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ULTIMATE LOAD OF LAMINATED PLATES SUBJECTED TO SIMULTANEOUS COMPRESSION AND SHEAR

This work deals with the analysis of elasto-plastic post-buckling state of rectangular laminated plates subjected to combined loads, such as uniform compression and shear. The plates are built of specially orthotropic symmetrical layers. The analysis is carried out on the basis of nonlinear theory of orthotropic plates involving plasticity. The solution can be obtained in the analytical-numerical way using Prandtl-Reuss equations. The preliminary results of numerical calculations are also presented in figures.

1. Introduction

In last decades, the fibrous composite materials are more widely used in manufacturing of thin-walled structures (bars, columns, plates and shells). Generally, fibrous composites are nonhomogenous and anisotropic. However, fibres are often laid in composite matrices in one or in two perpendicular directions. In such cases, the fibrous composite is usually considered as an orthotropic material .

The great advantage of composite materials consists in the fact that their material properties can be easily modelled in selected directions or regions [1]. It concerns especially orthotropic beams and plates. An important type of plate that appears as a subcomponent of advanced composite structure is the symmetrically laminated plate. The term "symmetrically laminated" refers to plates in which every lamina above the plate midplane has a corresponding lamina located at the same distance below the plate midplane, with the same thickness and material properties. In the case when the principal axes of

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orthotropy in every lamina are parallel to the plate edges, the plates are called "specially orthotropic laminated plates" [6].

Thin laminated plates are used in manufacturing of different forms of structures: columns and girders of closed and open cross-section, panels, etc. Frequently, the load carrying capacity of such structures subjected to increasing in-plane loading is determined by local buckling of component elements. In this case, the first step in understanding the behaviour of a thin-walled plate structure is to find the complete solution for an individual plate element.

In the analysis of nonlinear behaviour of thin laminated plates, it is usually assumed that the normal to the undeformed midplane remains straight and normal to the midplane after the deformation (classical laminated plate theory). The extensive review of works concerning the nonlinear analysis of composite laminated plate can be found in [2].

In recent years, many papers have been devoted to the problem of elastic and elastic/plastic buckling [8], [11] and post-buckling behaviour [12], [13] of composite plates under combined loading.

Among many methods of solution, the numerical methods (FEM) are undoubtedly the most powerful and the most general instrument for the analysis of the behaviour of thin-walled structures. However, they are rather expensive and time-consuming.

The advantage of analytical-numerical methods, in which the analytical solution of the elastic post-buckling state is used as a space base for the numerical solution in the elasto-plastic range, is very short time of computation and quite good accuracy in comparison with numerical methods.

In this work, the analytical solution describing the displacement field in the elastic postbuckling state is found out for specially orthotropic laminated plates subjected to simultaneous compression and shear. Next, the iterative approach to the problem is presented using a combination of analytical and numerical solutions based on the Rayleigh-Ritz variational method involving the relations of flow theory of plasticity.

1. Structural problem

The rectangular laminated plates subjected to uniform compression and shear, acting simultaneously, are considered (Fig. 1). The plates are simply supported and initially stress free. It is assumed that the plate edges remain straight during loading, and additionally the unloaded edges are restrained from pulling in.

The plates are built of orthotropic layers (lamina) that are symmetrically arranged about the plate middle surface. The layers are called specially orthotropic, what means that the principal axes of material orthotropy are parallel to the plate edges. In this case, neither shear or twist coupling nor bending-extension coupling exists.

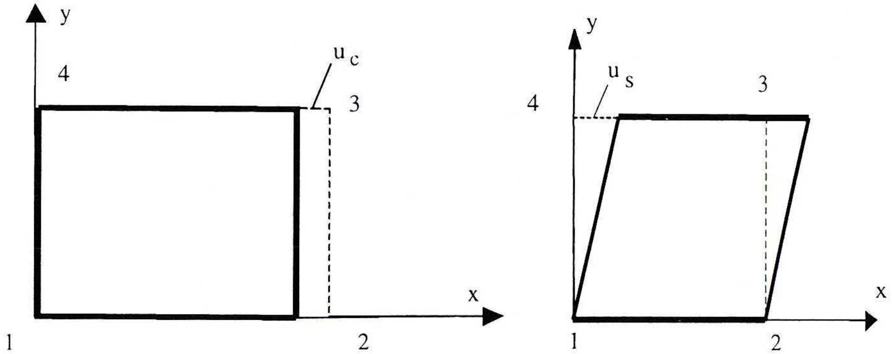

Fig. 1a. Rectangular plates subjected to compression and shear

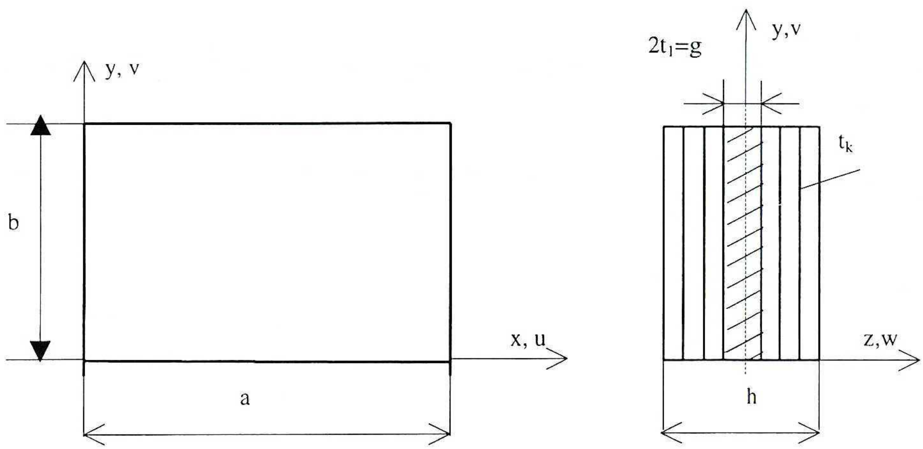

Fig. 1b. Geometry of a plate

The orthotropic material properties in the elastic range of k-th layer are determined by four independent constants: $E_k^x, \eta_k, G_k, \nu_k$, where:

$$\eta_k = \frac{E_k^y}{E_k^x}; \quad \nu_k = \nu_k^{yx}; \quad \nu_k^{xy} = \eta_k \nu_k .$$

The prebuckling displacement and stress fields of a plate are described by:
– a combination of its nodal displacements in the x-direction

$$u^o = u_c \frac{x}{a} + u_s \frac{y}{b} \tag{1}$$

– and additionally:

$$v^o = 0 \qquad \sigma_x^o = \text{const}, \qquad \sigma_y^o = \text{const}, \qquad \tau_{xy}^o = \text{const}. \tag{2}$$

1. Considerations in the elastic post-buckling state

According to the non-linear theory of thin plates, the relations among strains and displacements are as follows:

$$\varepsilon_{xm} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\frac{\partial w}{\partial x}\right]^2 - \frac{1}{2}\left[\frac{\partial w_0}{\partial x}\right]^2,$$

$$\varepsilon_{ym} = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\frac{\partial w}{\partial y}\right]^2 - \frac{1}{2}\left[\frac{\partial w_0}{\partial y}\right]^2,$$

$$\gamma_{xym} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}.$$

$$\varepsilon_{xb} = -z\frac{\partial^2(w - w_0)}{\partial x^2};$$

$$\varepsilon_{yb} = -z\frac{\partial^2(w - w_0)}{\partial y^2}; \qquad (3)$$

$$\gamma_{xyb} = -2z\frac{\partial^2(w - w_0)}{\partial x \partial y}.$$

In the postbuckling state, the initial geometrical imperfections are taken into account during the analysis. The forms of displacement fields in the elastic postbuckling state are found similarly as in the work of Masaoka et al. [9], by assuming the combination of selected eigen-functions (see also [4]).

The deflection w and initial out-of-flatness w_0 are assumed as:

$$w = \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \qquad (4)$$

$$w_0 = \sum_{i=1}^{m}\sum_{j=1}^{n} f_{0ij}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \qquad (5)$$

Substituting the above expressions in two in-plane equilibrium equations:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \qquad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \qquad (6)$$

where:

$$N_x = A_{11}\varepsilon_{xm} + A_{12}\varepsilon_{ym}, \quad N_y = A_{12}\varepsilon_{xm} + A_{22}\varepsilon_{ym}, \quad N_{xy} = A_{33}\gamma_{xym}; \qquad (7)$$

and

$$A_{rs} = \sum_{k=1}^{N}(Q_{rs})_k\,(z_k - z_{k-1}); \quad r,s = 1,2,3$$

$$Q_{11} = \frac{E_k^x}{1-\eta_k\nu_k^2}; \quad Q_{22} = \eta_k\frac{E_k^x}{1-\eta_k\nu_k^2}; \quad Q_{12} = \eta_k\nu_k\frac{E_k^x}{1-\eta_k\nu_k^2}; \quad Q_{33} = G_k. \qquad (8)$$

one can find the in-plane displacements u and v:

$$u = u^o + \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{C_1 \sin\frac{2i\pi x}{a} + B_1 \sin\frac{2i\pi x}{a}\cos\frac{2j\pi y}{a}\right\} \tag{9}$$

$$v = v^o + \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{C_2 \sin\frac{2j\pi y}{b} + B_2 \sin\frac{2j\pi y}{b}\cos\frac{2i\pi x}{a}\right\} \tag{10}$$

where:

$$C_1 = \frac{a\pi}{16i}\left(\frac{A_{12}}{A_{11}}\frac{j^2}{b^2} - \frac{i^2}{a^2}\right)\left(f_{ij}^2 - f_{0ij}^2\right),$$

$$C_2 = \frac{b\pi}{16j}\left(\frac{A_{12}}{A_{22}}\frac{i^2}{a^2} - \frac{j^2}{b^2}\right)\left(f_{ij}^2 - f_{0ij}^2\right),$$

$$B_1 = \frac{\hat{e}\hat{b} - \hat{c}\hat{d}}{\hat{a}\hat{d} - \hat{b}^2}\left(f_{ij}^2 - f_{0ij}^2\right),$$

$$B_2 = \frac{\hat{c}\hat{b} - \hat{e}\hat{a}}{\hat{a}\hat{d} - \hat{b}^2}\left(f_{ij}^2 - f_{0ij}^2\right),$$

$$\hat{a} = A_{11}\frac{i^2}{a^2} + A_{33}\frac{j^2}{b^2};$$

$$\hat{b} = \left(A_{12} + A_{33}\right)\frac{ij}{ab};$$

$$\hat{c} = \frac{-\pi i}{16a}\left[-A_{11}\frac{i^2}{a^2} + \left(A_{12} + 2A_{33}\right)\frac{j^2}{b^2}\right];$$

$$\hat{d} = A_{22}\frac{j^2}{b^2} + A_{33}\frac{i^2}{a^2};$$

$$\hat{e} = \frac{-\pi j}{16b}\left[-A_{22}\frac{j^2}{b^2} + \left(A_{12} + 2A_{33}\right)\frac{i^2}{a^2}\right].$$

The function w describing the post-buckled shape of a plate under combined loads is taken in a form:

$$w = C_c\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}^c \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} + C_s\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}^s \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \tag{11}$$

where:

f_{ij}^c, f_{ij}^s – coefficients of Fourier series for a buckled plate (of given ratio a/b and composed of N layers) under compression and shear acting separately,

C_c, C_s – free parameters.

To determine the buckling mode for a considered plate, the eigen-value problem has to be solved and then the coefficients of eigen-functions can be determined.

The solution of buckling problem of specially orthotropic laminated plate under pure compression or pure shear is well known ([6], [7]). The buckling loads are expressed as:

$$\bar{N}_x = \frac{\pi^2 \sum_i \sum_j f_{ij}^2 \left[D_{11} \left(\frac{i}{a} \right)^4 + 2(D_{12} + 2D_{13}) \left(\frac{ij}{ab} \right)^2 + D_{22} \left(\frac{j}{b} \right)^4 \right]}{\sum_i \sum_j f_{ij}^2 \left[\left(\frac{i}{a} \right)^2 + \nu \left(\frac{j}{b} \right)^2 \right]} \tag{12}$$

$$\bar{N}_{xy} = \frac{\pi^2 \sum_i \sum_j f_{ij}^2 \left[D_{11} \left(\frac{i}{a} \right)^4 + 2(D_{12} + 2D_{13}) \left(\frac{ij}{ab} \right)^2 + D_{22} \left(\frac{j}{b} \right)^4 \right]}{4 \sum_i \sum_j \sum_p \sum_q f_{ij} f_{pq} \frac{ijpq}{(i^2 - p^2)(q^2 - j^2)}} \tag{13}$$

where: $$D_{rs} = \frac{1}{3} \sum_{k=1}^{N} (Q_{rs})_k \left(z_k^3 - z_{k-1}^3 \right); \quad r,s = 1,2,3 \tag{14}$$

and i,j,p,q – natural numbers, such that $i \pm p$ and $j \pm q$ are odd numbers.

The coefficients f_{ij} should be chosen in such a way that they provide (with satisfactory accuracy) the smallest values of $\bar{N}_x$ and of $\bar{N}_{xy}$ for calculated values of stiffness matrix D_{rs} of a plate and for a given plate aspect a/b.

4. Main assumptions in the elasto-plastic range

In the studies concerning the stability of structures, it is essential to describe in analytical way the material characteristics in the elastic and in the elasto-plastic range. In practice, the strength tests are the simplest way to obtain the relations between stresses and strains.

As pointed out by Hill [5], there are four independent characteristics to be known for orthotropic material (in plane stress case). Three of them correspond to the uniaxial tensile stress-strain curves for principal and 45 degree directions of the strength plane of the material. The fourth characteristic corresponds to the pure shear test.

In the computations, it is convenient to represent the experimentally determined characteristics by empirical equations of suitable forms. In this work, it has been assumed that the characteristics of orthotropic materials used in the considered laminated plates are linear elastic – perfectly plastic.

In the analyses dealing with orthotropic materials, the anisotropic criteria of yielding have to be considered. For the engineering application, these criteria should describe adequately the behaviour of material, and should have simple forms allowing one to determine their parameters by simple strength tests.

In this work, Hill's yield criterion is applied, what means that the material characteristics obtained in tensile and compressive tests are the same.

All relations given here are valid in the case of plane stress state. Defining the plastic potential f, or effective stress $\overline{\sigma}$, in a similar manner to the Huber-Mises yield function for isotropic materials, Hill's criterion can be written in the form

$$f = \overline{\sigma}^2 = \overline{a}_1\sigma_x^2 + \overline{a}_2\sigma_y^2 - \overline{a}_{12}\sigma_x\sigma_y + 3\overline{a}_3\tau_{xy}^2 \tag{15}$$

where σ and τ are stress components, $\overline{a}_1 \div \overline{a}_3$ are parameters of anisotropy.

It is easy to notice that Hill's yield criterion is equivalent to the Huber-Mises yield criterion for isotropic material when parameters $\overline{a}_1 \div \overline{a}_3$ take the value 1.

The anisotropic parameters in (15) can be determined by four independent yield tests. The initial parameters (no hardening effects) are obtained by successively allowing all stress components to be zero in the yield function, except the one under consideration.

For a tensile test in x direction: $\overline{a}_1 = \overline{\sigma}_0^2 / \sigma_{10}^2$, where σ_0 is the uniaxial yield stress in the reference direction, σ_{10} is the uniaxial yield stress in the x direction.

Taking the x-direction as the reference direction, then $\overline{a}_1 = 1$, similarly: $\overline{a}_2 = \sigma_0^2 / \sigma_{20}^2$, $\overline{a}_3 = \sigma_0^2 /(3\tau_{120}^2)$, $\overline{a}_{33} = \sigma_0^2 / \sigma_{\theta 0}^2$, when $\theta = 45^0$ then $\overline{a}_{12} = \overline{a}_1 + \overline{a}_2 + 3\overline{a}_3 - 4\overline{a}_{33}$.

$\sigma_{20}, \sigma_{\theta 0}, \tau_{120}$ denote the initial yield stresses, respectively: in the uniaxial tensile tests in the y-direction, in the direction rotated by an angle θ to the x-direction, and in the pure shear test.

In the elasto-plastic range the following assumptions are made:

- the materials are orthotropic, elastic-perfectly plastic and obey Hill's yield criterion,
- all assumptions of non-linear plate theory still hold,
- the forms of displacement functions are the same in the elastic and elasto-plastic range but the amplitudes C can vary arbitrarily,
- according to the plastic flow theory, the increments of plastic strains are described by Prandtl-Reuss equations, where the infinitesimal increments are replaced by finite ones. Then the resulting expressions for elasto-plastic stress increments are:

$$\begin{aligned}
\Delta\sigma_x &= \frac{E_x}{(1-\eta\nu^2)}\left[\Delta\varepsilon_x + \nu\eta\Delta\varepsilon_y - \Lambda(S_{xx} + \nu\eta S_{yy})\right], \\
\Delta\sigma_y &= \frac{E_y}{(1-\eta\nu^2)}\left[\Delta\varepsilon_y + \nu\Delta\varepsilon_x - \Lambda(S_{yy} + \nu S_{xx})\right], \\
\Delta\tau_{xy} &= G_{xy}(\Delta\gamma_{xy} - \Lambda S_{xy}),
\end{aligned} \tag{16}$$

where

$$S_{xx} = \frac{1}{3}(2\bar{a}_1\sigma_x - \bar{a}_{12}\sigma_y),$$
$$S_{yy} = \frac{1}{3}(2\bar{a}_2\sigma_y - \bar{a}_{12}\sigma_x), \quad (17)$$
$$S_{xy} = 2\bar{a}_3\tau_{xy},$$

and the instant value of Λ has to be determined in each step of calculations [10]:

$$\Lambda = \frac{(S_{xx} + \nu\eta S_{yy})\Delta\varepsilon_x + \eta(S_{yy} + \nu S_{xx})\Delta\varepsilon_y + G^* S_{xy}\Delta\gamma_{xy}}{S_{xx}^2 + 2\eta\nu S_{xx}S_{yy} + \eta S_{yy}^2 + G^* S_{xy}^2}, \quad (18)$$
$$G^* = G(1-\eta\nu^2)/E_x.$$

5. Method of solution

The iterative approach to the problem is employed using the combination of analytical and numerical solutions based on the Rayleigh-Ritz variational method involving the relations of flow theory of plasticity. This method has been used in many works [3], [4], [10].

The analytical considerations are conducted until the expressions (1) for strains are determined in terms of eigen-value coefficients f_{ij} and free parameters C_c and C_s.

Next, the increment of total plate energy has to be found out:

$$\Delta W = \int_V \left[\left(\sigma_x + \frac{1}{2}\Delta\sigma_x\right)\Delta\varepsilon_x + \left(\sigma_y + \frac{1}{2}\Delta\sigma_y\right)\Delta\varepsilon_y + \left(\tau_{xy} + \frac{1}{2}\Delta\tau_{xy}\right)\Delta\gamma_{xy}\right] dxdydz, \quad (19)$$

where: V – denotes the volume of a plate, σ_x, σ_y,τ_{xy} denote the stresses before the displacement increment $\Delta\mathbf{U}$ is applied and $\Delta\sigma$, $\Delta\varepsilon$ are the increments of stresses and strains produced by $\Delta\mathbf{U}$.

The elastic and plastic energy increments are evaluated in a numerical way. In order to accomplish this, the discretisation of plate is performed. Each layer is divided equally to appropriate cubicoids. The energy values calculated for each cubicoids are summed up for the whole plate. Next, the numerical minimisation of ΔW (19) is performed versus independent parameters C. The discretisation of plate throughout thickness makes it possible to introduce different material properties for every k-th layer.

In each step of calculations the following aspects are taken into account: occurrence of active, passive and neutral processes, reduction, to the yield surface, of stresses that are strayed from it. The reduction is necessary because finite increments are used.

It should be emphasised that, during the analytical-numerical solution, the response of the plate to the increment of its nodal displacements (Fig. 1) is searched for. The values of the average shear stress and the average compressive stress corresponding to the nodal displacements of the plate are obtained numerically as a mean value of stresses along the edge $y = b$, summed up for all layers.

The procedure presented above allows us to build up a computer programme. It should be noted that, because of complication of the problem, a lot of quantities have to be stored in each step of calculations.

6. Preliminary results of numerical calculations

Although the theoretical considerations and the developed computer programme deal with orthotropic laminated plates, the preliminary numerical calculations were conducted for square plates $(a/b = 1)$ composed of three isotropic layers: steel-aluminium-steel.

It is assumed that the characteristics of component materials are linear in the elastic range and in the plastic range no hardening occurs.

Geometrical and material data are given in Table 1.

Table 1.

Geometrical and material data

Notation	a/h	Middle layer - aluminium	Exterior layers– steel
Case I	100	$E_{Al}=0.7*10^5$ MPa, $\nu=0.3$	$E_S=2*10^5$ MPa, $\nu=0.3$ $\sigma_{0S}=184$ MPa
Case II Case III	50 100	$\sigma_{0Al}=123$ MPa	$E_S=2*10^5$ MPa, $\nu=0.3$ $\sigma_{0S}=384$ MPa

The calculations were performed for different ratios of aluminium layer thickness (g) to the plate thickness (h) - Fig. 1b.

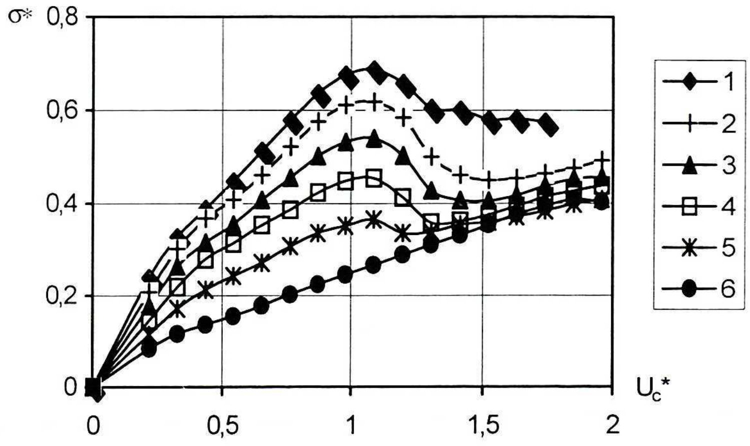

Fig. 2. Load versus shorterning curves for plates „I": 1 – g/h=0; 2 – g/h=0.2; 3 – g/h=0.4; 4 – g/h=0.6; 5 – g/h=0.8; 6 – g/h=1.0

The results of calculations are drawn in diagrams showing the following relations:

- the dimensionless average compressive stress $\sigma^*=\sigma_{av}/\sigma_{oS}$ versus the dimensionless shortening of a plate $U_c^*=(U_c/a)/(\sigma_{oS}/E_S)$;
- the dimensionless average shear stress $\tau^*=\tau_{av}/\tau_{oS}$ versus the dimensionless angle of rotation $U_s^*=(U_s/b)/(\tau_{oS}/G_S)$.

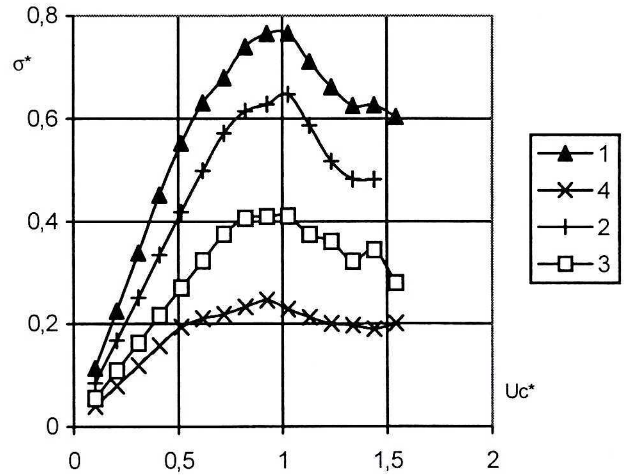

Fig. 3. Load-shortening curves for plates „II": 1 – g/h=0; 2 – g/h=0.2; 3 – g/h =0.6; 4 – g/h=1.0

In Figures 2, 3 and 4, the load -shortening curves are presented for three-layered plates subjected to pure compression. The corresponding material characteristics are shown in Fig. 5. It can be seen that in each case of considered plates the character of L-S curves is different.

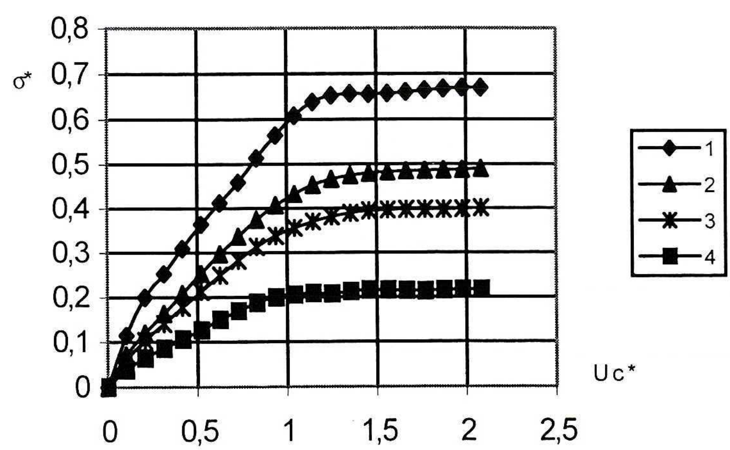

Fig. 4. Load-shortening curves for plates „III": 1 – g/h=0; 2 – g/h=0.4; 3 – g/h =0.6; 4 – g/h=1.0

In plates "II" and "III" (Figs. 3, 4), the external layers are made of steel of σ_{oS}=384 MPa. For plates of a/h =50 (width to thickness ratio) all curves gained the maximum value and for plates of a/h=100 the curves L-S have the "plateau". Let us notice that the yield limit in steel and aluminium is reached at similar value of shortening (Fig. 5a). Therefore, the ratio a/h governs the character of curves, and the plates of different ratio g/h behave as single layer plates of the constant a/h and of material parameters (Young's modulus and yield limit) located in the shaded area in Fig. 5a.

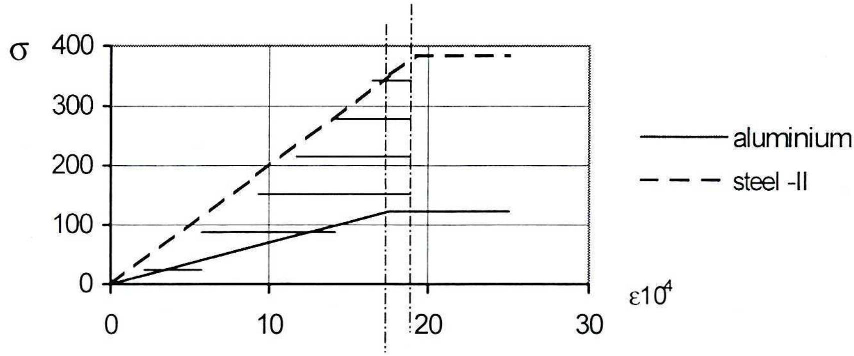

Fig. 5a. Material stress-strain relations for plates "II" and "III"

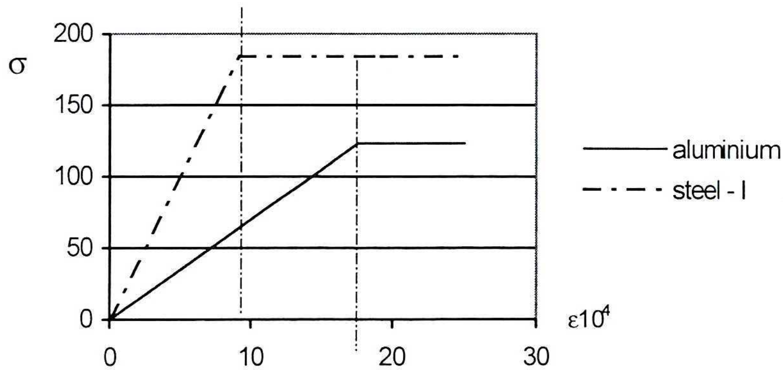

Fig. 5b. Material stress-strain relations for plates "I"

The character of curves for plates "I" (Fig. 2) is completely different. The material stress-strain characteristics for component layers are shown in Fig. 5b.

The ultimate load in the aluminium plate (Fig. 2- curve 6) is reached at the shortening almost twice higher than that for steel plate (curve 1). For U_c^* close to unity, the ultimate load in the steel plate appears while at this value of shortening the aluminium plate is still fully elastic, and is able to sustain further loading. A kind of regularity is observed in the behaviour of three layered plates. The average compressive stress reaches the extreme value for dimensionless shortening U_c^* slightly larger than 1. Next extreme value of σ^* appears at U_c^* close to 2, it is the value of shortening at which the ultimate load for aluminium plate is reached. The relation between these extreme values depends on the ratio of aluminium layer thickness to the thickness of a whole plate.

In Figures 6–10, the curves showing the relations $\tau^*=f(U_s^*)$ and $\sigma^*=f(U_c^*)$ are drawn for plates denoted as "III" (the material and geometrical parameters are given in Table 1, see also Fig. 5a.

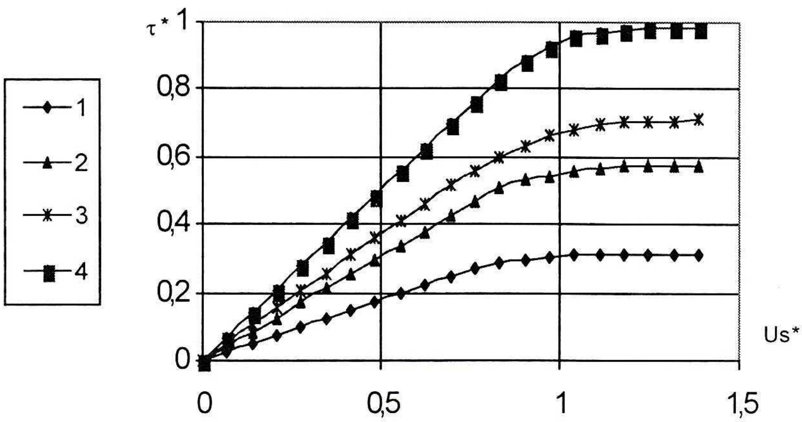

Fig. 6. Average shear stress versus angle of rotation for plates subjected to pure shear; 1 – g/h=0; 2 – g/h=0.4; 3 – g/h =0.6; 4 – g/h=1.0

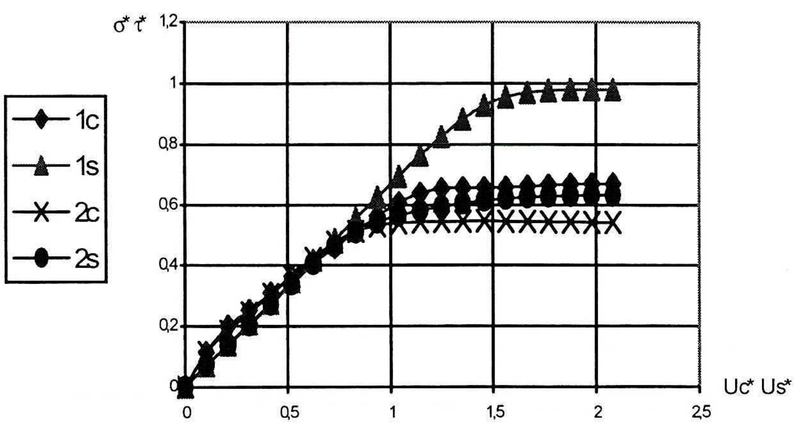

Fig. 7. Load-displacement curves for steel plates (g/h=0); 1c – pure compression; 1s – pure shear; 2 c, 2s – combined loading; $\Delta U_c^*/\Delta U_s^*=1$

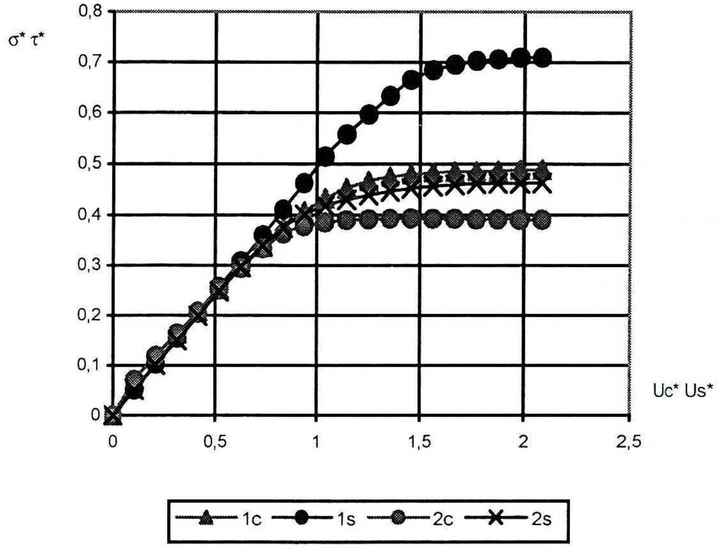

Fig. 8. Load-displacement curves for three layered plates (g/h=0.4); 1c – pure compression; 1s – pure shear; 2 c, 2s – combined loading; $\Delta U_c^*/\Delta U_s^*=1$

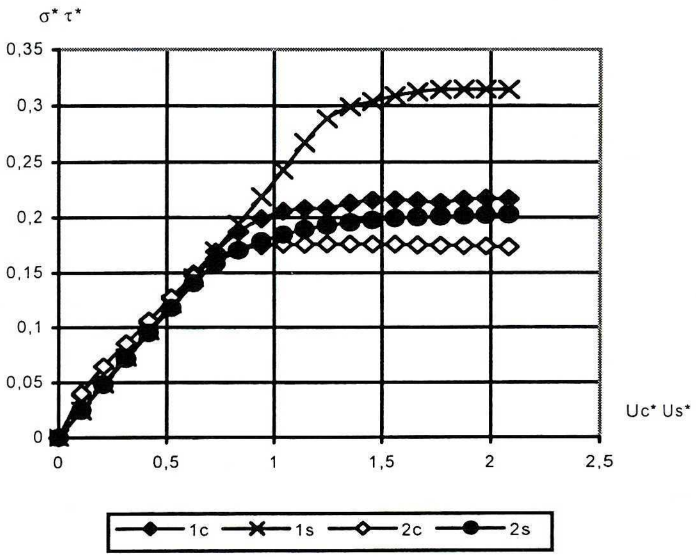

Fig. 9. Load-displacement curves for aluminium plates (g/h=1.0); 1c – pure compression; 1s – pure shear; 2 c, 2s – combined loading; $\Delta U_c^*/\Delta U_s^*=1$

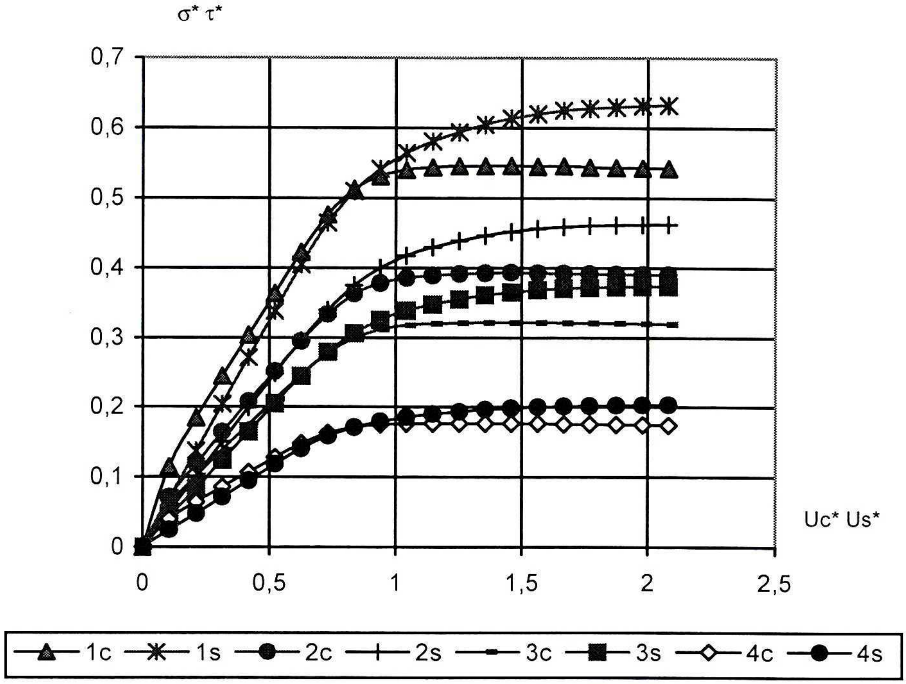

Fig. 10. Load-displacement curves for plates subjected to combined loading; 1 – g/h=0; 2 – g/h=0.4; 3 – g/h=0.6; 4 – g/h=1.0; (c – compression, s – shear), $\Delta U_c^*/\Delta U_s^*=1$

For plates subjected to simultaneous compression and shear ($\Delta U_c^*/\Delta U_s^*=1$), the decrease of average stress values relative to the values of these stresses in the case of single loads is observed in the elasto-plastic range. It can be noted that in this range (e.g. for $U_c^*=U_s^*=1.5$) the ratio of average compressive stress at combined loading to the average compressive stress at pure compression is almost constant for plates of different ratio g/h. The same remark concerns the ratio of shear stress values.

7. Final comments

The results of preliminary calculations seem to confirm that the presented method makes it possible to determine the postbuckling behaviour of the multilayered plates in the elastic and elasto-plastic range, when the plates are subjected to the combined loading.

It should be mentioned that some problems that appear in laminated plates (such as e.g. delamination, cracking) may provoke different behaviour of a plate subjected to in-plane loads. It is also known that for many composites the material characteristics obtained in tensile and compressive tests are not identical – in such a case the Hill's yield criterion should be replaced by Tsai-Wu criterion.

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Nośność graniczna wielowarstwowych płyt poddanych ściskaniu i ścinaniu

Streszczenie

W pracy przeprowadzono analizę stanu zakrytycznego w zakresie sprężysto-plastycznym prostokątnych wielowarstwowych płyt poddanych złożonemu obciążeniu, to znaczy jednoczesnemu ściskaniu i ścinaniu. Płyty zbudowane są z ortotropowych warstw o kierunkach ortotropii równoległych do brzegów płyty, warstwy umieszczone są symetrycznie. Analizę

przeprowadzono w oparciu o nieliniową teorię cienkich ortotropowych płyt z uwzględnieniem plastyczności. Rozwiązanie zagadnienia uzyskano w sposób analityczno-numeryczny wykorzystując równania Prandtla-Reussa. Wstępne wyniki obliczeń numerycznych pokazano w postaci wykresów.