

WAYS OF TALLYING UP VOTES

What kind of electoral system is the best? This question, easy to formulate in theory, has considerable practical significance. However, the answer is as widely known as it is disappointing: it has been shown that, given certain reasonable conditions, there is no such thing as a perfect voting system. This is the upshot of several different mathematical discoveries made in the latter half of the 20th century



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The first and most famous of these results is stated in what is called Arrow's impossibility theorem, demonstrated in 1951 by Kenneth Arrow, the youngest-ever recipient of the Nobel Prize in economics. If we assume that every voter can rank can-

didates from best to worst, a voting system should, according to Arrow, convert these individual preferences into a single aggregate ranking of candidates, possibly allowing for ties. Arrow's theorem states that if voters have to choose between three or more candidates, no voting system can possibly satisfy a certain set of seemingly natural criteria for what a fair system should look like. The most important of these requirements is referred to as the "independence of irrelevant alternatives": under this requirement, for a given set of individual preferences, if the group prefers candidate A over candidate B and individual preferences are modified in such a way that the relative ordering of A and B remains unchanged, the group will still prefer A. At first glance, this requirement seems modest. In reality, however, no electoral system can satisfy it.





But if we opted for a less challenging task, namely the choice of a single winner, could we find the perfect voting system? As it turns out, the answer is still “no.” In addition, if any “reasonable” voting scheme is used, any voter will be able to manipulate the system under certain conditions. This means that there is at least one preference profile in which a specific voter will prefer the outcome he can secure by misrepresenting his own preferences, over what would be an “honest” outcome. In other words, no reasonable system of voting is immune to strategic voting. Philosopher Allan Gibbard and economist Mark Satterthwaite demonstrated this fact independently in the early 1970s. In 1993, economist John Duggan and political scientist Thomas Schwarz extended the Gibbard-Satterthwaite theorem to include systems that allow ties.

The aforementioned theorems pertain to voting systems designed to choose a single winner or multiple winners, but only in plurality/majoritarian voting. Mathematics, as it turns out, also creates barriers that are impossible to surmount in a multiple-winner election based on proportional representation. That’s because such systems have a certain Achilles heel, namely the procedure used to allocate seats among parties. Two mathematicians, Michel Balinski from France and Hobart Peyton Young from the United States, demonstrated in 1982 that whenever there are three or more parties, there is no ideal method of apportionment that does not result in certain paradoxes under specific conditions.

If no perfect voting system exists, then, it is hardly surprising we can find numerous, subtly different solutions in use in various parliamentary democracies. The choice of a specific system depends on many cultural and political factors that also impact on how a given electoral system functions. Even so, voting systems naturally fall into several groups. In multiple-winner elections, the most important division is between proportional representation vs. plurality/majoritarian systems.

The apparent simplicity of proportional representation

In proportional representation, the rule indeed seems simple: the number of seats should be proportional to the number of votes cast for a specific party. Of course, exact proportionality is usually impossible, because seats in parliament cannot be split into fractions. In reality, the smallest representation requires a certain minimum share of voter support, which depends on two factors. The first of these is called the legal electoral threshold. In Poland, this is 5% of all valid votes for a party to make it into parliament (or 8% of the vote for coalitions), with the caveat that national minorities are not bound by this requirement. But there is also another threshold, referred to as the natural threshold, which attracts less attention. Many countries are divided into electoral districts, with each district being assigned a specific number of seats. Such constituencies may differ in magnitude: in Poland, for example, they range from seven representatives in Częstochowa to 20 in Warsaw. The fewer representatives are elected in a specific district, the more support a given party needs to garner in order to secure at least one seat; this minimum support is the natural threshold. A typical electoral district in Poland is represented by 10 or 11 members in the elected body. In this case, the natural threshold is comparable to the legal threshold. In Częstochowa, in turn, the natural threshold is higher than the legal threshold. In Warsaw, however, the opposite is true. For that reason, the substantial differences in the magnitude of electoral districts are a downside



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of the voting system in place in Poland, because specific constituencies have different natural thresholds.

But the Polish system has yet another disadvantage, one that is far more serious. Polish nationals residing abroad cast their votes in the Warsaw constituency. According to cautious estimates, more than 2 million voters have left the country since 2004, thereby significantly increasing the number of voters in the Warsaw district. Meanwhile, the number of parliamentary deputies elected in Warsaw has risen only slightly, from 19 to 20, over the past decade, which violates the constitutional principle of equality. This problem could be resolved if voters residing abroad could vote for candidates from their respective districts in Poland or had their own, separate representatives in parliament.

An important element of any system of proportional representation is how voter support gets converted into numbers of seats. There are several calculation formulas in use, but we shall restrict ourselves to the solution used in Poland, the D'Hondt method (named after Victor D'Hondt, who discovered it in Europe, although it had been described earlier in the United States by Thomas Jefferson). The method is illustrated in the table on the next page, using the distribution of 10 seats between five parties as an example. The first row shows the number of votes received by each party or, in a simplified variant, voter support expressed as a percentage (the percentages add up to 100%). The following rows show the result of dividing the voter support from the first row by divisors that correspond to a sequence of consecutive natural numbers. The 10 highest numbers in the table correspond to the 10 seats, with each party ultimately winning as many seats as there are winning entries in its column.

In this example, the parties will get 5, 3, 1, and 1 seat respectively. As we can see, the Purple Party wins no seats despite taking 5% of the vote, which illustrates how the natural threshold works. If there were only five seats in this district, these would be divided between the Blue Party (three seats) and the Greens (two seats). In this case, the Yellow Party would win no seat even though it took 12% of the vote! A smaller district means greater deviation from proportionality, although lawyers would still describe such a system as proportional representation. Reducing the number of seats assigned to a district means fewer parties in parliament and greater chances for a stable single-party government, at the expense of reduced representation in parliament. There is always a certain tradeoff.

Single-member districts: inevitable dominance

Stability is best reflected in plurality/majoritarian systems, which also come in many varieties. Recent discussions in Poland have focused on what is perhaps the simplest variant of plurality-based voting, namely

single-member districts (widely known in Poland by the Polish abbreviation "JOW"). Developed and used for many years in the United Kingdom and the United States, this method is indeed very simple: every electoral district corresponds to one seat in the elected body. The winner is the candidate with the highest number of votes, even if his or her overall share of the vote is not impressive. Practical experience shows that if used consistently for a long period, this system promotes the clear domination of two parties. In the United States, these are the Democrats and the Republicans. In the United Kingdom, the dominant parties are the Conservative Party and the Labour Party. This dependency is so clearly visible that it is referred to in political science as Duverger's law.

In single-member constituencies, it doesn't matter how great an advantage the winning candidate has over his or her rivals. As a result of this, single-member districts are susceptible to the practice of gerrymandering, or manipulating district boundaries. Let's imagine a region naturally divided into several constituencies in which two dominant parties have similar approval ratings. In such a situation, it is difficult to predict the outcome of the elections in each district. For example, it may depend on the intensity of campaigning or on campaign spending. However, if advocates of one of the parties redefine the boundaries of the constituencies, they may determine the outcome of the elections even before voters cast their votes. They just need to redraw the lines so that their opponent will win by a wide margin in one district, while their own candidates will maintain a slight advantage in the remaining constituencies. Of course, manipulation may help each of the two dominant parties. It is indeed a problem hard to combat.

A two-party system means that one party usually has an absolute majority in parliament. As a rule, this means that the government is formed by a single party and therefore very stable. In the years following WWII, a coalition government has been necessary in the UK only in 2010–2015 (the conservatives and the liberal democrats). It is worth adding that even single-party governments are not immune to crisis. Margaret Thatcher's idea of introducing a poll tax proved very unpopular, causing John Major to take over the Conservative Party and the cabinet in 1990. Similarly, after the Labour Party's Tony Blair was criticized for his rationalization of Britain's involvement in the war in Iraq, he was replaced as prime minister by Gordon Brown in 2007.

Domestic discussions in Poland often stress that single-member districts have the advantage of fostering stronger connections between members of parliament and their constituents, at the expense of weakening the central party bodies. This is indeed the case in the United States, but not in the UK, where the role of party leader is similar to that in Poland, because each



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Wędrowki przez krainę wyborów" [Every Vote Counts: A Journey Through the Land of Elections] (Wydawnictwo Sejmowe, Warsaw 2014), which attempts to answer the question of how to rationally translate individual votes into the decision of the group. The authors also describe other methods of counting votes, in addition to the D'Hondt method, presented in this article.

Party	Blue	Green	Yellow	Red	Purple
1	42.0	31.0	12.0	10.0	5.0
2	21.0	15.5	6.0	5.0	2.5
3	14.0	10.3	4.0	3.3	1.7
4	10.5	7.8	3.0	2.5	1.3
5	8.4	6.2	2.4	2.0	1.0
6	7.0	5.2	2.0	1.7	0.8
Number of seats	5	3	1	1	0

Example of using the D'Hondt method for the allocation of 10 seats between five parties.

party's central-level governing bodies choose candidates to represent the party in elections. In the UK, the party's governing bodies have the party's budget at their disposal. In the United States, candidates themselves are responsible for raising funds. When stressing the advantages of the links between members of parliament and their constituents, supporters of single-member districts often forget about the flip side of the system. In the United States, it is very difficult to enforce party discipline. It is sometimes possible to buy the support of members of Congress for a given bill by appending an measure that their constituents will find important but often remains unrelated to the subject matter of the bill itself. There is even a discernible difference in the level of discipline between the House of Representatives and the Senate, and understandably so: members of the House face reelection every two years, senators every six years.

Single-member constituencies may indeed narrow the political spectrum, which is clearly visible when we compare Poland's upper-house Senate, whose members are elected in single-member districts, and the lower-house Sejm, elected in proportional representation elections. In every newly-elected parliament, more parties are represented in the Sejm than in the Senate. Despite this, critics complain that the Sejm elections produce worse representation. On the other hand, even a parliamentary minority may have a significant impact on the political solutions adopted in parliament.

Different countries, different systems

Minority groups, which campaign on a platform of addressing new, significant problems, often stand no chance of securing a strong representation of their own in parliament, yet their proposals do frequently get picked up by one or both of the dominant parties. In the 1980s, for example, the public eye turned to environmental protection issues, which resulted in the emergence of various "green" parties in countries

with proportional representation. In Germany, the Greens even formed part of the ruling coalition. By contrast, green parties are very weak in the UK and practically non-existent in the United States. In those two countries, environmental issues have largely been taken up by one of the dominant parties: in the UK chiefly by the Labour Party, in the United States by Al Gore and the Democratic Party. Another example can be found in the UK Independence Party (UKIP), which has brought anti-EU sentiments into British politics in recent years. Despite taking 18% of the vote in recent parliamentary elections, the party won only one seat. Nigel Farage's personal defeat nonetheless did not mean the ultimate failure of his party's agenda. Prime Minister Cameron has taken up some of the UKIP ideas, after watering them down, and has promised a referendum on the UK's future membership in the EU.

In Poland, there is often talk that changing the voting system into one based on single-member districts will boost voter turnout. A glance at the English-speaking countries shows that this is not necessarily true, because some voters are discouraged by the absence of representation. In the UK, voter turnout is only slightly higher than 60%. In the United States, the elections to the US Congress that are held between the years of presidential elections have voter turnout rates of around 40%.

As we mentioned earlier, plurality/majoritarian systems come in different variants, depending on the country. France, for example, has a two-round system, which allows a majority of more than 50% of the vote to be won. In Australia, in turn, there is one round of elections, but voters have the right to rank candidates in their order of preference. Also, it is worth mentioning proportional representation systems in which some seats are distributed in single-member districts. The most famous and well-working example is the voting system in place in Germany.

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