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A Fast Fourier Transform Solution for 1D Unsteady Heat Transfer Model Bounded by Varying Temperature

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Abstract

Using the Fourier transform to solve a one-dimensional heat conduction model with temperature $f(t)$ as the boundary requires complex integral transformation operations. According to the property of the Fourier transform, $f(t)$ is regarded as a symbol in the process operation, and the universal theory resolution of such a problem is established without directly solving the $f(t)$ transformation. $f(t)$ is then substituted into the theoretical solution to obtain the solution of the actual model. Using the theoretical solution, the solutions of 3 types of common functions are given. Combined with the characteristics of the model, precautions during the solving process are proposed. The example application demonstrates the establishment and application process of inversing model parameters based on the inflection point of temperature variation over time.

Keywords: Thermal conduction; Fourier transform; Theoretical solution; Common functions; Inflection point

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1. Introduction

One-dimensional (1D) heat conduction model bounded by varying temperature is a classic problem [1,2]. In this problem, the edge temperature $f(t)$ is set as $\varDelta T_0$ (that is the instantaneous increase of boundary temperature $\varDelta T_0$ remains unchanged) [3,4]. This model can be directly solved by the Laplace and Fourier transform [5,6].

Numerical algorithms are more and more widely used in solving heat conduction models in complex calculation regions [7–12], boundary conditions [13–15] and source/sink terms [14–16]. Although the analytical solution of the model is strictly limited [17,18] and difficult to solve [19,20], the solution is an important theorem to discuss the mathematical laws of the model [18–20]. The analytical solution of the model corresponding to various conditions such as heat transfer in porous media [21–23] and pollutant transport [24–26] has always been difficult, and a hot spot in related fields. The Fourier transform and other integral transformation methods are basic tools for finding the solution [21–29]. For the solution of a 1D heat conduction model, a complex integral transformation operation is required to solve the problem with different $f(t)$ [5,6]. To avoid the above complex integral transformation and solve the unsteady model of unconfined groundwater near a canal, Wu et al. [30] proposed to make full use of the nature of Fourier transformation and a fast Fourier transformation solution method when $f(t)$ is not in transformation. Wei et al. [31] studied a 1D thermal conduction problem with $f(t)$ being $e^{-\lambda t}$ using fast solving methods.

Fast solution is a theoretical solution applicable to such problems based on the Fourier transform convolution theorem. Then,

Nomenclature

a – thermal diffusivity, m^2/s
erfc(u) – the complementary error function
f – boundary temperature, °C
F – Fourier transform operator
F^{-1} – inverse Fourier transform operator
$\bar{u}$ – image function for Fourier transform
s – Fourier operator
t – time, d
T – temperature of calculation point, °C
T_0 – boundary temperature, °C
x – distance of the calculation point from the boundary, m

Greek symbols

$\delta(t-t_{i-1})$ – Heaviside function
Δ – instantaneous change
λ – boundary temperature variation rate, °C/d
φ – temperature variation rate of the calculation point, °C/h
ω – the conversion factor

Subscripts and Superscripts

a – air
g – inflection-point
n – time step number
t – time

Abbreviations and Acronyms

1D – one-dimensional
* – convolution operator

$f(t)$ is substituted into the theoretical solution to obtain an example solution. This solving process can eliminate complex integral transformation, making it fast and convenient.

The establishment process of the fast Fourier transform method is systematically presented. The solutions of 3 type functions are given using the fast solution method, and the precautions during the process are discussed. Based on the example applications, a method is established and applied for calculating the parameters using the inflection point in the temperature variation process.

2. Basic model

The 1D thermal conduction model is shown in Figs. 1 and 2, assuming:

(1) There is a heat source $f(t)$ with the temperature changing over time at the edge ($x = 0$) of a homogeneous sheet with an infinite length;
(2) The temperature at x is $T(x, t)$, and $T(x, 0) = 0$;
(3) The surface of the thin plate has no thermal exchange outside, and the edge thermal source forms 1D heat conduction.

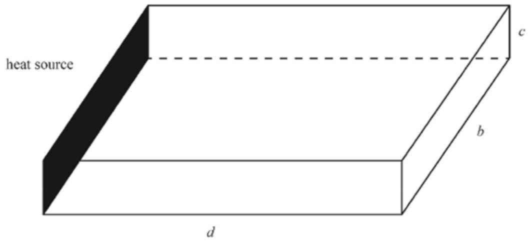

Fig. 1. Experimental materials.

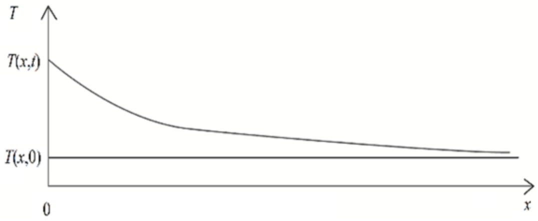

Fig. 2. Temperature change in the x direction.

The problem mentioned can be considered as the model (I):

$$
\text{(I)}\begin{cases}
\dfrac{\partial T}{\partial t} = a\dfrac{\partial^2 T}{\partial x^2} & (0 < x < +\infty, t > 0), \qquad \text{(10} \\
T(x,t)|_{t=0} = 0 & (x > 0), \qquad (2) \\
T(x,t)|_{x=0} = f(t) & (t \geq 0), \qquad (3)
\end{cases}
$$

where a [m^2/s] is the thermal diffusivity.

3. Theoretical solution

Let $u(x, t) = T(x, t) - T(x, 0)$, (I) be rewritten as the model (II):

$$
\text{(II)}\begin{cases}
\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} & (0 < x < +\infty, t > 0, \qquad (4) \\
T(x,t)|_{t=0} = 0 & (x > 0), \qquad (5) \\
u(x,t)|x = 0 = f(t) & (t \geq 0), \qquad (6)
\end{cases}
$$

Fourier transform about x is found for the model (II). In the model, x is a variable of $(0, +\infty)$ using a Fourier sine transform. From the Fourier laws, it can be obtained as follows:

$$F[u(x,t)] = \int_0^{\infty} u(x,t)\, sin\, \omega\, xdx = \bar{u}(\omega,t), \tag{7}$$

$$F\left\{\frac{\partial[u(x,t)]}{\partial t}\right\} = \frac{d[\bar{u}(\omega,t)]}{dt}, \tag{8}$$

$$F\left\{\frac{\partial^2[u(x,t)]}{\partial x^2}\right\} = \int_0^{\infty} \frac{\partial^2[\bar{u}(\omega,t)]}{\partial x^2} sin\, \omega\, xdx =$$

$$= \omega u|_{x=0} - \omega^2 \bar{u}(\omega,t), \tag{9}$$

$$F[u(x,t)|_{x=0}] = \omega f(t), \tag{10}$$

where $\bar{u}$ is the function after the Fourier transformation of x; ω is the conversion factor; F is the Fourier conversion, and F^{-1} is the Fourier inverse conversion.

From Eqs. (4), (8) and (9), we can obtain:

$$\frac{d[\bar{u}(\omega,t)]}{dt} = a[\omega u(x,t)|_{x=0} - \omega^2 \bar{u}(\omega,t)]. \tag{11}$$

The basic solution is derived from Eq. (11) and the boundary condition Eq. (10) is introduced as follows:

$$\bar{u}(\omega,t) = exp(-\omega a t) \int_0^t [\omega a f(\tau)\, exp(-\omega^2 a \tau)] d\tau. \tag{12}$$

The inverse Fourier transform of Eq. (12) is obtained as follows:

$$u(\omega,t) = F^{-1}[\bar{u}(\omega,t)] = \frac{2}{\pi}\int_0^\infty \left\{\exp(-\omega^2 at)\int_0^t [a\omega f(\tau)\exp(-\omega^2 a\tau)]d\tau\right\} sin\,\omega\,xd\omega =$$
$$= \frac{2a}{\pi}\int_0^t f(\tau)\left[\int_0^\infty \omega\,exp[-\omega^2 a(t-\tau)]\,sin\,\omega\,xd\omega\right]d\tau =$$
$$= \frac{2a}{\pi}\int_0^t \frac{f(\tau)}{2a(t-\tau)}\{-\exp[-\omega^2 a(t-\tau)]|_0^\infty + \int_0^\infty \exp[-\omega^2 a(t-\tau)]x\,cos\,\omega\,xd\omega\}d\tau =$$
$$= \frac{x}{\pi}\int_0^t \frac{f(\tau)}{t-\tau}\left\{\int_0^\infty exp[-\omega^2 a(t-\tau)]\,cos\,\omega\,xd\omega\right\}d\tau. \tag{13}$$

According to the characteristic function of the Fourier transform:

$$\int_0^\infty \exp(ax)\,cos\,\omega\,xdx = \frac{\sqrt{\pi}}{2\sqrt{a}}\exp(-\omega^2/4a). \tag{14}$$

From Eqs. (13) and (14), we have:

$$u(\omega,t) = \frac{x}{2\sqrt{\pi a}}\int_0^t \frac{f(\tau)}{(t-\tau)^{3/2}}\exp[-x^2/4a(t-\tau)]d\tau. \tag{15}$$

Eq. (15) is the solution of the 1D model under *f*(*t*). From Eq. (15), based on the convolution theorem, we have:

$$u(x,t) = \frac{x}{2\sqrt{\pi a}}\int_0^t \frac{f(\tau)}{(t-\tau)^{3/2}}exp[-x^2/4a(t-\tau)]\,d\tau =$$
$$= f(t) * \frac{x}{2\sqrt{\pi a t}}exp(-x^2/4at) = \tag{16}$$
$$= f(t) * \frac{d}{dt}\left[\frac{2}{\sqrt{\pi}}\int_{\frac{x}{2\sqrt{at}}}^\infty exp(-\tau^2)d\tau\right] = f(t) * \frac{d}{dt}\left[\text{erfc}\left(\frac{x}{2\sqrt{at}}\right)\right],$$

where * is the convolution operator and erfc(x) is a complementary error function.

According to the differential property of convolution, it is obtained that:

$$f(t) * \frac{d}{dt}\left[\text{erfc}\left(\frac{x}{2\sqrt{at}}\right)\right] + \text{erfc}\left(\frac{x}{2\sqrt{at}}\right)\bigg|_{t=0} f(t) =$$
$$= \text{erfc}\left(\frac{x}{2\sqrt{at}}\right) * \frac{d[f(t)]}{dt} + f(t)|_{t=0}\text{erfc}\left(\frac{x}{2\sqrt{at}}\right). \tag{17}$$

From the relationship between Eq. (16) and Eq. (17), $\text{erfc}(\frac{x}{2\sqrt{at}})\Big|_{t=0} = 0$, and $u(x,t) = T(x,t) - T(x,0)$. According to Eq. (17), it is obtained that:

$$T(x,t) = f(t)|_{t=0}\text{erfc}\left(\frac{x}{2\sqrt{at}}\right) + \int_0^t \left[\frac{d[f(t)]}{dt}\text{erfc}\left(\frac{x}{2\sqrt{a\tau}}\right)\right]d\tau. \tag{18}$$

Equation (18) is the solution of the model obtained without finding its Fourier transform image function of *f*(*t*). It is true for all boundary functions *f*(*t*) constituting the Dirichlet condition, that is Eq. (18) is the theoretical resolution of this problem. The application of Fourier and Laplace transform properties will facilitate the solution of the problem [17]. When *f*(*t*) of the practical problem is determined, a solution can be obtained by submitting *f*(*t*) into Eq. (18).

4. Solution for a common boundary function

Providing functional expressions applicable to actual boundaries is the foundation of mathematical models [32]. For the convenience of application, based on the theoretical solution, the solution is given when *f*(*t*) is a constant function, polynomial function and basic elementary common function.

4.1. Constant function

Function $f(t) = \Delta T_0$, where ΔT_0 is a constant. It is the boundary condition of the classical 1D model, that is when $t \to 0^+$, the boundary temperature will remain constant after an instantaneous change of T_0.

Substituting $f(0) = \Delta T_0$ into Eq. (18), due to $d[f(t)]/dt = d[\Delta T_0]/dt = 0$, we have:

$$T(x,t) = \Delta T_0\text{erfc}\left(\frac{x}{2\sqrt{at}}\right). \tag{19}$$

4.2. Multi-order function

In practical work, since the observation of the time-varying process *f*(*t*) of the target variable is discrete, piecewise functions are usually used to represent *f*(*t*) based on discrete measurement data. Among them, step functions and linear interpolation functions are commonly used forms in engineering technology.

1) Step function

For the boundary temperature $f(t_i,t_{i+1})$ between $t_i - t_{i+1}$ $(I \geq 2)$, when using the average temperature in the period $(f_i + f_{i+1})/2$ or the increment of $f_{i+1} - f_i$, we have:-

$$f(t) = \Delta T_0 + \sum_{i=2}^n (f_i - f_{i-1})\delta(t - t_{i-1}),$$
$$(t > t_{i-1}, i \in N^*), \tag{20}$$

when $t < t_{i-1}$, $\delta(t - t_{i-1}) = 0$ and when $t \geq t_{i-1}$, $\delta(t - t_{i-1}) = 1$.

Substituting Eq. (20) into Eq. (18), $\delta(t - t_{i-1})$ functionality and $f(0) = \Delta T_0$ are noted as follows:

$$T(x,t) = \Delta T_0\text{erfc}\left(\frac{x}{2\sqrt{at}}\right) +$$
$$+\sum_{i=2}^n (f_i - f_{i-1})\int_0^t \text{erfc}\left[\frac{x}{2\sqrt{a(\tau - t_{i-1})}}\right]d\tau. \tag{21}$$

Note: ΔT_0 means that variation occurs at $t \to 0^+$ and maintains for a long period of time. The time that remains unchanged is $t_1 - t_0$, and $i = 2 - n$.

2) Lagrange linear interpolation function

When the variables have certain changes in each test period $t_i - t_{i+1}$, linear interpolation methods could be used for *f*(*t*), representing as follows:

$$f(t) = \Delta T_0 + \sum_{i=2}^n (f_i - f_{i-1})\frac{t - t_{i-1}}{t_i - t_{i-1}}\delta\,(t - t_{i-1}). \tag{22}$$

Substituting Eq. (22) into Eq. (18), we obtain:

$$T(x,t) = \Delta T_0\text{erfc}\left(\frac{x}{2\sqrt{at}}\right) + \sum_{i=2}^n \frac{f_i - f_{i-1}}{t_i - t_{i-1}}\int_{t_{i-1}}^t \text{erfc}\left(\frac{x}{2\sqrt{a\tau}}\right)d\tau. \tag{23}$$

Based on the definition of ΔT_0, when establishing the *f*(*t*) piecewise function, attention should be paid to the expression methods of each time interval.

4.3. Basic elementary function

1) Exponential decay function.

When $f(t) = \Delta T_0 \exp(-\lambda t)$（$\lambda > 0$), substituting it into Eq. (18), we obtain the following formula:

$$T(x,t) = \mathit{\Delta} T_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{at}}\right) + \\ -\lambda \mathit{\Delta} T_0 \int_0^t \exp(-\lambda\tau) \operatorname{erfc}\left[\frac{x}{2\sqrt{a(t-\tau)}}\right] d\tau. \tag{24}$$

2) Logarithmic function.

When $f(t) = \Delta T_0 log_a(t)$, substituting it into Eq. (18), it can be obtained that:

$$T(x,t) = \mathit{\Delta} T_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{at}}\right) + \frac{1}{ln\, a}\int_0^t \frac{1}{\tau} \operatorname{erfc}\left[\frac{x}{2\sqrt{a(t-\tau)}}\right] d\tau. \tag{25}$$

3) Trigonometric function.

Taking $f(t) = \Delta T_0 \cos(t)$ for example, substituting it into Eq. (18), it can be obtained that:

$$T(x,t) = \mathit{\Delta} T_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{at}}\right) - \mathit{\Delta} T_0 \int_0^t sin(\tau) \operatorname{erfc}\left[\frac{x}{2\sqrt{a(t-\tau)}}\right] d\tau. \tag{26}$$

After the edge condition is determined, $f(t)$ is substituted into the equation, and the analytical solution of the model can be obtained more conveniently and quickly. The solutions of 3 function types are obtained, which are convenient for practical reference and application. When $f(t)$ is determined, the above solution may be further expanded by using the step-by-step integration. Numerical algorithms for analytical solutions can also be established based on [31], providing convenience for frequent applications.

5. Application of solution

One of the important purposes of researching problem-solving solutions is to establish inversion model parameters based on the analytical solution of the model and the observed data of the target variable changing over time [33−37]. Based on the solution of Eq. (21), when $i = 2$, the curve is shown in Fig. 3.

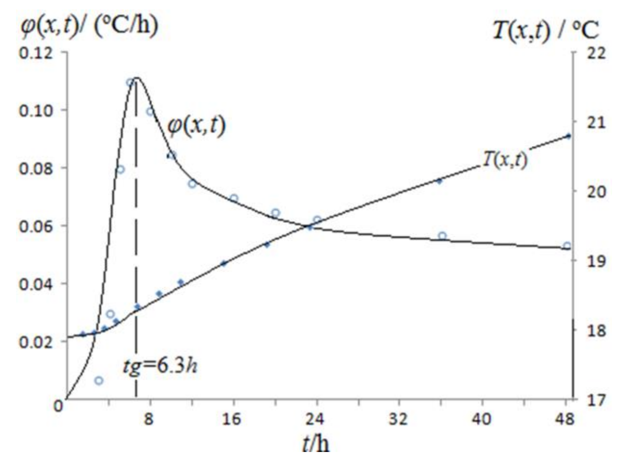

Fig. 3. Temperature variation rate at $x = 0.5$ m.

5.1. Calculation of model parameters

The partial derivative of T in Eq. (21) is calculated, $\varphi(x, t) = \partial T(x, t)/\partial t$, $\varphi(x, t)$ is the temperature variation rate at point x, then:

$$\phi(x,t) = \mathit{\Delta} T_0 \frac{2^{-3/2}}{2\sqrt{\pi a}} exp\left(\frac{x^2}{4at}\right) + \sum_{i=2}^{n} \frac{f_i - f_{i-1}}{t_i - t_{i-1}} \operatorname{erfc}\left[\frac{x}{2\sqrt{a(t-t_{i-1})}}\right]. \tag{27}$$

As $n = 2$, Eq. (27) can be rewritten as follows:

$$\phi(x,t) = \mathit{\Delta} T_0 \frac{t^{-3/2}}{2\sqrt{\pi a}} exp\left(\frac{x^2}{4at}\right) + \lambda \operatorname{erfc}\left(\frac{x}{2\sqrt{at}}\right), \tag{28}$$

where $\lambda = (f_2 - f_1)/(t_2 - t_1)$ expresses the slope change of the boundary temperature in the period of $t_1 - t_2$.

For Eq. (28), the partial derivative about t is found as follows:

$$\frac{\partial\phi(x,t)}{\partial t} = \frac{1}{2\sqrt{\pi a t^5}} \exp\left(-\frac{x^2}{4at}\right)\left[\mathit{\Delta} T_0\left(-\frac{3}{2} + \frac{x^2}{4at}\right) + \lambda t\right]. \tag{29}$$

At the inflection point of the curve in Fig. 3, $\partial\varphi(x,t)/\partial t = 0$, $t = t_g$. Two roots can be obtained from Eq. (29), where the mathematically and physically reasonable roots are expressed as follows:

$$\frac{\partial\phi(x,t)}{\partial t} = \frac{1}{2\sqrt{\pi a t^5}} exp\left(-\frac{x^2}{4at}\right)\left[\mathit{\Delta} T_0\left(-\frac{3}{2} + \frac{x^2}{4at}\right) + \lambda t\right]. \tag{30}$$

According to Eq. (28), the model parameter a can be obtained (at time t_g, ΔT_0, λ, x are known) as follows:

$$a = \frac{x^2}{2t_g(3 - 2\lambda t_g)/\mathit{\Delta} T_0}. \tag{31}$$

It should be noted that due to the differences in model boundary conditions [30,31,38] and source/sink term [14], the calculation formula for the parameter a at the inflection point method is also different.

5.2. Case study

A sample of silty mudstone obtained from the ground-source heat pump drilling site in Anhui Province with dimensions of $d = 3.0$ m, $b = 1.5$ m, and $c = 0.3$ m was used (refer to Fig. 1). A steel pipe with an outer diameter of 0.15 m was pre-installed at one end of this sample. In addition, a detailed insulation treatment was carried out on the surrounding environment as well as at the top and bottom surfaces of the steel pipes and concrete specimens. According to the standard protocol outlined in "Determination of Steady-State Thermal Resistance and Related Properties of Thermal Insulation Materials – Guarded Hot Plate Method (GB10294)", the specimen has undergone a guarded heat treatment procedure. The "steady-state method" is used to ensure a controlled and consistent thermal environment throughout the testing phase. A temperature measuring optical fibre was installed at 0.2 m from the steel pipe in the sample for continuous testing.

In a 2-day test, the initial temperature was $T(x, 0) = 18.06$°C. At the beginning, 36°C water was imported into the boundary slot, and then $T(x, t)$ was reduced slowly. At the end of the test, the boundary temperature was 35.5°C. So, $\Delta T_0 = 17.94$°C, and $\lambda = -0.25$°C/d.

The temperature measurements taken at 0.5 m from the heating device are shown in Table 1 (during the first two hours of the experiment, the temperature response was insensitive).

To ascertain the inflection point, the interpolation technique based on the recorded temperatures is used to calculate the temperature change rate $\varphi(x, t)$. To improve the calculation accuracy, it is recommended to increase the data density near the inflection point.

It can be seen from Fig. 3 that $\varphi(x, t) - t$, $t_g = 6.3$ h. According to Eq. (31), $a = 1.85 \times 10^{-6}$ m^2/s.

During the experiment, using forward or backward interpolation to calculate the temperature change rate $\varphi(x, t)$ will have a certain impact on determining the inflection point time. However, this impact can be effectively minimized by applying multiple encryption methods.

Table 1. Temperature at a distance of $x = 0.50$ m.

t / h	**2**	**3**	**4**	**5**	**7**	**9**	**11**	**15**	**18**	**24**	**36**	**48**
$T(x,t)$, °C	17.86	17.87	17.93	18.04	18.25	18.43	18.6	18.78	19.14	19.39	20.07	20.71
$\varphi(x,t)$, °C/h	0.006	0.009	0.059	0.109	0.104	0.089	0.084	0.069	0.064	0.062	0.056	0.052

6. Conclusions

Based on the research of the one-dimensional heat conduction model with variable temperature constraints, the following conclusions are drawn in establishing the fast Fourier transform method:

1) For the variable temperature boundary $f(t)$, a theoretical resolution of such models has been established using the differential properties of Fourier's law and the convolution theorem.
2) Based on theoretical solutions, there is an urgent need for practical models to avoid the tedious and complex Fourier transform.
3) There was an inflection point in the temperature over time, which can be used to invert the model parameter a.
4) Calculating the temperature change rate $\varphi(x,t)$ based on actual measured temperatures, using forward or backward interpolation can cause a degree of interference to the experimental results. To mitigate this effect, it is recommended to increase the frequency of data recording near the inflection point when determining the time of occurrence based on recorded data.

It should be noted that in the solving process, when the operator $f(t)$ is used for the Fourier transform, $f(t)$ should meet the specification of Fourier conversion [39], that is $f(t)$ should satisfy the Dirichlet condition: $f(t)$ is a succession function or has only a finite number of first-order discontinuities and only a finite number of extreme points.

The one-dimensional heat conduction model uses a fast Fourier transform solution that leverages the properties of the Fourier transform. This method simplifies the solution process by excluding boundary conditions from the complex transformation procedures. Using dynamic temperature monitoring data from the temperature field, the method can calculate the thermophysical properties of the experimental material [40] (i.e. thermal diffusivity or thermal conductivity coefficient of the model). This calculation represents a significant objective in studying such problems. In addition, this method has a significant reference for models controlled by the heat conduction equation, including groundwater flow models and pollutant diffusion models [40–44].

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