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# APPLICATION OF OPTIMISATION TO SCALING OF THE MATHEMATICAL MODEL OF THE WORKING CYCLE OF CLENGINE

The paper presents a method of choosing parameters of a mathematical model for simulation of a working cycle of compression-ignition engine on the basis of experimental measurements. In order to choose the parameters of the model, the Nelder-Mead method has been used. As a result of such an approach, a simplified mathematical model with very good numerical effectiveness can be used for simulation of the working cycle of the engine, while very good compatibility of numerical results and experimental measurements is ensured. Suitable algorithms and results of calculations are presented.

#### NOTATION

A – area.

 $B_0$  – fuel dose.

C – constant of the Wibe function,

 $c_p$  - specific heat of the medium at constant pressure,

 $c_v$  - specific heat of the medium at constant volume,

 $e_i$  – model parameter,

 $\bar{e}_i$  – approximate model parameter,

 $f_i$  – measured value of emission or soot,

 $\bar{f}_i$  – approximate value of emission or soot,

*h* - coefficient of heat exchange,

k – index.

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*m* – mass of the medium in the cylinder,

m+1 – exponent of combustion dynamic of Wibe function,

p – pressure of the medium,

R – individual gas constant of the medium,

T – temperature of the medium,

 $T_s$  – average temperature of cylinder walls,

t – time,

V - cylinder volume,W - fuel caloric value,

x - fraction of fuel dose burnt,
 x<sub>i</sub> - engine control parameter,
 y - degree of fuel dose used,
 φ - position angle of crankshaft.

 $\varphi_z$  – angle of combustion start,

 $\Delta \varphi_s$  – combustion angle,

 $\varphi_w$  - ignition advance angle,  $\mu$  - valve flow coefficient.

#### **SUBSCRIPTS**

c – cylinder, d – inlet, w – outlet

#### 1. Introduction

The aim of modelling a working cycle of engines enables us to predict engine performance without having to conduct tests, and to deduce the performance of parameters that could be difficult to measure in tests. However, because of the complexity of thermodynamic and chemical processes taking place in the cylinder, the modelling is very difficult. Based on the enormous development in computer methods and technology, simulation research into combustion engines is carried out in a lot of research centres [1], [2], [3]. The numerical models used for working cycle simulations usually involve many simplifying assumptions based on formulae and relations obtained from experimental measurements [4]. For example, the turbocharger performance and the valve flow characteristics are determined from steady-flow tests and as tables can be built into the working cycle engine model along with appropriate interpolation routines. Empirical correlation are also used for predicting the heat transfer process, the ignition delay and the burn rate.

In general, two extreme approaches to design and control engines and supply systems can be distinguished: experimental and numerical ones. Both have their

advantages and disadvantages [5], [6]. The approach based on experiments is expensive and achieving optimal solutions is very time-consuming, since every change in design involves experimental verification. The second approach, which leads to a formulation of complicated mathematical models, requires participation of research teams of specialists of mechanics, thermodynamics and computer methods. Moreover, complicated mathematical models require long computational time and their use in optimisation of the parameters is limited.

Simple mathematical models of working process have also been used [7]. Such an approach can lead to significant differences between calculation results and experimental measurements. Those differences can be minimized by an appropriate choice of flow coefficients through the valves, heat transfer coefficients and Wibe function parameters. The values of those parameters usually were taken into model directly from references: [8], [9], [10], i.e. in paper [2] where the author used the published data of heat transfer coefficient. Other approach is applied in paper [7], where an additional coefficient for heat transfer is formulated in order to obtain a better correlation between model results and measurements.

A good correspondence can be achieved by calibrating (scaling) the simplified mathematical model. This means that all model parameters have to be estimated by special methods. One of them is the regular search method used to obtain valve flow coefficients [2], [11] or Wibe function parameterisation [7], [12], however this approach is time consuming. A solution to the problem of estimating the model parameters can also be obtained by one of more sophisticated optimisation methods. In this case, the estimated parameters can be the independent variables of the optimisation task. Both methods require the experimental set of data to be given in order to evaluate the response of the model to a given set of model parameters. Calibration of the mathematical model using the experimental data and optimisation methods ensures good correspondence of calculation results and measurements.

It seems that the most reasonable option would be to formulate a simplified mathematical model, which should be numerically effective. Numerical effectiveness of the model is essential when the research is carried out in order to optimise parameters of the engine together with the supply system, especially when the model is used for controlling the engine in real time.

This paper presents an application of optimisation methods to determine parameters of the numerical model using the experimental measurements taken for a given set of control engine parameters. In order to calculate estimated parameters of the model (valves flow coefficients, heat transfer and Wibe function parameters) for any set of control parameters, an approximation task is also formulated. Such an approach enables us to reduce the number of

experimental measurements. A sort of verification of the procedure described will also be presented. The process of calibrations of the model is the main step of research concerned with emission control in real time.

#### 2. Measurements

Measurements have been carried out on a compression-ignition engine supercharged by a turbo compressor with direct injection equipped with an electronically controlled injection unit with a distributing injection pump. Basic technical parameters of the engine are presented in Table 1.

Technical data of the engine

Table 1.

Type of the engine	Compression-ignition engine with turbosupercharger, with air cooling system, exhaust gas recirculation and catalyst		
Structure / Number of cylinders	In-line / 4		
Valves per cylinder	4		
Cylinder diameter [mm]	79		
Piston stroke [mm]	86		
Cubic capacity [cm <sup>3</sup> ]	1668		
Compression ratio	18.4		
Maximum power [kW]	55 at 4400 rpm		
Maximum torque [Nm]	165 at 1800–3000 rpm		

Experimental measurements were carried out on a special test stand built in Department of Combustion Engines and Vehicles of University of Bielsko-Biała. The scheme of the test stand is presented in Fig. 1.

As a result of the experiments, the influence of the following basic control engine parameters:

- rotational speed,
- fuel dose,
- ignition advance angle,
- temperature of the medium in the suction manifold,
- pressure of the medium in the suction manifold,
- degree of exhaust gas recirculation,

on the pressure profiles in the cylinder, as well as emission of nitrogen oxide  $(NO_x)$ , carbon monoxide (CO), hydrocarbons (HC) and smoke levels (D) has been determined. Pressure profiles and emission and smoke of the exhaust gas were registered for different values of control parameters (Table 2).

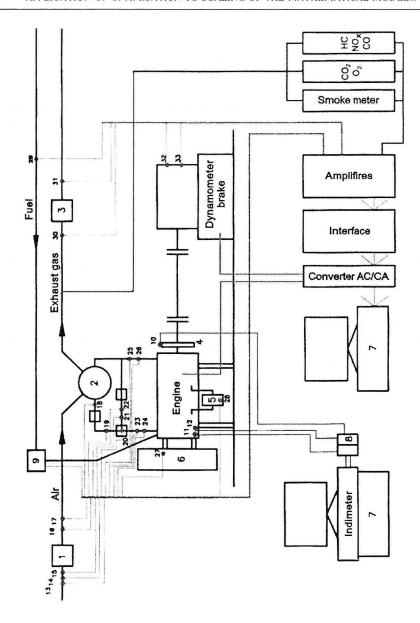


Fig. 1. Scheme of the test stand and measurement paths

1 — air filter, 2 — turbo compressor, 3 — catalyst, 4 — flywheel, 5 — air cooler, 6 — coolant cooler, 7 — PC computer, 8 — instrumentation amplifiers, 9 — fuel consumption meter, 10 — position sensor of crankshaft, 11 — pressure transducer in the cylinder, 12 — pressure transducer in the injection conductor, 13 — temperature of the environment, 14 — air pressure at the outlet, 15 — air humidity, 16 — air mass flow, 17 — air temperature at the outlet, 18 — temperature of the charged air, 19 — temperature of the air after the cooler, 20 — temperature of EGR controller, 21 — temperature of EGR, 22 — flow of EGR, 23 — temperature of the load in the suction manifold, 24 — pressure of the load in the suction manifold, 25 — temperature of the exhaust gas in the exhaust manifold, 26 — pressure of the exhaust gas in the exhaust manifold, 27 — temperature of the coolant, 28 — oil temperature, 29 — fuel temperature, 30 — exhaust gas temperature before the catalyst, 31 — exhaust gas temperature after the catalyst, 32 — rotational speed, 33 — torque

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Synthetic results of experimental measurements

No		Cont	trol param	eter	S	Registered			Profile $p(\varphi)$	
	$n(x_1)$		$\varphi_w(x_i)$	Ī	$X_{EGR}(x_6)$	$NO_{x}(f_{1})$	$CO(f_2)$	$HC(f_3)$	$D(f_4)$	
0										$p_0(\varphi)$
3										1
k										$p_k(\varphi)$
:										
m										$p_m(\varphi)$

Averaged pressure profiles p in the cylinder for given values of control parameters were written in files and then smoothed by Fourier analysis. Fig. 2 shows profile  $p \varphi$ ) registered and smoothed by the Fourier series:

$$p_F(\varphi) = a_0 + \sum_{i=1}^{n_F} \left[ a_i \cos i \frac{\varphi}{4\pi} + b_i \sin i \frac{\varphi}{4\pi} \right]$$
 (1)

where:  $n_F$  – the number of series components.

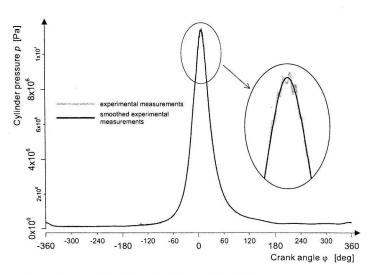


Fig. 2. Registered and smoothed ( $n_F = 50$ ) cylinder pressure profile

The results of measurements of  $f_1 
div f_p$  can be generalised by means of approximation functions. In this paper, it is assumed that quantities  $f_1 
div f_p$  are approximated by power functions using the least squares method. It has been assumed that:

$$\bar{f}_s(x_1, ..., x_r) = \delta_0^{(s)} x_1^{\delta_1^{(s)}} \cdot ... \cdot x_r^{\delta_j^{(s)}} \cdot ... \cdot x_r^{\delta_r^{(s)}}$$
(2.1)

After logarithmic transformation we obtain

$$\ln \bar{f}_s = \ln \delta_0^{(s)} + \delta_1^{(s)} \ln x_1 + \dots + \delta_r^{(s)} \ln x_r + \dots + \delta_r^{(s)} \ln x_r \tag{2.2}$$

If we denote

$$F_s^{(k)} = \ln f_s^{(k)} = \ln \left[ f_s(x_1^{(k)}, ..., x_r^{(k)}) \right]$$
 (3.1)

$$a_0^{(s)} = \ln \delta_0^{(s)}, \quad a_i^{(s)} = \delta_i^{(s)} \ (i = 1, ..., r)$$
 (3.2)

$$X_{i}^{(k)} = \begin{cases} 1 & i = 0, \\ \ln x_{i}^{(k)}, & i = 1, ..., r \end{cases}$$
 (3.3)

we can formulate the sum of the squares of residuals:

$$\Omega_s = \sum_{k=0}^m \left[ \sum_{i=0}^r a_i^{(s)} X_i^{(k)} - F_s^{(k)} \right]^2$$
 (4)

where m is the number of measurement points.

The sufficient and necessary conditions for minimising  $\Omega_s$  are as follows:

$$\frac{\partial \Omega_s}{\partial a_i^{(s)}} = 0 \quad \text{for} \quad i = 1, ..., r.$$
 (5)

Conditions (5) lead to the formulation of the system of r+1 linear algebraic equations with r+1 unknowns. Its solution allows determination of coefficients  $a_i^{(s)}(i=0,...,r)$ , and from (3) we can calculate coefficients  $\delta_i^{(s)}(i=0,...,r)$  of function  $\bar{f}_s$  approximating  $f_s$  according to (2.1).

The next figures (Figs 3, 4 and 5) present hypersurfaces obtained using the approximating procedure for s = 1, 2 and 3. Two independent (control) parameters are constant:  $x_4 = T_d = 60^{\circ}\text{C}$  and  $x_5 = p_d = 0.15$  MPa.

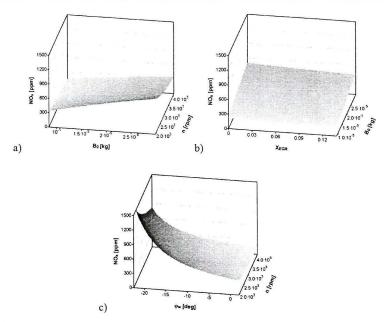


Fig. 3. The nitrogen oxide emission in dependency on a) rotational speed (n) and fuel dose ( $B_0$ ) at degree of recirculation  $X_{EGR} = 4\%$  and ignition advance angle  $\varphi_w = -11.5^\circ$ , b) degree of recirculation ( $X_{EGR}$ ) and fuel dose ( $B_0$ ) for rotational speed n = 3000 rpm and ignition advance angle  $\varphi_w = -11.5^\circ$ , c) rotational speed (n) and ignition advance angle ( $\varphi_w$ ) for fuel dose  $B_0 = 1.8 \cdot 10^{-5}$  kg and degree of recirculation  $X_{EGR} = 4\%$ 

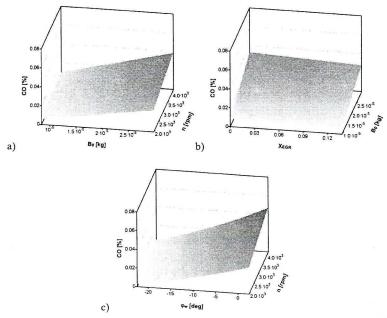


Fig. 4. The carbon monoxide emission in dependency on a) rotational speed (n) and fuel dose ( $B_0$ ) at degree of recirculation  $X_{EGR} = 4\%$  and ignition advance angle  $\varphi_w = -11.5^\circ$ , b) degree of recirculation ( $X_{EGR}$ ) and fuel dose ( $B_0$ ) for rotational speed n = 3000 rpm and ignition advance angle  $\varphi_w = -11.5^\circ$ , c) rotational speed (n) and ignition advance angle ( $\varphi_w$ ) for fuel dose  $B_0 = 1.8 \cdot 10^{-5}$  kg and degree of recirculation  $X_{EGR} = 4\%$ 

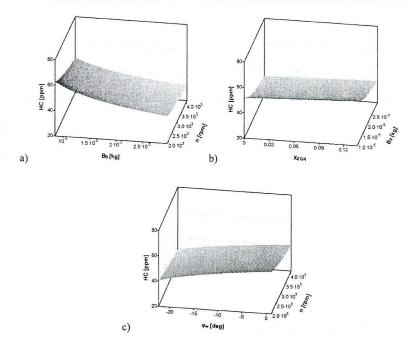


Fig. 5. The hydrocarbon emission in dependency on a) rotational speed (n) and fuel dose ( $B_0$ ) at degree of recirculation  $X_{EGR} = 4\%$  and ignition advance angle  $\varphi_w = -11.5^\circ$ , b) degree of recirculation ( $X_{EGR}$ ) and fuel dose ( $B_0$ ) for rotational speed n = 3000 rpm and ignition advance angle  $\varphi_w = -11.5^\circ$ , c) rotational speed (n) and ignition advance angle ( $\varphi_w$ ) for fuel dose  $B_0 = 1.8 \cdot 10^{-5}$  kg and degree of recirculation  $X_{EGR} = 4\%$ 

#### 3. Mathematical model

The phenomena occurring in the engine cylinder can be described by a set of partial differential equations [4]. Some research require a simple and numerically effective model, especially if the calculation time is the impact factor.

The model of working cycle of CI engine considered includes inlet and outlet processes. The flow by inlet and outlet valves was described as isotropic flow by convergent nozzle [13]. The phenomena occurring in the engine cylinder can be described by two non-linear ordinary differential equations in the following form [2], [14]:

$$\frac{dm_c}{dt} = \frac{dm_d}{dt} = \frac{dm_w}{dt} + B_0 \cdot \frac{dx}{dt}$$
 (6.1)

$$c_{vc} \cdot T_c \cdot \frac{dm_c}{dt} + c_{vv} \cdot m_c \cdot \frac{dT_c}{dt} + p_c \cdot \frac{dV}{dt} =$$

$$y \cdot B_0 \cdot W \cdot \frac{dx}{dt} + h \cdot A_c \cdot (T_s - T_c) + c_{pd} \cdot T_d \cdot \frac{dm_d}{dt} \cdot c_{pc} \cdot T_c \cdot \frac{dm_w}{dt}$$

$$(6.2)$$

Moreover, the additional relations defining quantities appearing in equations (6) are assumed according to references [2], [7], [14], [15] as follows:

$$p_c \cdot V = m_c \cdot R_c \cdot T_c \tag{7.1}$$

$$x = 1 - \exp\left[C\left(\frac{\varphi - \varphi_z}{\Delta \varphi_s}\right)^{m+1}\right]$$
 (7.2)

$$\frac{dm_d}{dt} = \mu_d \cdot A_d \cdot p_d \cdot \sqrt{\frac{2}{R_d \cdot T_d} \cdot \frac{k_d}{k_d - 1} \left(\beta_d^{\frac{2}{k_d}} - \beta_d^{\frac{k_d + 1}{k_d}}\right)} \quad (7.3)$$

$$\frac{dm_w}{dt} = \mu_w \cdot A_w \cdot p_c \cdot \sqrt{\frac{2}{R_c \cdot T_c} \cdot \frac{k_w}{k_w - 1} \left(\beta_w^{\frac{2}{k_w}} - \beta_w^{\frac{k_w + 1}{k_w}}\right)} \quad (7.4)$$

where 
$$\beta_d = \frac{p_c}{p_d} \le 1$$
,  $k_d = \frac{c_{pd}}{c_{vd}}$ ,  $\beta_w = \frac{p_w}{p_c} \le 1$ ,  $k_w = \frac{c_{pc}}{c_{vc}}$ .

In further analysis, index c will be omitted for simplification.

Having numerically integrated equations (6) and used relation (7), one can calculate the profiles of ([13], [16]):

$$\begin{cases}
m(t) \\
p(t) \\
T(t)
\end{cases}$$
(8)

Apart from control parameters, the following also characterise the model:

 $e_1$  - combustion angle  $(\Delta \varphi_s)$ ,

 $e_2$  – angle of combustion start  $(\varphi)$ ,

 $e_3$  - exponent of combustion dynamic of Wibe function (m+1),

 $e_4$  - degree of use of the fuel dose (y).

The values of parameters  $e_1 \div e_4$  can be estimated on the basis of complement analysis or approximated according to literature [4], [14]. The profiles (8) essentially depend on the values assumed for those parameters. Fig. 6 presents the influence of these parameters on pressure profiles, when the same values of parameters  $x_1 \div x_r$  are assumed.

It can be seen that the choice of values of parameters  $e_1 \div e_4$  has an essential significance for compatibility of calculation and experimental results. These parameter values have to be known for each set of control engine parameters. Values of parameters given by different authors cannot be

applied in our model. Now we will present optimisation task used for searching for values of model parameters.

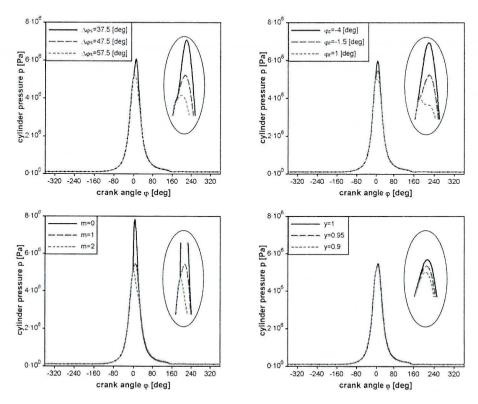


Fig. 6. Influence of values of parameters e<sub>1</sub>÷e<sub>4</sub> on pressure profiles
a) combustion angle φ<sub>z</sub> = -1.5°, m = 1, y = 1,
b) angle of combustion start, Δφ<sub>s</sub> = 47.5°, m = 1, y = 1,
c) exponent of Wibe function, φ<sub>z</sub> = -1.5°, Δφ<sub>s</sub> = 47.5°, y = 1,
d) coefficient of fuel dose use, φ<sub>z</sub> = -1.5°, Δφ<sub>s</sub> = 47.5°, m = 1

### 4. Application of optimisation

In this paper, dynamic optimisation is used in order to choose appropriate values of parameters  $e_1 \div e_4$  for each combination of variables  $x_1 \div x_r$  (according to Table 2). The approach proposed is described below.

Equations of the mathematical model (6), (7) can be symbolically written as:

$$M_i[X, E, p, V, m, T] = 0$$
 for  $i = 1, ..., l$  (9)

where:  $M_i$  – differential operator or function, l – number of equations (6) and (7),

 $X = [x_1 ... x_r]^T$  – vector of control parameters,  $E = [e_1 ... e_4]^T$  – vector of model parameters.

Let us assume that the desired values  $e_1 \div e_4$  have to fulfil the following conditions:

$$e_{i\min} \le e_i \le e_{i\max} \tag{10}$$

for i = 1, ..., 4 where  $e_{i\min}$ ,  $e_{i\max}$  are minimal and maximal permissible values respectively. Values  $e_{i\min}$  and  $e_{i\max}$  can be determined using technical premises of the modelled system or on the basis of data in the literature [4], [14].

The optimisation task is to choose values  $e_1 \div e_4$  in order to minimise the expression:

$$\Omega(X, E) = \int_{0}^{4\pi} \left[ p_E(\varphi) - p_F(\varphi) \right]^2 d\varphi \tag{11}$$

where:  $p_E$  – pressure profile obtained on the basis of the mathematical model described by equations (9),

 $p_F$  – smoothed profile from experimental measurements. In the formulated problem, calculation of the objective function  $\Omega$  each time requires integration of equations (9). As a result of optimisation, values  $e_1 \div e_4$  should ensure the minimal difference between the measured and calculated pressure profile in the cylinder. A procedure ensuring proper mass charge in the cylinder has been applied. According to (7.1), an error of the temperature calculated depends on the error of the pressure and mass in the cylinder.

Since the presented mathematical model is numerically effective [16] (time of integration of equations (6) using the Runge-Kutta method for a single set of parameters does not exceed 0.5 s on IBM PC – 2.4 GHz), any classical optimisation method [17] can be used for solving the task (11). In this paper, the downhill simplex method (Nelder-Mead) [18] is used. Fig. 7 presents some results of calculations; the following notations are used:  $p_F$ : solid line – smoothed measured profile,  $p_E^{(beg)}$ : broken line – calculated profile  $p(\varphi)$  for initial values  $e_1 \div e_4$ ,  $p_E^{(opt)}$ : broken point line – calculated profile  $p(\varphi)$  for values  $e_1 \div e_4$  obtained from the optimisation task.

Since calculation times of optimal values of parameters  $e_1 \div e_4$  for each set of control parameters  $x_1 \div x_r$  do not exceed 10–15s on IBM PC – 2.4 GHz, values  $e_1 \div e_4$  have been determined on the basis of ca. 300 sets of control parameters  $x_1 \div x_r$ . The procedure of choosing parameters  $e_1 \div e_4$  of the

mathematical model is called scaling of the model. The results are presented in the form of Table 3.

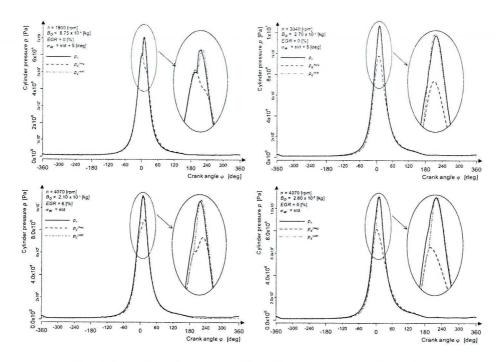


Fig. 7. Comparison of pressure profiles of the medium in the cylinder for:

- a) rotational speed n = 1800 rpm, fuel dose  $B_0 = 8.75 \cdot 10^{-6}$  kg, degree of recirculation  $X_{EGR} = 0\%$  and accelerated advanced injection angle,
- b) rotational speed n = 3340 rpm, fuel dose  $B_0 = 2.70 \cdot 10^{-5}$  kg, degree of recirculation  $X_{EGR} = 0\%$  and accelerated advanced injection angle,
- c) rotational speed n = 4070 rpm, fuel dose  $B_0 = 2.10 \cdot 10^{-6}$  kg, degree of recirculation  $X_{EGR} = 6\%$  and standard advanced injection angle,
- d) rotational speed n = 4070 rpm, fuel dose  $B_0 = 2.80 \cdot 10^{-6}$  kg, degree of recirculation  $X_{EGR} = 0\%$  and standard advanced injection angle

Synthetic scaling results

Table 3.

No	Control parameters Optimal parame		imal paramet	ters of the model				
No	$n(x_1)$		$\varphi_w(x_i)$	 $X_{EGR}(x_6)$	$e_1$	e2	e 3	e 4
0								
:								
k								
:								
m								

After the scaling process (having determined optimal values of parameters  $e_1 \div e_4$  for some set of control parameters), it is possible to define approximating functions in the following form:

$$\bar{e}_{k}(x_{1},...,x_{r}) = \delta_{0}^{(s)} x_{1}^{\delta_{1}^{(s)}} \cdot ... \cdot x_{r}^{\delta_{j}^{(s)}} \cdot ... \cdot x_{r}^{\delta_{r}^{(s)}}$$
(12)

which will be defined by means of the least squares method. This approximation enable us to calculate the values of model parameters and to model the working cycle for all possible values of engine control parameters.

The next figures (Figs. 8 and 9) present hypersurfaces obtained by approximating functions  $\bar{e}_k(x_1,...,x_r)$  for k=1 and 3 ( $e_1$  – combustion angle,  $e_3$  – exponent of combustion dynamic of Wibe function). Two independent (control) parameters are constant:  $x_4 = T_d = 60^{\circ}\text{C}$  and  $x_5 = p_d = 0.15$  MPa.

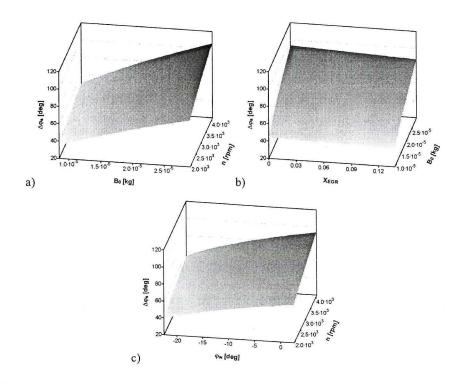


Fig. 8. The combustion angle in dependency on a) rotational speed (n) and fuel dose  $(B_0)$  at degree of recirculation  $X_{EGR} = 4\%$  and ignition advance angle  $\varphi_w = -11.5^\circ$ , b) degree of recirculation  $(X_{EGR})$  and fuel dose  $(B_0)$  for rotational speed n = 3000 rpm and ignition advance angle  $\varphi_w = -11.5^\circ$ , c) rotational speed (n) and ignition advance angle  $(\varphi_w)$  for fuel dose  $(B_0) = 1.8 \cdot 10^{-5}$  kg and degree of recirculation  $(B_0) = 1.8 \cdot 10^{-5}$  kg and  $(B_0) =$ 

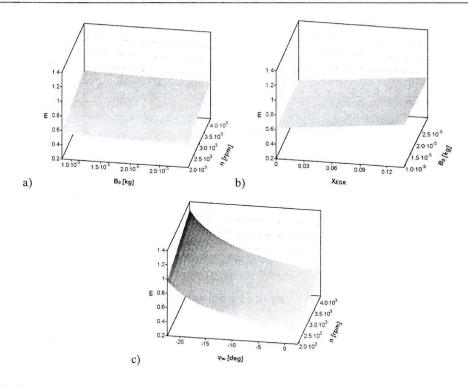


Fig. 9. The exponent of combustion dynamic of Wibe function in dependency on a) rotational speed (n) and fuel dose ( $B_0$ ) at degree of recirculation  $X_{EGR} = 4\%$  and ignition advance angle  $\varphi_w = -11.5^\circ$ , b) degree of recirculation ( $X_{EGR}$ ) and fuel dose ( $B_0$ ) for rotational speed n = 3000 rpm and ignition advance angle  $\varphi_w = -11.5^\circ$ , c) rotational speed (n) and ignition advance angle ( $\varphi_w$ ) for fuel dose  $B_0 = 1.8 \cdot 10^{-5}$  kg and degree of recirculation  $X_{EGR} = 4\%$ 

#### 5. Model validation and final remarks

The presented approach enables us to scale the mathematical model of the working cycle of engine using the results of experiments. In order to choose parameters for scaling the model, we propose an application of optimisation methods. As a result, a mathematical model is obtained for which the results of calculations are as close as possible to those of experiments.

In order to verify the calibrated model, some additional experimental measurements have been carried out. In this step, we take such values of control parameters which are different from those taken in previous measurements (see Table 1), and all model parameters are calculated according to the approximation formulae. The values of control engine parameters taken into account are presented in Table 4.

Table 4.

200	~	7.2			
Data	tor	model	1/2	100	tion

$x_1 \div x_6$	Case I	Case II
n [rpm]	2600	2600
B <sub>0</sub> [kg]	9.85 · 10 <sup>-6</sup>	1.64 · 10-5
$\varphi_w$ [deg]	-5	-7
X <sub>EGR</sub> [%]	8	8
$T_d$ [°C]	55	55
p <sub>d</sub> [hPa]	1380	1460

Comparison of pressure profiles calculated and measured are presented in Figures 10 and 11.

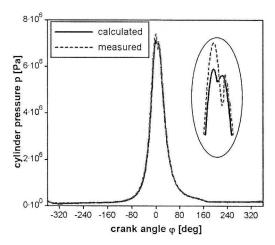


Fig. 10. Comparison of pressure profiles of the medium in the cylinder for case I

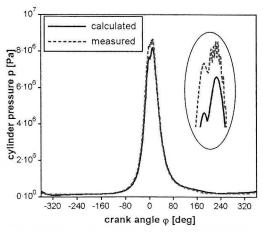


Fig. 11. Comparison of pressure profiles of the medium in the cylinder for case II

Table 5.

We can see an acceptable correlation of the pressure profiles obtained from the model and those of measurements. Table 5 presents emission, soot and mean effective pressure, maximal pressure and thermal efficiency calculated and obtained from the experiment.

Comparison of calculation results with experimental data

	Ca	ise I	Case II		
	Calculations	Measurements	Calculations	Measurements	
NO <sub>x</sub> [ppm]	153	172	296	300	
HC [ppm]	65	61	62	68	
CO [%]	0.022	0.019	0.014	0.012	
D [°]	0.7	0.6	0.6	0.5	
p <sub>i</sub> [MPa]	0.47	0.47	0.76	0.76	
ης	0.46	0.45	0.47	0.47	
p <sub>max</sub> [MPa]	7.53	7.40	8.69	8.66	

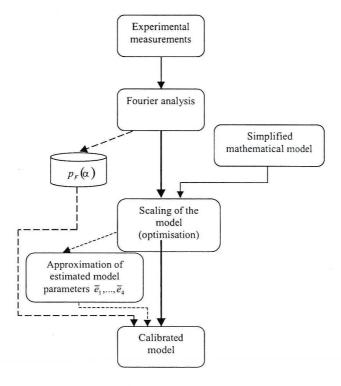


Fig. 12. General algorithm for formulation of the mathematical model of the working cycle of compression-ignition engine

The results of validation presented enable us to formulate the following conclusions:

- a limited set of measurement data enable the calibration of working cycle model,
- optimisation task can be used as an effective method for working cycle model calibration,
- the calibrated model of working cycle, apart from its simplicity, ensures good accuracy of results and it is effective numerically,
- the number of measurements considerably influences the quality of the elaborated model.

The general algorithm for formulation of the extended mathematical model of the working cycle of engine has the form as in Fig. 12.

The advantage of the obtained model is that the values of thermodynamic  $e_1 \div e_4$  parameters are determined on the basis of experimental measurements. The model, which includes thermodynamic processes, enables us to formulate the main task of the engine design i.e. the optimisation task. The result of this optimisation would be a choice of control parameters  $x_1 \div x_r$  that would ensure reduction or minimisation of emission of exhaust gas components.

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## Zastosowanie optymalizacji do skalowania modelu matematycznego cyklu roboczego silnika ZS

#### Streszczenie

W pracy przedstawiono, jak w oparciu o wyniki badań doświadczalnych silnika o zapłonie samoczynnym, można dobrać parametry modelu matematycznego pozwalającego na symulację cyklu roboczego tego silnika. Do doboru parametrów modelu (skalowania), zastosowano metodę optymalizacji Nelder-Meadsa. W wyniku tego postępowania zapewniono, iż uproszczony model matematyczny, cechujący się dużą efektywnością numeryczną, umożliwia symulację cyklu roboczego silnika, zapewniając możliwie dużą zgodność wyników obliczeń i pomiarów. Postępowanie zilustrowano odpowiednimi algorytmami i wynikami obliczeń.