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SCALING OF FLOW PHENOMENA IN CIRCULATING FLUIDIZED BED BOILERS

Paweł Mirek

Częstochowa University of Technology, Faculty of Environmental Protection and Engineering,
Department of Heating, Ventilation and Air Protection,
ul. Dąbrowskiego 69, 42-200 Częstochowa, Poland

The paper presents an overview of scaling models used for determining hydrodynamic parameters of Circulating Fluidized Bed boilers. The governing equations and the corresponding dimensionless numbers are derived and presented for three different approaches to the scaling law of fluidized beds: classical dimensional analysis, differential equations and integrated solutions and experimental correlations. Some results obtained with these equations are presented. Finally, the capabilities and limitations of scaling experiments are discussed.

Keywords: scaling, fluidized bed, hydrodynamics

1. INTRODUCTION

A Circulating Fluidized Bed (CFB) boiler is a device for generating steam by burning fossil fuels in a furnace operated under special hydrodynamic conditions: where fine solids (Geldart group A or B) are transported through the furnace at a velocity exceeding the terminal velocity of average particles, yet there is a degree of refluxing of solids adequate to ensure uniformity of temperature in the furnace (Basu and Fraser, 1991). CFB boilers have a number of unique features, which make them more attractive than other solid fuel fired boilers. These features include: fuel flexibility, high combustion efficiency, efficient sulfur removal, smaller furnace cross-section, low NO_2 emission, fewer feed points, good turndown and load-following capability. Owing to a large number of CFB boilers' advantages it is expected that the capacity will increase in the future and an increasing number of plants will be in place in the next few years. This has been made possible by focused research and development (R&D) effort over the last forty years, involving CFB units of various sizes, ranging from a few centimeters in diameter to demonstration size plants. Unfortunately, owing to its large complexity a complete description of CFB boiler hydrodynamics is not yet possible after more than two decades of enthusiastic research. Moreover, much of the R&D work can be still done only in small-scale units, mostly of laboratory size. Thus, in the development of CFB technology much attention has been paid to dimensionless analysis.

It should be kept in mind, that in a circulating fluidized bed reactor three distinct phenomenological sub-processes influencing each other take place (i.e. hydrodynamic gas-solid contact, heat and mass transfer and chemical reactions) (Horio, 1997). Therefore, scaling of CFB processes is connected with establishing such a balance among hydrodynamics, chemistry and heat and mass transfer, so that the total output from the system satisfies a large-scale demand, maintaining the quality established in

*Corresponding author, e-mail: pmirek@neo.pl

small-scale units (Horio, 1997). In other words, in this situation, in addition to geometric similarity, key-parameters are similar between small and large and cold and hot units.

In most cases, scaling of CFB processes is limited to hydrodynamics (Horio, 1997). In that situation, there are three different approaches to the scaling law of fluidized beds:

- classical dimensional analysis,
- differential equations,
- integrated solutions and experimental correlations.

In the case of circulating fluidized beds there is always a coexistence of flow structures of three different length scales (Horio, 1997), i.e.:

- the macroscopic issue of radial and axial time-average velocity and voidage distributions,
- the mesoscale issue of clusters and velocity fluctuations,
- the microscopic issue of the flow field around each particle.

Because achieving a complete similarity of flow for all the different scales is difficult and very often unnecessary, in many practical cases similarity can be limited to micro- and mesoscale ranks.

2. DERIVATION OF SCALING MODELS

2.1. Dimensional analysis

In the dimensional analysis approach for deriving scaling relationships the main assumption is that the ratio of all the important forces acting on particles is the same in the model as it is in the full-scale bed. The additional conditions are that the scale model uses geometry similar to that of the larger fluidized bed and that the distribution of particle sizes (PSD) is similar between the model and a commercial unit. Gas flow in circulating fluidized beds can be characterized by viscous, inertia and particle-to-gas forces. The forces acting on particles include gravity, particle inertia, collision forces between particles and between particles and wall, gas interaction with the particles such as drag, electrostatic and adhesion forces, i.e. particle surface forces. There are many difficulties connected with the quantification of particle surface forces for small particles (Geldart group A). However, for larger particles (Geldart group B and D) these forces can be neglected. In Table 1, a list of the most important forces acting on a particle in a circulating fluidized bed boiler is presented.

Table 1. Ratio of forces acting on a particle in a circulating fluidized bed

$\mathrm{Re}_d = U_0 d_p \rho_s / \mu$	Particle inertia/gas viscous force
$\mathrm{Re}_D = U_0 D_h \rho_f / \mu$	Gas inertia/Gas viscous force
$Fr = U_0^2 / (g D_h)$	Inertia/gravity force
ρ_s / ρ_f	Soild inertia/gas inertia force
Debatable form	Surface forces – collision, adhesion

As follows from Table 1, the form of the surface force ratio is still debatable. There are limited results which suggest that within the normal range of particle and wall coefficient of restitution and particle-to-particle sliding friction, surface forces play an insignificant influence on the bed dynamics (Litka and Glicksman, 1985; Yang, 2003). Besides the correlations presented in Table 1, there are some additional important ratios, i.e. solid recycle volumetric flow/gas volumetric flow rate $G_s / (\rho_s U_0)$ and bed height/bed diameter *L/D*.

When all the force ratios are the same for a scale model and a commercial bed then the scale model should reflect dynamic behaviour of a gas-solid mixture occurring in the commercial bed. This state is described as a full hydrodynamic similarity of CFBs and the corresponding scaling relationships can be written as follows

$$\frac{U_0 d_p \rho_s}{\mu}, \frac{U_0 D_h \rho_f}{\mu}, \frac{U_0^2}{g D_h}, \frac{\rho_s}{\rho_f}, \frac{G_s}{\rho_s U_0}, \frac{L}{D}, PSD, \varphi, \text{model geometry} \tag{1}$$

A similar set of similitude invariants derived on the basis of π-theorem of the dimensional theory has been proposed by (Teplitskiy and Ryabov, 1999). Hydrodynamics of two CFBs units will be similar if the six following groups in them are equal to each other, respectively

$$Ar, Fr = \frac{(U_0 - u_T)^2}{gH}, \mathrm{Re} = \frac{U_0 d_p}{\nu_f}, \frac{d_p}{D}, \frac{\rho_s}{\rho_f}, \frac{H_{mf}}{H} \tag{2}$$

On the basis of dimensional groups of set (2) the basic laboratory characteristics of a model can be calculated, i.e.: d_p, D, U_0, ρ_s, H, H_{mf}. It is worthy to note that in the literature on dimensional analysis of circulating fluidized beds it has been common to neglect electrostatic forces and cohesion, particle shape, particle size distribution, particle to particle and particle to wall coefficients of friction and restitution (Glicksman et al., 1993).

2.2. Differential equations

An alternate approach to formulate scaling relationships is nondimensionalization of momentum equations for gas and solids. For Anderson and Jackson two-fluid model (Anderson and Jackson, 1967), given by equation of motion for gas:

$$\rho_f \in \left[\frac{\partial u}{\partial t} + (u \cdot \nabla) u\right] = \in \rho_f \vec{g} - \in \nabla p - R \tag{3}$$

equation of motion for solids:

$$\rho_s (1 - \in)\left(\frac{\partial v}{\partial t} + (v \cdot \nabla) v\right) = (1 - \in) \rho_s \vec{g} - (1 - \in) \nabla p + R + \nabla \cdot P_s \tag{4}$$

where R denotes:

$$R = \beta(u - v) + (1 - \in) M \rho_f \left[\frac{D(u - v)}{Dt}\right] \tag{5}$$

and the gas-solid drag factor β can be expressed for homogeneous dilute suspensions (Horio, 1997):

$$\beta = (\rho_s - \rho_f)(1 - \in) g / u_T \in^{n-2} \qquad (\in_{mf} \ll \in \le 1) \tag{6}$$

for incipiently fluidized suspensions (Horio, 1997):

$$\beta = (\rho_s - \rho_f)(1 - \in_{mf}) \in_{mf}^2 g / U_{mf} \qquad (\in \cong \in_{mf}) \tag{7}$$

The dimensionless expression of momentum Equations (3) and (4) have been proposed by [2] in the following forms

$$\frac{\rho_f}{\rho_s} \in \left[\frac{\partial \hat{u}}{\partial \hat{t}} + (\hat{u} \cdot \hat{\nabla}) \hat{u} - \frac{\vec{g} l}{U_0^2}\right] + \in \hat{\nabla} \hat{p} + \frac{\beta l}{\rho_s U_0}\left(\hat{u} - \left(\frac{v_0}{U_0}\right) \hat{v}\right) = 0 \tag{8}$$

$$(1-\in)\left[\left(\frac{U_0}{v_0}\right)\frac{\partial\hat{v}}{\partial\hat{t}}+\left(\hat{v}\cdot\hat{\nabla}\right)\hat{v}+\left(\frac{U_0}{v_0}\right)^2\left(-\frac{\vec{g}l}{U_0^2}+\hat{\nabla}\hat{p}\right)\right]$$
$$-\frac{\beta l}{\rho_s U_0}\left(\frac{U_0}{v_0}\right)^2\left(\hat{u}-\left(\frac{v_0}{U_0}\right)\hat{v}\right)-\hat{\nabla}\cdot\hat{P}_s=0 \tag{9}$$

Where $\hat{t}\equiv t/(l/U_0),\hat{u}\equiv u/U_0,\hat{v}\equiv v/U_0,\hat{p}\equiv p/\left(\rho_s U_0^2\right),\hat{P}_s\equiv P_s/\left(\rho_s U_0^2\right),\hat{\nabla}\equiv l\nabla$

In most cases $\rho_f / \rho_s << 1$, therefore, Equation (8) can be simplified to

$$\in\hat{\nabla}\hat{p}+\frac{\beta l}{\rho_s U_0}\left(\hat{u}-\hat{v}\right)=0 \tag{10}$$

The term $\beta l /(\rho_s U_0)$ can be expressed with the help of Equations (6) and (7) correspondingly as follows

$$\frac{\beta l}{\rho_s U_0}\cong\frac{gl}{U_0^2}\frac{U_0}{u_T}(1-\in)\in^{n-2} \qquad (\in_{mf}\ll\in\leq 1) \tag{11}$$

$$\frac{\beta l}{\rho_s U_0}=\frac{gl}{U_0^2}\frac{U_0}{U_{mf}}(1-\in_{mf})\in_{mf}^2 \qquad (\in\cong\in_{mf}) \tag{12}$$

The scaling relationships derived from Equations (8), (9) and (10) can be written in the following form:

$$\frac{gl}{U_0^2},\frac{v_0}{U_0},\frac{u_T}{U_0},\frac{\rho_s}{\rho_f}, \text{model geometry} \tag{13}$$

where l is the reference length scale. If the scale factor $k = l / l^m$ between a geometrically similar model and the full scale-bed is introduced to the set (13), the scaling relationships can be rewritten as follows

$$\begin{aligned}&\frac{U_0}{v_0}=\frac{U_0^m}{v_0^m}=\sqrt{k}\\&\frac{u_T}{u_T^m}=\sqrt{k} \qquad (\in_{mf}\ll\in\leq 1)\\&\frac{U_{mf}}{U_{mf}^m}=\sqrt{k} \qquad (\in_{mf}\ll\in\leq 1)\end{aligned} \tag{14}$$

It is necessary to keep in mind, that the scaling relationships derived from differential equations can be only as good as the primary model is. Moreover, many researchers argue that the main limitation of differential equations is the lack information on macroscopic flow fields. Therefore, as the third approach to the derivation of scaling law of fluidized beds integrated solutions and experimental correlations are considered.

2.3. Integrated solutions and experimental correlations

In the fluidized bed literature there is a large number of experimental and semi-experimental correlations to predict either fluid dynamic or overall process behavior. Assuming that the particle acceleration ratio is dependent on the drag force, gravity force and buoyancy force acting on the particle, a new similarity dimensionless number called particle-Froude-number can be derived (Shi and Reh, 2005).

$$Fr_p = Fr_m - Fr_{m,t} = \frac{U_0 - u_T}{\sqrt{gd_p}} \sqrt{\frac{\rho_f}{\rho_s - \rho_f}} \tag{15}$$

Because the vertical profile of cross-section-momentum solids volume fraction in CFB strongly depends on a bed inventory, particle size distribution and secondary air ratio, a modified form of particle-Froude-number is introduced as follows

$$Fr_p^* = Fr_p \cdot \left[4 \cdot \left(1 - \frac{\dot{Q}_{SA}}{\dot{Q}_{Tot}}\right)\right]^{m_1} \cdot \left(6.5 \cdot \frac{z_{mf}}{z_b}\right)^{m_2} \cdot \left(\frac{d_{50}}{d_m}\right)^{1/2} \tag{16}$$

Where m_1, m_2 and d_m are to be determined from experimental data. The axial profiles of cross-section-momentum solids volume fraction are almost the same if their respective particle-Froude-numbers are similar, even though their Archimedes numbers and superficial gas velocities may be totally different (Shi and Reh, 2005). Although the particle-Froude-number is a promising similarity dimensionless number for gas-solid flow in CFB, it is necessary to carry out further investigations, especially between two geometrically similar units.

A derivation of scaling relationships using integrated solutions and semi-empirical correlations can give a similar set of nondimensional numbers to that of differential equations. In comparison to differential equations the main difference lies in physical aspects of CFB flow structure. In this respect, integrated solutions give a better insight into and understanding that structure. According to the core-annulus model for flow in circulating fluidized beds the necessary conditions for macroscopic flow structure are stated as follows (Horio, 1997)

$$\frac{U_0}{U_0^m} = \frac{v_0}{v_0^m} = \frac{u_{T,cl,C}}{u_{T,cl,C}^m} = \sqrt{k} \tag{17}$$

The set (14) satisfies the requirements of similar distribution of solids between core and annulus, equal dimensionless core radius, same voidage distribution and similar division of gas flow to core and annulus. For geometrically similar CFB units the cluster size will be similar if

$$\frac{d_{cl}}{d_{cl}^m} = k \qquad \text{if} \qquad \rho_f/\rho_s \ll 1 - \epsilon_{lean}, \qquad \rho_s = \rho_s^m \tag{18}$$

or

$$\frac{d_{cl}}{d_{cl}^m} = k \qquad \text{if} \qquad \rho_f/\rho_s \ll \left(\rho_f/\rho_s\right)^m \tag{19}$$

Additional requirements of the same voidage in cluster phases is fulfilled when

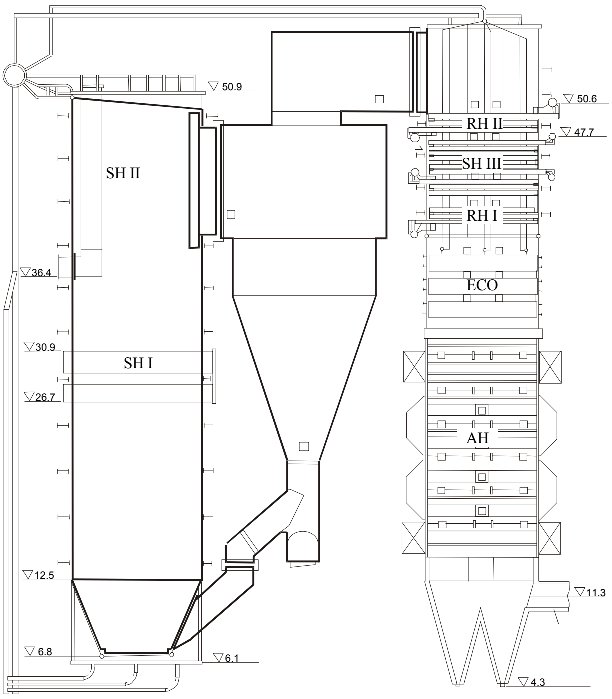
$$\frac{u_{sl,cl}}{u_{sl,cl}^m} = \frac{u_T}{u_T^m} = \sqrt{k} \tag{20}$$

As can be seen, the set of Equations (17), (18), (19) and (20) is the same as the set of Equations (14).

3. CALCULATION OF THE PARAMETERS OF A LABORATORY CFB MODELING INSTALLATION

As an example of using nondimensional numbers for scaling CFB hydrodynamics the data from a large commercial unit CFB 670 working in the Turow Power Plant, Poland, has been utilised. A schematic diagram of the boiler as well as its basic operational parameters have been presented in Figure 1.

The most popular case for reflecting hydrodynamic conditions of a large CFB boiler in a small scale laboratory unit with a preserved geometric similarity comes from carrying out experiments with air close to atmospheric pressure and temperature which yields a gas density of 1.2kg/m^3. In this case, full hydrodynamic similarity may be obtained using the set of Equations (1), (2) or (13). This leads to evaluation of the following set of parameters: U_0, G_s, d, ρ_p and D. As follows from Table 1, conducting experiments with air viscosity of $1.8{\cdot}10^{-5}$ Pa·s is connected with using a very high density (2700·3.891=10506 kg/m^3) and very small particles (66.3 μm) in a small unit. An additional problem is the dimension of the small unit, which is only four times smaller than the commercial one.



a)

CFB 670 operational parameters

Electric power	235	MW
Efficiency	90	%
Useful heat output	529	MW
Fuel consumption	71.3	kg/s
Live steam mass flow	185.4	kg/s
Live steam pressure	13.2	MPa
Fuel	Brown coal	

Combustion chamber

Height	44.8	m
Cross-section – grid level	21.2 x 5.2	m^2
Cross-section	21.2 x 9.9	m^2

b)

Fig.1. Schematic diagram (a) and operational parameters (b) of CFB 670 boiler working in Turow Power Plant, Poland

Table 2. Operational parameters of large-scale CFB 670 boiler and small-scale corresponding equivalents evaluated for 5, 4, 3 and 2 dimensionless groups

Parameter	CFB 670MW	Number of dimensionless groups			
		5	4	3	2
U_0, m/s	5.5	$0.47U_0$	$0.224U_0$	$0.224U_0$	$0.248U_0$
G_s, kg/m^{2}s	9.3	$1.829G_s$	$0.870G_s$	$0.215G_s$	$0.239G_s$
d, μm	300	$0.221d$	$0.05d$	$0.306d$	**70**
ρ_p, kg/m^3	2700	$3.891\rho_p$	$3.891\rho_p$	**2600**	**2600**
D, m	13.5	$0.221D$	**0.675***	**0.675***	**0.675***
ρ_f, kg/m^3	0.31	**1.20**	**1.20**	**1.20**	**1.20**
μ, Pa·s	$4.456{\cdot}10^{-5}$	**$1.8{\cdot}10^{-5}$**	**$1.8{\cdot}10^{-5}$**	**$1.8{\cdot}10^{-5}$**	**$1.8{\cdot}10^{-5}$**
U_{mf}, m/s	0.038	**0.0179**	**0.0009**	**0.0085**	**0.0049**
U_t, m/s	0.98	**1.037**	**0.069**	**0.519**	**0.325**
Ar	**111**	**111**	**1.29**	**74**	**33**
Re_d	**11.46**	**11.46**	**1.23**	**7.56**	**6.4**
Re_D	**515607**	**515607**	**55530**	**55530**	**61708**

*Scale factor k = 20; Gas density 1.2 kg/m^3

When experiments on a small-scale are carried out on a test-stand of a free scale factor, the condition d_p/D may be neglected and the four following parameters U_0, G_s, d_p and ρ_p will be evaluated. Employing four groups, the set of dimensionless numbers can be written according to (Horio et al. 1989) as follows

$$\frac{gl}{U_0^2}, \frac{u_T}{U_0}, \frac{\rho_s}{\rho_f}, \frac{G_s}{\rho_p U_0} \tag{21}$$

As follows from experimental verification, Set (21) can be successfully applied over the whole CFB flow regime (Glicksman et al., 1993).

Owing to the problems in using a polydispersed mixture of a very high density particles in a small-scale unit, in most cases the fulfillment of the condition ρ_s / ρ_f is practically impossible. As can be seen in many experiments, neglecting this group would give dissimilarity of the sizes of clusters of particles in the riser, but would retain the macroscopic flow field (Van der Meer, 1997).

The least restrictive case in scaling CFB hydrodynamics is when only two dimensionless groups have been employed. As follows from Table 1, in such a case there is no problem with using particles of any density and diameter as well as carry out experiments on a small-scale unit with a free scale factor. Unfortunately, when the density ratio is neglected roughly similar axial solids volume fraction profile is to be expected. In this case, the level of dissimilarity will depend on the degree to which the condition ρ_s / ρ_f is different for the commercial and laboratory unit though the fluidization regime will be similar.

4. PROBLEMS WITH VERIFICATION OF SCALING MODELS IN CFB BOILERS

Scaling laws validation has been successfully carried out by many authors (Ake and Glicksman, 1988; Horio et al, 1989; Ishii and Murakami, 1991). In scaling experiments, attention must be paid to several aspects resulting from the unique nature of CFB process. As can be seen in Table 1, the particle size in a laboratory unit sometimes has to be reduced so much that a smaller unit operates with a different Geldart group of particles (Knowlton et al., 2005). In this situation, a commercial bed can operate with group B solids, but the small bed will operate with group A solids. As a result different fluidization behaviour of two different groups of particles can be observed in the big and small units. Moreover, during scaling experiments special attention must be paid to maintain the same flow regime in two CFBs. Therefore, a successful application of scaling laws is connected with keeping the same particle Geldart group and flow regime in both units.

Most scaling experiments have been carried out on two geometrically similar but still laboratory size CFBs units. Moreover, most of them have been conducted with using a monodispersed fraction of solids. Thus, it is necessary to carry out further scaling experiments which aim at reflecting hydrodynamics conditions of a real size CFB boiler on a small-size unit. This kind of a lab test is connected with using a polydispersed mixture of inert material instead of a monodispersed one. The verification of scaling laws using solids with a distribution of sizes opens discussion on the issue of a place, where a sample of inert material in a large-scale CFB boiler has to be collected. This kind of research work entails further investigations to elucidate.

Another problem with scaling experiments concerns adaptation of a small-scale unit to work with a very small particles, which in most cases cannot be separated in its cyclones. The problem can be solved with additional separators arranged outside the test-stand, but it should be kept in mind, that from that moment very small particles are not a part of the flow occurring in a main circulation loop of the test-stand.

5. CONCLUSIONS

The performance of a circulating fluidized bed boiler is controlled to a large extent by the bed's hydrodynamics. Details of the bed's hydrodynamic behaviour as well as its overall performance can be successfully studied with the help of cold experimental models with geometrical similarity to the large commercial bed being preserved and with the use of scaling relationships. A full hydrodynamic similarity of CFBs requires at least five dimensionless groups, but in the inertia limit their number can be limited to four or three in the viscous limit. In case of two dimensionless groups, the level of dissimilarity will depend on the degree to which the condition of density ratio is different from the commercial and laboratory unit. In general, the greater the number of dimensionless groups, the lesser the number of parameters which can be freely assumed in experiments on a small-scale unit.

Most scaling experiments have been carried out using monodispersed particles. Research should be undertaken to verify scaling relationships for:

- CFBs with a particle distribution of sizes,
- large-scale (i.e. commercial) and model circulating fluidized beds.

Additionally, an issue for further investigations is the identification of scaling parameters for beds fluidized with very small and very low density particles.

SYMBOLS

Ar	Archimedes number, $d_p^3\, \rho_f(\rho_s - \rho_f)g/\mu^2$, dimensionless
D	column diameter, m
d_{50}	mass mean particle diameter, m
D_h	riser hydraulic mean diameter, m
d_m	particle diameter at which the cumulative undersize mass is m%, m
d_p	particle diameter, m
Fr	Froude number
Fr_m	modified Froude number, dimensionless
$Fr_{m,t}$	modified Froude number calculated with terminal velocity, dimensionless
Fr_p	particle Froude number, dimensionless
g	acceleration of gravity, m/s^2
$\vec{g}$	gravity acceleration vector, (0,0,-g), m/s^2
G_s	external solids circulation flux, kg/m^{2}s
H	riser height, m
k	scale factor, dimensionless
L	riser height, m
l	length scale, m
M	virtual mass coefficient, dimensionless
m_1, m_2	experimental coefficients, dimensionless
n	constant in Richardson-Zaki correlation, dimensionless
p	pressure, Pa
P_s	particle-particle interaction stress tensor, Pa
PSD	particle size distribution, dimensionless
$\dot{Q}_{Tot}$	total gas volume flow, m^3/s
$\dot{Q}_{SA}$	secondary gas volume flow, m^3/s
R	gas-solid interaction term,
Re_d	particle Reynolds number, dimensionless
Re_D	Reynolds number based on hydraulic diameter, dimensionless

t	time, s
u	fluid velocity, m/s
U_0	superficial gas velocity, m/s
u_{sl}	slip velocity, m/s
u_T	terminal velocity of particle, m/s
v	particle phase velocity, m/s
v_0	superficial solid velocity, G_s/ρ_p, m/s
z	bed height, m
z_b	effective CFB risers height, m

Greek symbols

φ	particle sphericity, dimensionless
$\in$	void fraction, dimensionless
β	gas-solid drag factor, kg/m^{3}s
ν_f	fluid kinematic viscosity, m^2/s
μ	gas viscosity, Pa·s
ρ_f	gas density, kg/m^3
ρ_s	particle density, kg/m^3

subscripts

c	core
cl	cluster, cluster phase
lean	lean (i.e. cluster-free) phase
mf	minimum fluidization

superscripts

$\wedge$	dimensionless
m	reference scale model

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