Lowering the risk of financial investments

# Risk and the Market

Prof. Łukasz Stettner is a mathematician who studies stochastic control and financial mathematics

ŁUKASZ STETTNER
Institute of Mathematics, Warsaw
Polish Academy of Sciences
Academy of Finance
stettner@impan.gov.pl

**As risk is an inevitable part of all human activity, we naturally seek to be able to better measure it, estimate it, and even reduce it**

Harry Markowitz's proposed method for measuring the risk of financial investments in terms of portfolio value variance (i.e. the standard deviation of a portfolio's value from its expected value) stirred mixed feelings at first. For economists it represented a work of theoretical mathematics, while mathematicians saw it as concerning a linear programming problem with quadratic limits, a sort that was then already standard. Yet although Markowitz's method did not contribute anything new mathematically, it has became a fundamental tool for the financial world.

Breaking the standard financial canons then in force, the genius of Markowitz's idea was to take risk into account by defining it as the standard (or "root-mean-square") deviation of a portfolio from its expected value. While this was a simple idea, simple ideas only become so once someone discovers them.

Without a doubt, financial institutions have indeed "risen to the challenge" of coping with risk since then, as the pursuit of ever-greater profits frequently led to undesirable situations. That is why, despite the initial reluctance, Markowitz's methodology has dominated financial markets ever since the 1950s.

## Modeling risk

The notion of "risk" carries connotations of chaos, the unexpected and undesired behavior of an observed phenomenon. It is difficult to anticipate how a chaotic model will behave, since - as the name itself indicates - such a model does not have predictable dynamics. But while it is hard to say how a chaotic process will behave at any specific moment, things are quite different if we take a longer-

Bartosz Bobkowski, Agencja Gazeta

Success in stock exchange investments frequently hinges upon being familiar with refined mathematical methods

Rafał Guz/Fotorzepa

The Warsaw Stock Exchange building in Warsaw

term perspective, chiefly looking at the mean value of certain functions that hinge on the process. For example, the investors of a bank or insurance company are much more interested in overall market performance than in individual transactions they might stand to lose on. The point is to be sure they "come out on top" in the appropriate long-term perspective, regardless of short-term fluctuations.

Calculating risk requires a certain language (or mathematical model) to describe the observed phenomena. Random fluctuations in continuous time are described using the concept of Brownian motion. Such motion is characterized by continuous trajectories and independent increments, which have normal distribution, zero expected value and variance equal to the time increment being considered. If we give up continuous trajectories and assume independent and stationary increments (the latter dependent only upon the difference between the moments measured) we arrive at the definition of more general class, called Lévy processes.

Mathematical modeling may employ two different approaches: static and dynamic. The static approach rests upon the postulate that the model does not change quickly (if it changes at all). We assume that based on historical data we can predict its behavior at the next moment. That is an important limitation which considerably simplifies the model's use.

## Seeking an optimal portfolio

The classic Markowitz approach seeks to identify an investment portfolio, i.e. a proportion for investing capital into certain shares, that maximizes the rate of return in the next moment while assuming a set level of risk (or variance value). The "optimal return vs. risk" pair forms what is called an "effective frontier," on which we would like our portfolio to fall.

This mathematical problem can be solved by estimating - such as by using historical data - the matrix of covariances between shares and the expected rate of return on each individual share, i.e. a quite standard mathematical task. The problem, however, lies in the practical application of this method. Insofar as the covariance matrix can be estimated well, expected rates of return are very unstable and difficult to estimate. This can be aided by a certain modification introduced by Black and Litterman in the 1990s, which forecasts expected rates of return based on expert opinions. This has become the fundamental mathematical tool used in the analysis of financial investments.

We should consider, however, whether variance is in fact the "right" way to measure risk. It assigns the same significance to fluctuations which are positive for the investor as to negative ones, where the portfolio drops below the expected value. This observation suggested a different approach, which measures the negative effects only, in the form of semivariance - although this is more difficult to study from the mathematical standpoint.

Bankers predominantly employ the concept of "value at risk" (VaR), i.e. using the value exposed to risk as a measure of risk itself. This means the greatest loss that might be suffered assuming a set probability, called the "confidence level." The value-at-risk approach has a range of shortcomings, not only in view of the difficulties inherent in its calculation. Above all it gauges not the value of losses in a portfolio, but rather their likelihood. This shortcoming is avoided by a modification known as the "conditional value at risk" method, which gauges the conditional expected value of losses to the portfolio on condition that this loss is smaller than the VaR.

What requirements should the "right" measure of risk meet? First and foremost, it should be monotonic - meaning that larger investments involve smaller risk. Increasing an investment by a deterministic constant will decrease the risk by the same constant. The risk of a sum of investments is less than the sum of the risks of the investments considered separately. The value of a portfolio, in turn, does not have any impact on risk estimation. A portfolio which consists of multiple sets of the same instruments will be a multiple of the risk of the set of instruments.

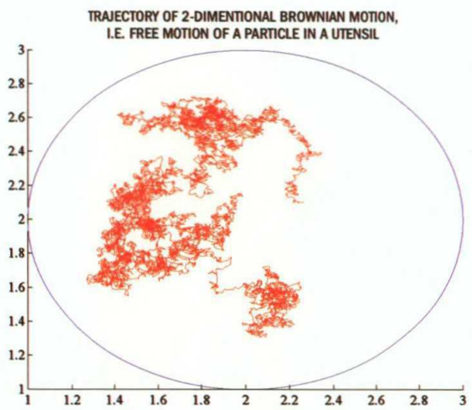
These and similar requirements are used in defining various risk measures considered to be model. Not all measures of risk meet such requirements. Variance and semivariance, for instance, are more measures of deviation. Nevertheless, they do gauge unfavorable situations with better or worse success, encouraging investors to behave less riskily.

## Optimal investing

The goal of investing optimally, with risk as a limit, faces us with a two-criterion problem: on the one hand we want to maximize the rate of return on a portfolio, while at the same time we want to keep risk at the lowest possible level. Problems of this sort, i.e. striving to maximize one function while minimizing another, are known as "minimax" problems and are inherently difficult to solve. A sort of alternative approach can be found in risk-sensitive functions, which can measure the rate of return on a portfolio while at the same time assigning risk a certain weight.

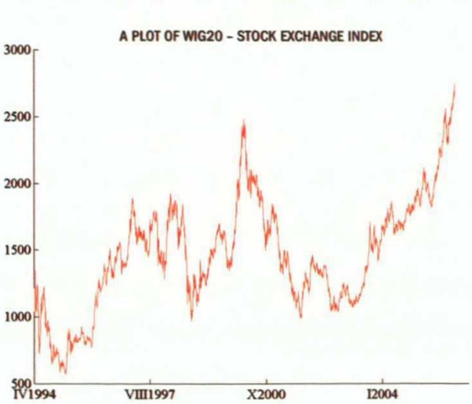
In such cases, the optimization problem reduces to a single function, which is a significant simplification, although it is usually difficult to find an optimal solution to the problem in analytical form. In practice, this difficulty can be overcome using the Monte Carlo method, introduced in 1949 by the Hungarian-American mathematician John von Neumann and the Polish mathematician Stanisław Marcin Ulam, working on the first computing machines in Los Alamos (and, incidentally, also involved there in work on the US nuclear program).

Modeling fluctuations in continuous time using Brownian motion, or more generally as a Lévy process, forms the basis for analyzing complex time-variable phenomena, enabling us to develop a dynamic rather than static model. Dynamic modeling allows us to describe fluctuating share prices, by assuming that they are the solution to a certain stochastic differential equation. However, finding the proper coefficients for the equation, known as calibrating the model, poses a serious problem.




Grzegorz Hałaj

**In 1827, the English botanist Robert Brown observed the chaotic motion of flower pollen particles in a liquid. This stochastic process was given a mathematical description by Norbert Wiener in 1923, now used to describe financial market behavior, for example**




Grzegorz Hałaj

**Investors are more interested in overall market performance than in individual transactions they might stand to lose on. The point is to make sure they "come out on top" in the long-term perspective. That is why they keep close tabs on stock exchange indexes**

## Eliminating risk?

So far we have concentrated on how to measure risk. But human activity naturally strives to eliminate risk, even when its estimation poses a problem.

The financial instrument known as options offers one way of doing so. Let's assume that we are planning to travel from Poland to the United States five months from now. We need \$3000 for the trip, but it is still too early to purchase so many dollars. On the other hand, we are afraid that the exchange rate (which now stands at 1 USD = 2.75 PLN) could shift in our disfavor. How can we guard against that risk?

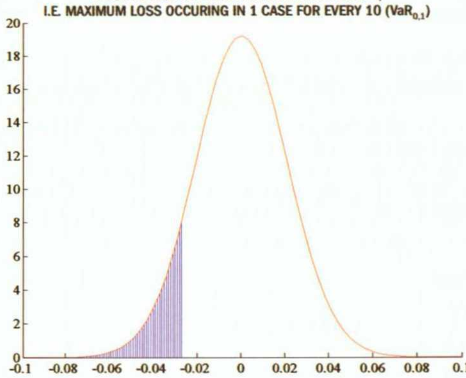
We can simply purchase an option to buy \$3,000 five months from now at a set price per dollar, say 1 USD = 2.8 PLN. Such an option will cost a certain amount, say 100

PLN, but it will guarantee us the ability to purchase dollars just before departure at the agreed rate of 1 USD = 2.8 PLN. If at that time the current rate in fact stands lower than 2.8 PLN, we can decide not to use the option and simply buy at the current rate. But we have the certainty that if the rate does go higher than 2.8, we can still buy the dollars at 2.8 PLN using the option. Thus by spending 100 PLN five months in advance, we guarantee ourselves the ability to purchase the currency we need just before departure without worrying about the exchange rate risk.

This raises the question of how the option should be properly valuated, in other words whether 100 PLN is in fact the right price for such a service. That question is even more crucial for companies which sell options. They need to identify what minimum price would be economically viable for them - called the "fair price." Calculating such a fair price is one of the fundamental problems of financial mathematics. The analytical formula for calculating the fair price of an option, derived by Black and Scholes in 1973 (with certain simplified assumptions, assuming for example logarithmically normal rates of return), has become one of the basic tools used by financial institutions. Although it was later noted that this model of share prices or currency rates is in fact too broad a simplification of reality, the fact that everyone uses this method (in view of its simplicity) has made it more "grounded in reality."

Grzegorz Hałaj

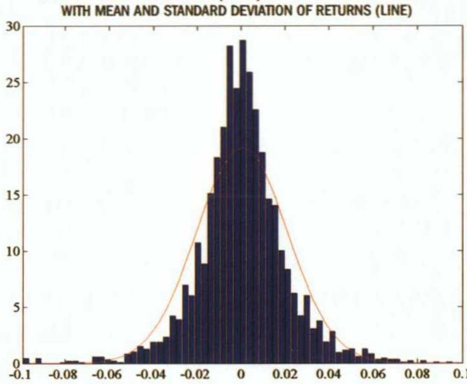
VALUE AT RISK OF RETURNS FROM WIG20 INDEX, I.E. MAXIMUM LOSS OCCURING IN 1 CASE FOR EVERY 10 ($VaR_{0.1}$)




**Bankers predominantly employ the concept of "value at risk" (VaR), i.e. using the value exposed to risk as a measure of risk itself. This means the largest loss that might be suffered assuming a set probability, called the "confidence level"**

Grzegorz Hałaj

DAILY RETURNS FROM WIG20 (BARS) AND NORMAL DISTRIBUTION WITH MEAN AND STANDARD DEVIATION OF RETURNS (LINE)




**We often try to model the behavior of daily stock exchange indexes or currency exchange rates by comparing them to the known distributions of the random variable: normal or Student's t**

## Types of risk

While the above examples of risk illustrate market-related financial risks, there are also many other sorts of risk. If a certain party to a loan agreement might not meet their repayment obligations, we speak of credit risk. Liquidity risk entails losing the ability to make payments due on time, while operational risk describes the risk of errors made in the operations of a company. We can talk about the risk of legal changes or political shifts, the risk of natural catastrophe, etc. A considerable share of such risks can be modeled and gauged using mathematical methods based upon statistics and the broad field of probability calculus. State-of-the-art computer techniques are also now bringing many models once considered purely theoretical into practical application.

Of course, the quantitative methods I am now describing must be approached with a certain amount of reserve. They can supply significant data, on condition that we choose the right model and estimate its parameters well. That is why the language of mathematics and quantitative methods is helpful, but cannot fully take the place of experts able to make correct qualitative decisions based on calculations plus their own knowledge.

Nevertheless, there is no doubt that risk is a factor that can and should be gauged in various situations. Researchers who study risk in the broad range of quantitative sciences have a lot of work still in store for them. ■

**Further reading:**

Dempster M.A.H. (Ed.) (2002). *Risk Management: value at risk and beyond.* Cambridge University Press.

Foellmer H., Schied A. (2002). *Stochastic Finance: An Introduction in Discrete Time.* Walter de Gruyter.

Jakubowski J., Palczewski A., Rutkowski M., Stettner L. (2006). *Financial Mathematics - Derivatives* [in Polish]. WNT.

Shiryaev A.N. (2001). *Essentials of Stochastic Finance. Facts, Models, Theory.* World Scientific.