a stiff free body

A modified Kelvin model [20] in a nonlinear form, similar to the Hertz model, in which the normal impact force occurring between the bodies is described by the equation:

$$N = b_1(m_1 k_1)^{0,5} x_1^{0,25} \dot{x}_1 + k_1 x_1^{1,5} \tag{3}$$

where:

m_1 – load mass,

b_1 – non-dimensional damping coefficient referred to as restitution coefficient of the colliding bodies, seems to be a proper model for the analysis of the process, considered in this paper, in which the load impacts against a fence clutched with the drive system through a flexible element.

In this model, the force of elasticity is proportional to the displacement of the body, while the force of damping is proportional to both the displacement and the displacement velocity. Such a method of description of the impact energy dissipation phenomenon makes it possible for the force of damping value ($t = 0$) to rise always from the initial zero value at the impact initiation moment, regardless of the body's damping properties (Fig. 3).

3. Sorting process model

The proper choice of damping and stiffness coefficients K_2 and B_2 is of great importance, when we design a manipulator with a flexible element connecting the fence with the drive system. These parameters should ensure that the maximal deflection of the fence referred to its equilibrium position would not exceed the permitted value, which results from the manipulator constructional constraints, requirements of scraping all the loads from the conveyor belt, and the possibility that the highest impact energy dissipation may occur. For this purpose, the kind and values of constraints superimposed on the fence must be defined, and their influence on the fence movement must be taken into account.

The fence is put in working motion through a drive system by a kinematic constraint (Fig. 4a). Constraints loads imposed on the fence result from the requirements concerning realisation of the continuous sorting process. Having in mind the experiments of numerical simulation of the model of load scraping process that utilises the active fence, described in previous publications by the authors ([12], [13], [14]), it is now accepted that the sorting process can be modelled as a sequence of characteristic discrete states of the load:

- S1 – oblique load and fence impact and rotation of the load around the corner brushing against the fence,
- S2 – load free motion,
- S3 – load motion along the fence,
- S4 – scraping the load placed closest to the fence rotation axis (one of load corners moves onto the belt conveyor band, and another one – along the fence).

During model simulation of the sorting process, the quantity and order of the realized characteristic states depends on the initial parameters of the sorting process (arrangement of the load on the conveyor at the moment of the fence activation, conveyor belt velocity v, fence work cycle time t_c) and the fulfilment of conditions controlling the load position.

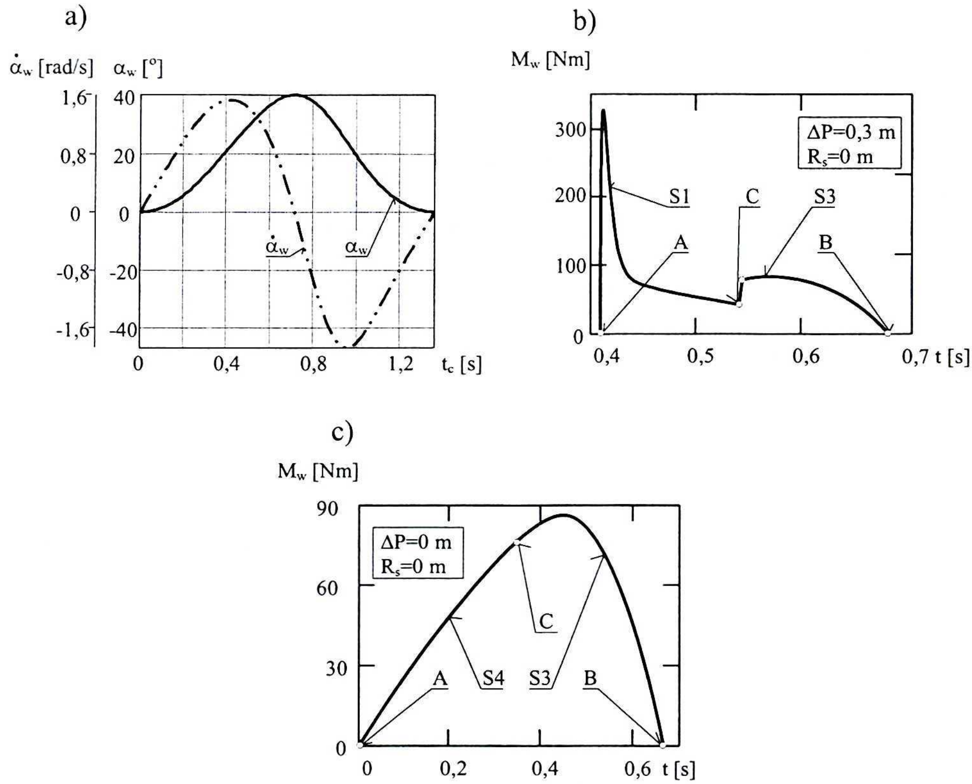

Fig. 4. Examples of the fence input signals: a) the drive system kinematic, b) and c) dynamic action of the manipulated object; A, B – initiation and loss of the load contact with the fence, C – point separating the sorting process stages, S1÷S4 – sorting process stages; sorting process parameters: t_c = 1,36 s, v = 2 m/s, axb = 0,7×0,1 m, m_1 = 15 kg, m_2 = 5 kg

On the basis of observation of the sorting process, performed on a research stand, we can conclude that the fence load can take its course according to two main scenarios:

- if the load is distant from the rotation axis ($\Delta P > 0$), the fence is subjected to constraints caused by its collision with the load and brushing of the load corner against the fence (S1 stage) as well as by the load movement along the fence towards the slide (S3 stage) – Fig. 4b,
- if, while being transported, the load is at the closest distance from the rotation axis ($\Delta P = 0$, $\phi = 0$), the fence is loaded by the moment occurring during the load unit impactless slide along the fence (S4 and S3 stages) – Fig. 4c.

The examples of the fence loads exerted by the manipulated object, presented in Fig. 4b÷c, were determined for the assumed extreme-loading working conditions of the manipulator (m_l = 15 kg, t_c = 1,36 s, v = 2 m/s and B_2 = 72 Nms/rad, K_2 = 570 Nm/rad).

In the stages S2÷S4, the manipulated loads are treated as undeformable bodies. In the case of collision of the load with the fence (S1 stage) these are treated as a body with nonlinear springy-damping properties. Loads represented by undeformable bodies apply to the manipulator work stages in which the fence acts on the load in an impactless way. The possible load deformation in these stages of movement should not have significant influence on the main course of the moving objects. Mathematical description of the proposed sorting process stages is presented in the Appendix to this paper.

Physical phenomena predominant in the load sorting process are dry slide friction and inelastic oblique impact of the bodies. The dry friction phenomenon in translatory and flat motions is modelled as Coulomb friction according to the instructions proposed in the works [7], [8]. A modified nonlinear Kelvin model was used for the description of the impact phenomenon, as it was discussed in the previous part of the paper.

4. Model analysis

4.1. Assessment of the object overloads during sorting and free fall

For the assessment of overloads exerted on the sorted object during its impact against the fence, the impact process was arranged in such a way that maximum dynamic interactions could be generated. Before the impact, the load on the conveyor was positioned and oriented so as to contact the fence with that part which was the most distant one from the fence rotation axis, and the load gravity centre C_s placed on the impact normal. In such an arrangement of the load in relation to the fence (gravity centre on the impact normal), the mathematical form of the maximum impact force is the same as for the impact of a material point against an obstacle. The loss of the load kinetic energy in favour of rotary motion is then avoided.

Investigations of the impact phenomenon were focused on the occurrence of maximum dynamic interaction, in order to relate them to the permitted loads (accepted in this paper as overloads reached during the load free fall from the height H = 0.3 m on an undeformable surface) and to be able to assess the influence of the manipulator structure and exploitation parameters on the level of overloads exerted on the sorted load units.

Fig. 5a shows the results of numerical experiments on the object overloads during sorting (oblique lines: 1 ,2, 3) and during its free fall on an undeformable surface (horizontal, dashed lines). The results pertain to the previously accepted settings. Dynamic action is expressed by a non-dimensional force – normalized in relation to the permitted overloads determined during free fall of the object from the height H = 0.3 m.

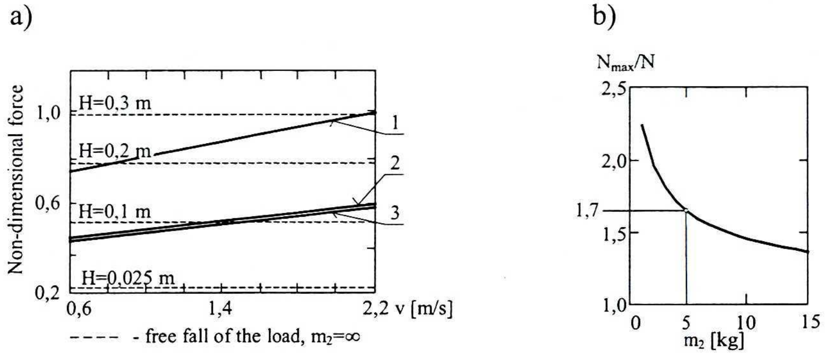

Fig. 5. Dynamic interactions of the load: a) during sorting with the fence and during a free fall, b) quotient of maximum force values N_{max} ($I_2 = \infty$) and N ($I_2 = f(m_2)$) reached during sorting with the fence; the fence work cycle time t_c = 1,36 s; (1) - $m_2 = \infty$, (2) – m_2 = 5 kg, B_2 = 72 Nms/rad, K_2 = 570 Nm/rad, (3) – m_2 = 5 kg, B_2 = 0, K_2 = 0

Fig. 5b shows the quotient of maximum values of forces occurring during the load impact (the load is treated as a material point) against the fence for the case when the fence mass is infinitely large ($I_2 = \infty$) and its moment of mass inertia is the function of mass (($I_2 = m_2 l^2/3$, l = 1.2 m – the fence length), and the fence can freely move during the impact ($B_2 = 0$, $K_2 = 0$). The value of inertia mass moment has a significant influence on the dynamic overloads exerted on the manipulated loads – the lower the fence inertia moment, the lower the overloads. For example, if we accept the arm mass m_2 = 5 kg, the dynamic interactions appearing in the load can be 1.7 times lower (if the extreme load sorting case is accepted, i.e.: m_1 = 15 kg, R = 1.2 m, v = 2 m/s). It means that the level of dynamic loads can be decreased (for the examined sorting parameter ranges: the scraper work cycle time t_c and the belt conveyor velocity v, Fig. 5a) to the level of overloads reached during a free fall from the height of about H = 0.1 m. These results refer, however, to an impact against a free fence – i.e. a fence with disconnected drive when its rotary motion during the impact against the load is not constrained by the drive system. Dynamic interactions generated in such conditions in the load determine the 'idealized' overload level. When the impact relative velocity has the normal direction, one can not achieve any reduction of these overloads for the given configuration of bodies by modifying the damping B_2 and stiffness K_2 coefficients of the fence drive.

Introduction of an element with maximal springy-damping parameters (B_2 = 72 Nms/rad, K_2 = 570 Nm/rad – defined in the further parts of the paper) into the fence drive system results only in a slight increase of maximal dynamic interactions existing in the manipulated objects during the impact as compared with a free fence application (Fig. 5a, Fig. 3b(2), (Fig. 3b(3)).

4.2. Discussion of the influence of the fence external load and the drive system springy-damping features on the load units sorting process course

The ability of the springy-damping element to dissipate energy improves when the value of the system relative damping, described by $\Psi = B_2/\left(2\sqrt{K_2 I_2}\right)$ [5], becomes greater. However, if the relative damping exceeds unity (the so-called critical damping value), the system does not return to the equilibrium position after deflection. If a series of the fence operations is assumed, the optimal relative damping value should approach and not exceed one. The fence should be in a neutral initial position before it starts its work cycle. Such a position of the fence releases the presently realized sorting process course from the influence of previous work cycles and provides good conditions for undisturbed load flow in the case of transporting them to manipulators placed farther on the conveyor.

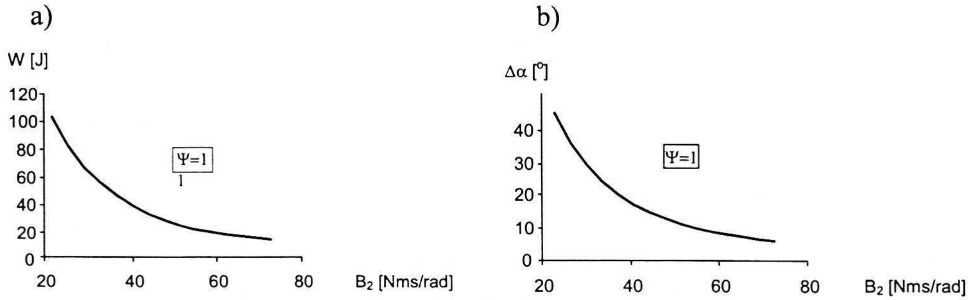

Fig. 6. Properties of the springy-damping system: a) energy absorbed by damping system, b) maximal deflection of the fence from the equilibrium position

In Fig. 6a the influence of damping coefficient B_2 on the level of dissipated energy has been presented. Because of the level of energy dissipation (for the accepted Ψ = 1, Fig. 6a) possibly the lowest values of damping coefficient B_2 should be chosen. On the other hand, increasingly smaller values of damping coefficient result in bigger (as for the same constraints) relative fence deflections $\Delta\alpha$ – Fig. 6b. Thus, it can be said that the proper choice of damping coefficient ultimately depends on the value of permitted relative deflection of the fence $\Delta\alpha_{max}$ liable to the manipulator construction constraints and the requirement of the sorting process reliable course.

Assessment of the choice of properties for the springy-damping element with respect to constructional criteria of the manipulator can be made on the basis of an analysis of graphs from Fig. 7. They show the fence complete responses to the input signals coming from its kinematic constraints (Fig 4a) and the sorting process defined by loads in Fig. 4b and Fig. 4c, respectively.

The condition of the sorting process reliability demands for all the loads to be correctly and safely guided to proper slides – new transport directions. The most difficult problems are caused by loads with boundary dimensions (in the analyzed example: 0.2×0.1 m and 0.7×0.1 m – dimensions in the project onto the conveyor belt surface) when before the scraping they are placed in extreme positions on the conveyor belt, i.e. if:

- the load with the smallest boundary dimensions (0.2×0.1 m) is transported in the longest distance from the fence rotation axis,
- the load with boundary dimensions (0.7×0.1 m) is transported in the shortest distance from the fence rotation axle ($\Delta P = 0$, $\phi = 0$).

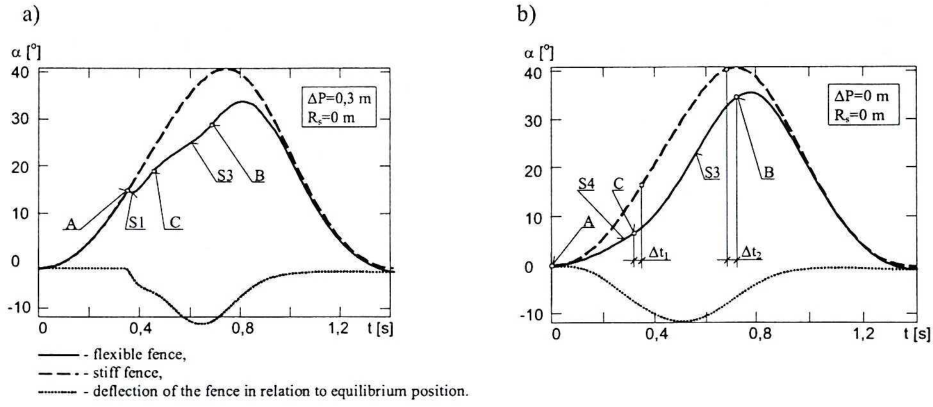

Fig. 7. The fence response to: a) load according to Fig. 4b, b) load according to Fig. 4c; A, B – respectively initiation and loss of the load and fence contact, C – point separating the sorting process stages, springy-damping element parameters: B_2 = 72 Nms/rad, K_2 = 570 Nm/rad

The choice of parameters for the sorting process (t_c, v, R_s) should account for both cases of the load unfavourable positions. Determination of these parameters can be performed according to the following scheme. First, parameters for successful scraping for the first unfavourable load position are determined (with boundary dimensions 0.2×0.1 m), then it is checked if these parameters allow sorting loads from the second case. For a fence stiffly connected with the drive system [13], it was determined that for such loads the sorting process optimal parameters (not exceeding permissible overloads during the load free fall from the height H = 0.3 m, assuming the conveyor belt width P_{max} = 0.7 m) should be: t_c = 1.36 s, v = 2 m/s, R_s = 0 m. Then, for these parameters a simulation of sorting with the use of a fence supplied with a drive of springy-damping characteristics was performed, as well.

On the basis of the performed numerical experiments, it was established that despite different characteristics of the fence movement (caused by a possibility of deflection from the equilibrium position) is possible to satisfy the condition of the load and fence contact occurrence for particular load positions on the whole accessible width of the conveyor belt, and load units can be successfully scraped to the proper slides. The flexible element parameters (B_2 = 72 Nms/rad, K_2 = 570 Nm/rad) that satisfy the sorting process reliability condition were established by the trial-error method.

While the loads are scraped with the use of a flexible fence, they remain in contact with it for a longer time (time Δt_2 – Fig. 7b) then are transported toward the slide for a longer time. In the case of scraping a load positioned near the fence rotation axis (second case of the load unfavourable position: ΔP = 0, R_s = 0, ϕ = 0 – Fig. 7b, Fig. 8), the load enters the fence faster, and it earlier finds itself in its operation area (by the time Δt_1 – Fig. 7b), so the next stage of the process – the S3 stage – is accelerated. The essential difference between the scraped load motion paths obtained with the use of a stiff fence and that related to a flexible fence is different displacement of the load in the direction along the conveyor belt (Fig. 8). However, from the point of view of the sorting process reliability, the most important thing is to make it possible for the loads to cover the whole belt conveyor width P_{max}.

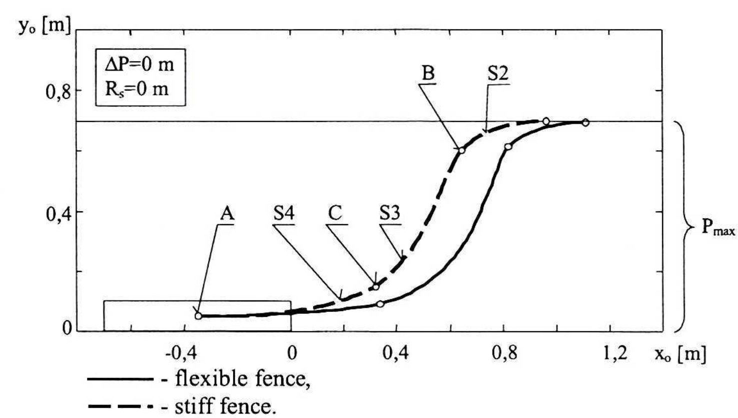

Fig. 8. The load motion path; parameters of the sorting process: t_c = 1.36 s, v = 2 m/s, axb = 0.7×0.1 m, m_1 = 15 kg, m_2 = 5 kg, B_2 = 72 Nms/rad, K_2 = 570 Nm/rad

5. Conclusion

In this paper we proposed a method of modelling of load unit stream sorting process with the use of a stiff active fence clutched with the drive system by a flexible element. The analysis of the elaborated model shows that a properly chosen flexible element makes it possible to achieve significant mitigation of dynamic interactions exerted on the sorted load units and guarantees that the sorting process reliability is maintained. In the result of introduction of a springy-damping element, the fence can deflect from the equilibrium position (due to interactions in the manipulated object) and does not follow precisely the path that would result from kinematic constraint in the drive system. The change of the fence motion characteristics leads to an increase of the load and fence contact time in comparison to that of a manipulator without flexibility features. Thanks to this effect, the load is displaced sufficiently far in the direction transverse to the conveyor axis, which makes it possible for the manipulated object to reach the proper slide, even if the fence have not reached its full working deflection.

APPENDIX

the load motion characteristic states and basic equations of the mathematical model

Analytic equations defining the load motion during the sorting process are determined on the basis laws of equilibrium of forces (d'Ambert's law – S1, S2 and S4 stages and Lagrange's of II kind – S3 stage). In the case when the equation systems created on the basis of the d'Ambert's law had fewer equations that unknown quantities – the missing equations were completed by building equations of bilateral geometrical constrains (S4 stage). For determination of the physical model equations, two reference systems were accepted: the main (immovable) Ox_oy_o connected with the manipulator frame, and an auxiliary (movable) Oxy connected with the fence. Moreover, during the process of object and fence impact analysis (Sl stage), the object motion was described by polar coordinates: the object gravity centre position C_s was described by a radius-vector r and polar angle α_p. Anticlockwise direction was assumed as the positive turn direction.

S1 – oblique load impact against the fence and rotation of the load around the corner brushing against the fence

During the inelastic oblique impact process, the fence makes a rotary motion and the load a flat one (Fig. 9). Taking into consideration accepted

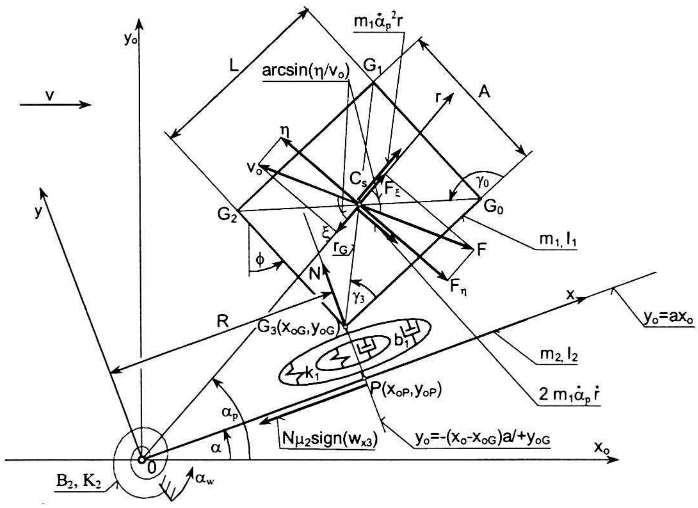

Fig. 9. Scheme of forces acting on the load during the impact against an active fence and turning the load around its corner brushing against the fence

assumptions, one can describe the load and fence impact process (according to Fig. 9) by the following equations:

$$\begin{cases} m_1\ddot{r} = F_\xi - N\left(\mu_2 \mathrm{sign}(w_{xj})\cos(\alpha_p - \alpha) - \sin(\alpha_p - \alpha)\right) + m_1\dot{\alpha}_p^2 r \\ m_1 r\ddot{\alpha}_p = N\left(\mu_2 \mathrm{sign}(w_{xj})\sin(\alpha_p - \alpha) + \cos(\alpha_p - \alpha)\right) - F_\eta - 2m_1\dot{\alpha}_p\dot{r} \\ I_1\ddot{\phi} = -r_G N\left[\cos\left(\gamma_j - \alpha + \phi\right) + \mu_2 \mathrm{sign}(w_{xj})\sin\left(\gamma_j - \alpha + \phi\right)\right] - M\ \mathrm{sign}(\dot{\phi}) \\ I_2\ddot{\alpha} = B_2(\dot{\alpha}_w - \dot{\alpha}) + K_2(\alpha_w - \alpha) - NR \end{cases} \tag{1}$$

where: $j = 0,1,2,3$ – number of the corner contacting the fence.

The force of interaction N between the load and the fence in the normal direction of impact that appears in the equation system (1) is a sum of elasticity and damping forces:

$$N = q(m_1 k_1)^{0,5}(D)^{0,25}(\dot{D}) + k_1(D)^{1,5} \tag{2}$$

The load deformation in the contact point G_j (in the impact normal direction) appearing in equation (2) is the shortest distance between the fence arm and the load corner (according to Fig. 9, point G_3). This distance is determined by a segment connecting points G_j & P:

$$D = \sqrt{(x_{oG} - x_{oP})^2 + (y_{oG} - y_{oP})^2} \tag{3}$$

Point P is at the crossing of two straight lines:

- straight line determining the fence position (the fence direction):

$$y_o = ax_o, \text{ where: } a = \mathrm{tg}\alpha \tag{4}$$

- straight line perpendicular to the fence and passing through point the G_j:

$$y_o = -(x_o - x_{oG})/a + y_{oG} \tag{5}$$

The load displacement velocity arising during impact (equation (2)) is expressed as follows:

$$\dot{D} = \frac{d}{dt}(D). \tag{6}$$

Depending on which of the load G_j corners is in contact with the fence, the load gravity centre angle γ_j can be described by the following formula (with the assumption of a rectangular load cross-section and density homogeneously distributed in its volume):

$$\gamma_j = \begin{cases} \mathrm{arctg}\dfrac{L}{A} & j = 1,3 \\ \pi - \mathrm{arctg}\dfrac{L}{A} & \text{otherwise} \end{cases} \tag{7}$$

The radius-vector r_G linking the corner G_j with the load gravity centre C_s equals a half of the load diagonal (assuming uniform distribution of the load density).

The distance between the corner G_j and the fence rotation axis is expressed by equation:

$$R = r\cos(\alpha_p - \alpha) - r_G\cos(\gamma_j - \alpha + \phi) \tag{8}$$

The components of the load friction against the conveyor belt F_ξ i F_η appearing in equation system (1), as well as the friction moment M in the flat motion of the load can be determined, for example, on the basis of models proposed in [7], [8]:

$$F_\xi = \frac{\xi F_{max}}{v_o\sqrt{1 + \left(\frac{M_{max}\dot{\phi}}{F_{max}v_o}\right)^2}},\ F_\eta = \frac{\eta F_{max}}{v_o\sqrt{1 + \left(\frac{M_{max}\dot{\phi}}{F_{max}v_o}\right)^2}},\ M = \frac{M_{max}}{\sqrt{1 + \left(\frac{F_{max}v_o}{M_{max}\dot{\phi}}\right)^2}} \tag{9}$$

where:

$F_{max} = m_1 g \mu_1$, $M_{max} = \frac{mg}{S}\mu_1 \int_S \rho \, ds$ – maximal friction force and maximal friction moment occurring in the cases of pure translatory and rotary motion of the load, respectively,

S – the load and conveyor belt contact area,

ds – elementary friction area,

ρ – distance between the elementary friction area and the load gravity centre,

$\xi = v \cos \alpha_p - \dot{r}$, $\eta = v \sin \alpha_p + r\dot{\alpha}_p$ – components of the load slide velocity in relation to the conveyor belt,

moreover:

$w_{xj} = \dot{r}\cos(\alpha_p - \alpha) + r_G \dot{\phi} \sin(\gamma_j - \alpha + \phi)$ – the load corner slide velocity in relation to the fence.

S2 – load free motion

After the interaction forces between the load and the fence disappear (when the load is still in the operation area of the fence), the load drifts on the conveyor belt and makes free flat motion described by equations (according to Fig. 10):

$$\begin{cases} m_1 \ddot{x}_o = F_\xi \\ m_1 \ddot{y}_o = -F_\eta \\ I_1 \ddot{\phi} = -M \, \mathrm{sign}(\dot{\phi}) \end{cases} \tag{10}$$

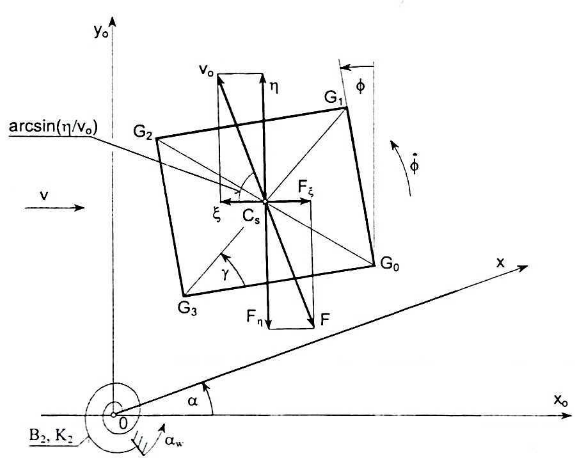

Fig. 10. Scheme of forces acting on the load in its free motion

The fence free from interactions from the manipulated object realizes its motion:

$$I_2\ddot{\alpha} = B_2(\dot{\alpha}_w - \dot{\alpha}) + K_2(\alpha_w - \alpha) \tag{11}$$

The forces of friction (F_x i F_y) and friction moment (M) components of the load slide velocity in relation to the conveyor belt, which appear in the equations take the form:

$$\xi = v - \dot{x}_o, \quad \eta = \dot{y}_o \tag{12}$$

S3 – load motion along the fence

Lagrange's equation of II kind was applied for the description of the load motion along the fence (characterized by two degrees of freedom):

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} = Q_i \tag{13}$$

Kinetic energy of the load motion (necessary for determination of Lagrange's equations of II kind, according to Fig. 11) takes the form:

$$E_k = \frac{1}{2}m_1\left[\dot{x}^2 + \left(R^2 + i_1^2\right)\dot{\alpha}^2\right] \tag{14}$$

where:
i – the arm of load mass inertia moment.

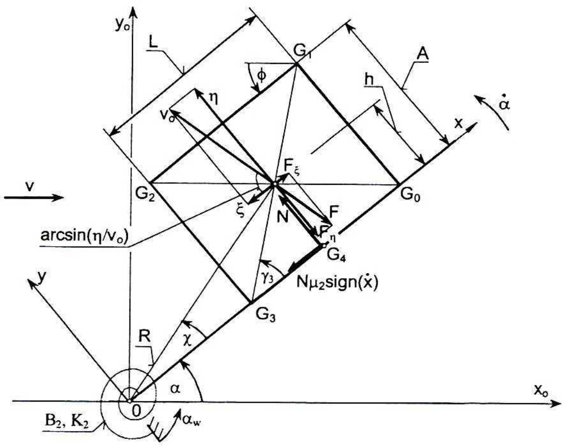

Fig. 11. Scheme of forces acting on the load moving along the fence

Lagrange's equation components:

- for a generalized coordinate x:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}}\right) = m_1\ddot{x},\ \frac{\partial E_k}{\partial \dot{x}} = m_1\dot{x},\ \frac{\partial E_k}{\partial x} = m_1 x\dot{\alpha}^2,\ Q_x = F_\xi - N\mu_2 \mathrm{sign}(\dot{x})$$

- for the generalized coordinate α:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\alpha}}\right) = 2m_1 x\dot{x}\dot{\alpha} + m_1\left(x^2 + h^2 + i_1^2\right)\ddot{\alpha},\ \frac{\partial E_k}{\partial \dot{\alpha}} = m_1\left(x^2 + h^2 + i_1^2\right)\dot{\alpha}\,,$$

$$\frac{\partial E_k}{\partial \alpha} = 0,\ Q_\alpha = (N - F_y)x.$$

After ordering the components of Lagrange's equations and taking into account the fence movement, one obtains a system of two dynamic equations formed in the following way:

$$\begin{cases} m_1\ddot{x} = F_\xi + m_1 x\dot{\alpha}^2 - N\mu_2 \mathrm{sign}(\dot{x}) \\ I_2\ddot{\alpha} = B_2(\dot{\alpha}_w - \dot{\alpha}) + K_2(\alpha_w - \alpha) - Nx \end{cases} \tag{15}$$

where:

$$N = 2m_1\dot{x}\dot{\alpha} + m_1\ddot{\alpha}\left(x + \frac{h^2 + i_1^2}{x}\right) + F_\eta,\ \xi = v\cos\alpha + \dot{\alpha}R\sin\chi - \dot{x},$$

$$\eta = v\sin\alpha + \dot{\alpha}R\cos\chi.$$

S4 – scraping the load placed closest to the fence rotation axis (one of load corners moves onto the belt conveyor band, and another one – along the fence)

In the case when a load unit transported by a belt conveyor is positioned right near its edge (on the side where the fence is fixed), one can encounter the situation, in which the load brushes against the conveyor band with one of its corners, and against the fence – with another one. Separation of the load from the stream is realised due to impactless slide of the load along the fence. The equilibrium equation system of forces and moments acting on the load in this stage of sorting (according to Fig. 12) can be presented as follows:

$$\begin{cases} m_1\ddot{x}_o = F_\xi - N_{G_0}\left(\mu_2 \mathrm{sign}(w_{xG_0})\cos\alpha + \sin\alpha\right) - N_{G_3}\mu_2 \mathrm{sign}(w_{xG_3}) \\ m_1\ddot{y}_o = -F_\eta - N_{G_0}\left(\mu_2 \mathrm{sign}(w_{xG_0})\sin\alpha - \cos\alpha\right) + N_{G_3} \\ I_1\ddot{\phi} = r_G N_{G_0}\left[\cos\left(\alpha + \gamma_3 - \phi\right) - \mu_2 \mathrm{sign}(w_{xG_0})\sin\left(\alpha + \gamma_3 - \phi\right)\right] + \\ \quad -r_G N_{G_3}\left[\cos\left(\gamma_3 + \phi\right) + \mu_2 \mathrm{sign}(w_{xG_3})\sin\left(\gamma_3 + \phi\right)\right] - M\mathrm{sign}(\dot{\phi}) \\ I_2\ddot{\alpha} = B_2(\dot{\alpha}_w - \dot{\alpha}) + K_2(\alpha_w - \alpha) - N_{G_0}R \end{cases} \tag{16}$$

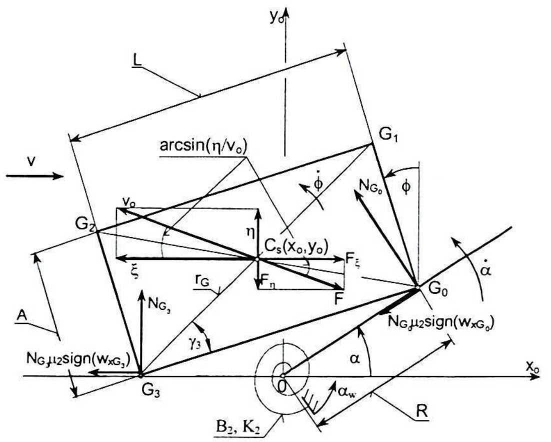

Fig. 12. Scheme of forces acting on the load brushing with one of its corners against the conveyor band and against the fence with the other one

Four equations and six unknown values – $x_o(t)$, $y_o(t)$, $\phi(t)$, $\alpha(t)$, $N_{G0}(t)$, $N_{G3}(t)$ appear in the equation system. Two missing equations are determined by double differentiation of geometrical constrain equations which define the load gravity centre co-ordinates, x_o and y_o:

$$\Phi_1 = x_o - L\frac{\sin\phi}{\sin\alpha}\cos\alpha + r_G\cos(\gamma_3 - \phi) = 0 \tag{17}$$

$$\Phi_2 = y_o - r_G\frac{c}{2}\sin(\gamma_3 + \phi) = 0 \tag{18}$$

moreover:

$$\eta = \dot{y}_o, \quad \xi = v - \dot{x}_o,$$

$$w_{xG_0} = \dot{y}_o\sin\alpha + \dot{x}_o\cos\alpha + r_G\dot{\phi}\sin(\alpha + \gamma_3 - \phi), \quad w_{xG_3} = \dot{x}_o + r_G\dot{\phi}\sin(\gamma_3 + \phi).$$

Manuscript received by Editorial Board, February 22, 2006;
final version, December 20, 2006.

REFERENCES

[1] Akella S., Huang W. H., Lynch K. M., Mason M. T.: Parts feeding on a conveyor with a one joint robot. Algorithmica v 26, 2000, pp. 313÷344.
[2] Akella S., Mason M. T.: Posing polygonal objects in the plane by pushing. The International Journal of Robotics Research, v 17, 1998, pp. 70÷88.
[3] Berretty R. P., Goldberg K. Y., Overmars M. H., Stappen A. F.: Computing fence designs for orienting parts. Computational Geometry, vol. 10 Elsevier Science, 1998, pp. 249÷262.

[4] Berretty R. P., Overmars M. H., Stappen A. F.: Orienting polyhedral parts by pushing. Computational Geometry, v 21, 2002, pp. 21÷38.
[5] Cannon R. H.: Dynamics of Physical Systems. McGraw-Hill, 1967.
[6] Gilardi G., Sharf I.: Literature survey of contact dynamics modelling. Mechanism and machine theory. 37, 2002, pp. 1213÷1239.
[7] Goyal S., Ruina A., Papadopoulos J.: Planar sliding with dry friction. Part 1. Limit surface and moment function. Wear v 143, 1991, pp. 307÷330.
[8] Goyal S., Ruina A., Papadopoulos J.: Planar sliding with dry friction. Part 2. Dynamics of motion. Wear, v 143, 1991, pp. 331÷352.
[9] Huang H., Dallimore M. P., Pan J., McCormick P.G.: An investigation of the effect of powder on the impact characteristics between a ball and a plate using free falling experiments. Materials science & engineering, A241, 1998, pp. 38÷47.
[10] Mason M. T.: Mechanics and planning of manipulator pushing operations. The International Journal of Robotics Research, v 5, 1986, pp. 53÷71.
[11] Mason M. T.: Progress in nonprehensile manipulation. The International Journal of Robotics Research, v 18, 1999, pp. 1129÷1141.
[12] Piatkowski T., Sempruch J.: Model and Analysis of Selected Features of Scraping Process. Tenth World Congress on the Theory of Machines and Mechanisms, Oulu 1999, pp. 159÷168.
[13] Piątkowski T., Sempruch J.: Sorting process of load units – dynamic model of scraping process. The Archive of Mechanical Engineering, v 49, 2002, pp. 23÷46.
[14] Piątkowski T., Sempruch J:. Modelling of curvilinear scraper arm geometry. The Archive of Mechanical Engineering, vol. LII, 2005, pp. 221÷245.
[15] Rajalingham C., Rakheja S.: Analysis of impact force variation during collision of two bodies using a single-degree-of-freedom system model. Journal of Sound and Vibration, v 229(4), 2000, pp. 823÷835.
[16] Piątkowski T.: Aktywna zastawa obrotowa z napędem pneumatycznym. Pneumatyka, 4(47) 2004, s. 24÷27.
[17] Salvarinov A., Payandeh S.: Flexible part feeder: Manipulating parts on Conveyer Belt by Active Fence. IEEE International Conference on Robotics and Automation, v 1, 1998, pp. 863÷868.
[18] Stappen A. F., Goldberg K., Overmars M. H.: Geometric eccentricity and the complexity of manipulation plans. Algorithmica v 26, 2000, pp. 494÷514.
[19] Stronge W. J.: Unravelling paradoxical theories for rigid body collisions. ASME Journal of Applied Mechanics, v 58, 1991, pp. 1049÷1055.
[20] Zhang D., Whiten W. J.: The calculation of contact forces between particles using spring and damping models. Powder technology, v 5, 1996, pp. 59÷64.
[21] PN-O-79100/02:1992, Complete filled transport packages, Quantitative data, (PL).

Zastawa aktywna z elementem podatnym w układzie napędowym

Streszczenie

W artykule przedstawiono analizę dynamiczną procesu sortowania ładunków jednostkowych transportowanych na przenośnikach taśmowych realizowanego przy pomocy zastawy aktywnej o jednym stopniu swobody. Podczas badań analitycznych manipulowane ładunki traktowano jako ciała nieodkształcalne bądź w przypadku zderzenia ładunku z zastawą jako ciała o nieliniowych właściwościach sprężysto-tłumiących. W napędzie zastawy stanowiącej sztywną i obrotowo zamocowaną belkę zastosowano element podatny opisany liniowym modelem Kelvina. Odpowiednie kształtowanie parametrów konstrukcyjnych tego elementu pozwala wpływać na złagodzenie oddziaływań dynamicznych wywieranych na sortowane ładunki.