Searching for the hidden order in the world around us

Random Phenomena

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Our recognition and quantitative characterization of real-word phenomena cannot be complete (indeed being impossible in many situations) unless we account for the great role played by randomness

The randomness of events and phenomena has troubled mankind since the earliest times; its hidden essence has been a point of interest for philosophy and the methodology of science, whereas its intrinsic regularities have been brought to light by mathematics and empirical sciences. Various kinds of chance are well-known to every one of us from our everyday experience: the outcome of a coin-toss or die-roll, the length of time spent waiting in line, how meteorological phenomena will proceed. In all such situations, we are unable to predict the outcome of an "experiment" or the future course of a process. The cause of the difficulty is generally our incomplete information about all the factors at work driving the phenomenon at hand, although it may also lie in the unknown degree of accuracy in the empirical data we possess about it. Randomness may also stem from the excessively complex nature of a phenomenon, which thus precludes a clear-cut deterministic description. This is the case in statistical physics, for example, or more precisely in kinetic gas theory, which looks at very large sets of gas particles (on the order of 10^{23} particles – Avogadro's number),

Earth Views

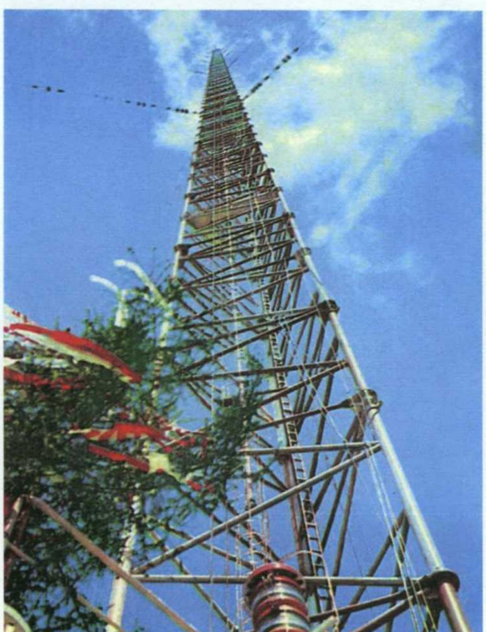
How might we capture the possible regularities in the propagation of acoustic ways within a sea full of fish? What laws govern the "refraction" of waves at the confluence of two media where the boundary is uneven, highly irregular, with a variable random geometry (such as the rough surface of the sea)? We do not need to stress that the above questions (and the answers to them) are highly important in practical terms which are in permanent, disorderly/chaotic motion and are subject to a vast number of collisions with each other every second. And although it is "theoretically possible" to write equations describing the movement of the individual particles, as I. Stewart writes: "...that would take an amount of paper comparable to the surface of the region delimited by the orbit of the Moon," and so it would be senseless to imagine solving them.

It is not wholly obvious what sort of uncertainty or incomplete information about a phenomenon should be considered to constitute randomness. Is the randomness involved in the breakdown of a radioactive element (considered a hallmark case) of the same sort as that involved in ocean waves? Yet regardless of the difficulty in rigorously pinpointing the causes and nature of chance/randomness, there has long been a strong need - even though phenomena are indeed random - to try to recognize the orderliness inherent in them, to describe them in mathematical terms and analyze them quantitatively. This is what is done by the theory (or "calculus") of probability, a vast and still expanding field of mathematics closely linked to various applications.

Two notions lie at the base of probability theory: the *random event* and *probability*. Probability theory defines these concepts, studies the relationships between the probabilities of various random events, identifies when events are independent of one another, introduces rules for transforming probabilities, etc., etc. In other words, probability theory constructs and analyzes general models of random events, whereby *probability* is a quantitative characterization of the potential for specific events to occur. Developing first from the characterization of "static" events, probability theory expanded its methods to include random events that are dependent on time (the theory of random/stochastic processes) and to phenomena which vary in space and are linked to geometric objects (random field theory, stochastic geometry).

This raises the following natural questions: In what way do probability theory and empirical sciences seek to uncover the regularities that lie hidden in randomness? What does the recognition and explanation of random phenomena involve? What do we mean when we discuss predicting/forecasting the future behavior of a random process? And finally, what sort of useful information about a phenomenon may be gleaned from probabilistic analysis (i.e. by employing probability theory)?

Such questions, although very natural, are certainly not trivial. Answering them would require insight into the entire broad diversity of the ways randomness is manifested in natural, technical, and economic phenomena, etc., as well as the quite advanced mathematical language of probability theory. Nevertheless, we can shed at least some light on these problems by looking more closely at two examples.



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The field of stochastic dynamics, emerging in recent decades, has developed methods for solving dynamical equations including random elements, therefore making it possible to characterize the reaction of various real systems to the kind of random stresses that many man-made structures are subject to - such as this long-wave broadcasting antenna of Polish Radio 1 in Konstantynowo, in its day the tallest such structure in history

Stochastic Waves

Let us turn our attention to spatial random processes, i.e. those which manifest their variability not only in time, but also (or often chiefly) in space. Here we are concerned with real processes whose irregularity and randomness are generated by complexities in the physical and geometrical properties of the spatial domains in which they play out.

Let's consider the phenomenon of wave propagation, known to each of us from day-to-day experience (as well as school education). In general, waves constitute disturbances in a material medium which propagate through space at a finite speed, transporting energy and information. In the language of mathematics, waves are described by a function

Krzysztof Miller/Agencja Gazeta

646.38 meters tall, the antenna came online in 1974. Conservation work that was begun in 1991 disturbed the statics of the antenna core, which collapsed on 8 August 1991. The job of broadcasting Polish Radio 1 was taken over by two antennas in Solec Kujawski near Bydgoszcz, only 330 and 289 meters tall

dependent on a spatial variable (i.e. the point in space *r*) and on time *t*. If the temporal dependency of such disturbances is "fixed" - for example, periodic or what is called "harmonic" - then we are only interested in the disturbances' dependence upon the spatial variable, or more precisely, upon complexities in the wave-bearing medium.

One common aspect of classical wave analysis (i.e. the kind familiar to us from school or university classes) is that there is a deterministic mathematical model assumed to describe wave propagation (e.g. the equations used in acoustics or radio physics). More precisely, the wave-bearing medium is considered *ideal*, i.e. its parameters are invariant (note the dielectric *constant*, the *constant* of magnetic permeability, and of course the medium's *constant* density), and the obstacles which waves encounter *en route* are of ideal shape. We may also recall Snell's law, which describes the reflection of waves from the boundary between two different media (of course, this boundary was seen as a flat plane (!), and so the law was simple). Yet reality turns out to be not so ideal in many situations. The existence of many diverse and random factors that determine real wave processes requires other mathematical descriptions to be sought. How should we describe and analyze the propagation of waves through a turbulent atmosphere (with strong spatial fluctuations of pressure, medium density, magnetic properties, etc.)? How might we capture the possible regularities in the propagation of acoustic ways within a sea full of fish? Finally, what laws govern the "reflection" of waves at the confluence of two media where the boundary is uneven, highly irregular, with a variable random geometry (such as the rough surface of the sea)? What regularities are there in the propagation of seismic disturbances through the Earth's highly heterogeneous crust? We do not need to stress that the above questions (and the answers to them) are highly important in practical terms. Yet we hope that readers will recognize the beauty hidden here for the researcher who tries to work out the basic dependencies and express them in the language of mathematics.

The complexity of real wave processes is above all a consequence of the random heterogeneity and indefiniteness in the structure of most wave-bearing media and the random unevenness of the surfaces which delimit regions of differing properties. The irregular and complex properties of many real media (such as the turbulent atmosphere, the Earth's crust, or composite materials) do not lend themselves to description in terms of ordinary "deterministic" mathematical tools, thus necessitating the use of probability theory - essentially the language of random field theory (i.e. random functions of several variables). To study such phenomena, we

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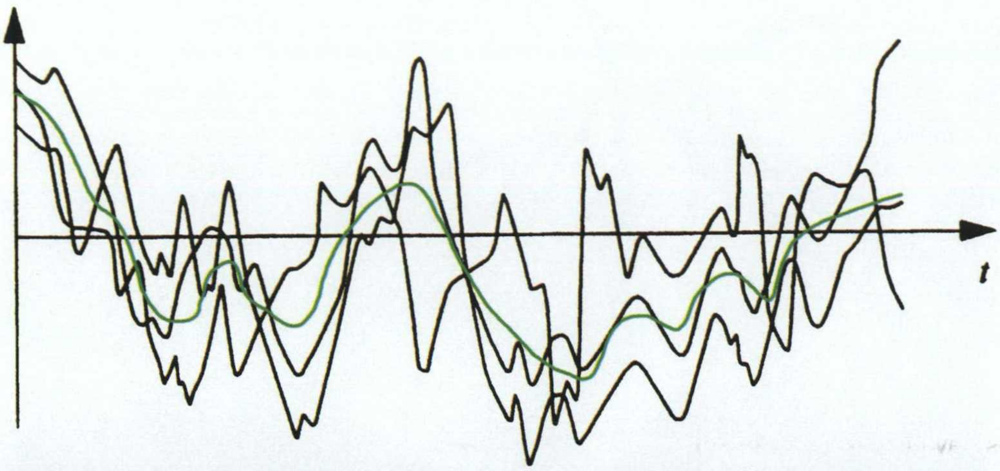


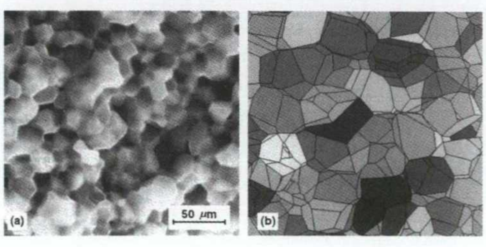
K. Sobczyk

Wind vs. Man-made Structures

How can we characterize the motion of the wind? Instead of a function of the sort $V(t)$, such as we know from school, we introduce a random function $V(t, \gamma)$, which depends not only on time t, but also on chance (γ symbolizes an "elementary event"). Such a function is in essence a family (set) of various possible realizations - i.e. deterministic velocity functions $v(t)$ that could occur. We derive basic information about this random function from observations and statistically-processed measurements. The simplest characterization of the function $V(t, \gamma)$ is its mean value (frequently written $m_V(t)$), which describes the average value of all the possible speeds at any given instant. Of course, they way that materials react to displacement or stress at a given point is likewise random (more frequently we say "stochastic") and a different random process $Y(t, \gamma)$ is used to characterize such processes. If we know the basic characteristics of the reaction, i.e. the process $Y(t, \gamma)$, we can estimate the probability of damage to the material or endurance. We shouldn't have to point out how important such information is for the practice of engineering!

need to introduce a random function Φ (r, γ) of the spatial variable r so as to capture the random heterogeneity of the medium (or the random unevenness of surfaces); the basic characteristics of this function must be determined using empirical data.

One chief physical phenomenon that attracts researchers' interest in this situation is the *scattering* of waves at "heterogeneities" of the medium. When an initial (incoming) wave reaches a point in the medium with different properties than its surroundings, it becomes a source of new waves - scattering waves. These overlap with the initial wave and cause random total field fluctuations (fluctuations in phase, amplitude, etc.); as a result the wave may be attenuated (a decay in amplitude), retarded, depolarized, or undergo a range of other phenomena. The objective of stochastic wave analysis is to quantitatively characterize such phenomena. Of course, in view of the difficulties, stochastic wave analysis is based on many physical hypotheses and mathematical assumptions that are subsequently empirically verified. One premise that facilitates effective analysis involves introducing a parameter into the mathematical model to describe the ratio of wavelengths to the size (scale) of random spatial heterogeneities. Stochastic wave analysis is nowadays not only its own quite theoretically advanced field; it has also proven quite useful in such fields as radio physics, geophysics, astrophysics, and acoustics.



K. Sobczyk

The true microstructure of aluminum Al2O3 and a simulated model of it using a random tessellation

Random microstructures

When we pick up a piece of metal (such as steel) and look at it closely, as long as its surface has been given the proper treatment, we have no doubt that it is a very *homogenous* material and it seems natural that basic science books on mechanics and strength of materials work with such metals. Indeed, for a long time, when the point was to characterize the global, macroscopic properties of materials, internal structure was not of interest. But when we want to glean a more profound knowledge of the basic construction of various materials used in technology (such as in electronics), it becomes important to pin down their microstructure, i.e. their morphology on the microscopic level (i.e. on scales much smaller than the scale of our direct sensory observations). Nowadays such macroscopically uniform materials are well known to have very complex microstructure e.g. poly-crystal bodies, that is aggregates of a vast number of anisotropic crystals, whose crystallographic axes are randomly oriented in space.

And so, an important problem has emerged for both research and development: how can we mathematically characterize the empirically observed, complicated and random microstructures of materials, and how can potential models of them be used in analyzing microscopic-level phenomena that occur in materials? Here again, the theory of probability comes to our aid, or more precisely a specific branch of this theory: stochastic geometry, which studies various geometrical objects and structures with random properties. One type of these highly interesting structures consists of random tessellations (or mosaics) – a family (set) of adjoining random polyhedra which fill the space. These polyhedra (or polygons, if in two dimensions) are random, because they have a differing, random number of vertices, edges, and sides, and are also of random volume. Although such tessellations first appeared in pure mathematics long ago (Dirichlet - 1850, Voronoi - 1908), they became a point of interest for a great many researchers only once applications arose. They have turned out to serve as adequate models for many real random cellular structures – including the grain microstructure of metals and ceramics.

When such a tessellated model is constructed for a given metallic microstructure under study, and its parameters are related to empirical data, we can then identify deformation and stress for the model, which in turn are necessary for characterizing such phenomena as the initiation of cracking. This is of course a random phenomenon, and so we express our expectation that it will occur in terms of probability.

It is not easy to construct a general model for a phenomenon such as cracking that would take account of the complex microstructure and its randomness. The micro-scale mechanisms of cracking have not yet been recognized. And although the stress field is the basic driving force here, just like in macroscopic mechanics, in micromechanics cracking is greatly complicated by the complexity of the microstructure, i.e. by its high spatial variability and randomness - here this is a hierarchical variability, evident in the relation between stress and the medium's properties on various spatial scales. Without a doubt, the problems outlined above in researching the phenomena in random microstructures (also in biological microstructures) have a place among the challenges faced by both contemporary applied mathematics and materials science, but also in a certain sense they belong to a more general *complexity science*. The present author is pleased to be able to take active part in pursuing such challenges. ■

Further reading:

Sobczyk K. (1985), *Stochastic Wave Propagation*, Amsterdam: Elsevier.

Sobczyk K., Spencer B.F. (1992), *Random Fatigue: From Data to Theory*, Boston: Academic Press.

Sobczyk K., Kirkner D.B. (2002), *Stochastic Modelling of Microstructures*, Boston: Birkhauser.

Sobczyk K. (2003), Reconstruction of random material microstructures:patterns of maximum entropy, *Probabilistic Eng. Mechanics (Elsevier)*, Vol. 18, 279-287.

The forces or stresses acting on modern technological constructions are frequently very irregular and develop randomly over time. This pertains not only to the tornado-torn buildings portrayed here, but also to tall radio or television antennas and open-sea drilling platforms. The stresses they sustain from gusts of wind or sea waves have a very complicated development over time, in the face of which traditional means of describing them (e.g. as periodic functions of time) break down

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