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THE EQUATION DESCRIBING THE FILTRATION PROCESS WITH COMPRESSIBLE SEDIMENT ACCUMULATION ON A FILTER MESH

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Abstract: Filtration process is one of the basic and essential processes in technological systems for treatment of municipal, community and industrial wastewater treatment. Filtration process is a subject of numerous published research and theoretical elaborations. This publication concerns theoretical analysis with basic character, and is a verification of theoretical analysis and physical equations describing process of filtration aided with empirical formulas.

INTRODUCTION

Filtration is one of the basic and essential processes in city, municipality, and industrial sewage treatment processing systems.

The filtration process is the subject of a great amount of published research as well as theoretical work, conducted both by domestic [4, 10, 17–44] – as well as foreign [5–9, 11–14, 17, 45–50, 52, 53] research centers.

As a result, the work cited in the present paper can only be viewed as a considerably incomplete overview of some of the publications by some of authors dealing with the issue. This provides a specific justification for the systematization of publications (provided below) related to the filtration process.

The work can be divided into two groups:

- Group 1 – Typical laboratory or large-scale research whose ultimate purpose is practical implementation (formulas for conducting the process in specific conditions and with the use of a specific suspension mixture) often described with the use of analytical-empirical formulas, e.g. [1, 2, 7, 18–25, 34, 35].
- Group 2 – Theoretical analyses and rudimentary research attempting to verify those theoretical analyses and physical equations describing the process, sometimes accompanied with empirical formulas, thus, providing a description of the process

both by means of classical (physical) and empirical equations [3–6, 8, 9, 11–16, 23, 26, 27, 36, 37, 45–53].

According to the classifications provided above, the present work belongs to the second group.

The most commonly and widely used equation describing the flow of a liquid through a porous layer for the purpose of analyzing the theoretical basis of the filtration process is the basic Darcy's equation.

Very often, the most general form of Darcy's equation:

$$\dot{V}=\frac{\Delta p}{R}=\frac{\Delta p}{\alpha\cdot\frac{L}{A}}=\frac{\Delta p}{\frac{\mu}{k}\cdot\frac{L}{A_F}} \tag{1}$$

is referred to as the filtration equation, which, of course, is not precise and even incorrect to some extent. The flow of a liquid which, by definition, is a single-phase flow (of liquid or gas) through a porous barrier cannot be referred to as a filtration process because, to put it simply, any filtration process consists in the retention of solid-phase particles flowing through a porous barrier as a liquid mixture on or inside the barrier (the so-called colmatage). Solid- phase particles are, however, often retained both on the porous barrier, that is in the form of sediment accumulating on it, and inside the pores of the barrier, that is through colmatage. An adaptation of Darcy's equation for the purpose of describing the filtration process has already been proposed by T. Piecuch in his works [29–32] where he included the flow of a mixture of a liquid and a solid, which he also described many times in his other works [26–28, 30, 37]. Nevertheless, the reference publications for the present consideration are T. Piecuch's publication [28] as well as an important extension of this and other J. Piekarski's publications [39]. What is more, for many equations, J. Piekarski has created algorithms and computer programs based on them [38–44].

In equation (1) permeability coefficient is expressed by empirical relation [4, 6, 17, 45–50, 52, 53]:

$$k=\frac{b_O}{\Delta p^{s_O}} \tag{2}$$

but T. Piecuch assumes that pressure Δp (equation 2) corresponds to filtration pressure [28].

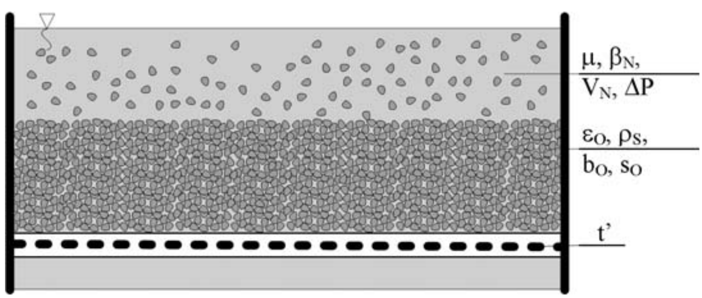

Fig. 1. A diagram of the filtration process with the use of a filter mesh

Referring and relating to T. Piecuch's publication [28], one can find a general equation describing emulsion sewage filtration with the use of a filter mesh (Fig. 1) based on Darcy's equation [4–9, 11–14, 17, 46–53] which has the following form:

$$\dot{V}=\frac{\Delta P}{t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\Delta P^{so}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}} \tag{3}$$

or the above-mentioned equation (3) with the following form:

$$\frac{V}{t}=\frac{\Delta P}{t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\Delta P^{so}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}} \tag{4}$$

The equation is the general filtration equation according to Piecuch [28], which can be referred to as stable and is characterized with a constant flow (V/t = const) and constant pressure (ΔP = const). This is the theoretical form of the filtration equation because the accumulation of sediment on a filter mesh leads to the increase of the general resistance value which, in turn, means that conditions V/t = const and ΔP = const are practically impossible.

Thus, the filtration process can occur when the flow value is fluctuating and the pressure value is constant, that is:

$$\frac{dV}{dt}=\frac{\Delta P}{t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\Delta P^{so}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}} \tag{5}$$

or when the pressure value is fluctuating and the flow value is constant, that is:

$$\frac{V}{t}=\frac{dP}{a'_t\cdot\left[t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\Delta P^{so}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}\right]dt} \tag{6}$$

where:
a'_t – constant time unit correction coefficient (dimensional), $a'_t = 1$ [s^{-1}].

The equation is simplified in different scientific works published so far, e.g. Piecuch [28] and others [4, 6, 9, 13, 46–53], assuming that s = 0, which means no compression, and thus the equations provided in (4) and (5) take the following form:

$$\frac{dV}{dt}=\frac{\Delta P}{t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}} \tag{7}$$

and:

$$\frac{V}{t}=\frac{dP}{a'_t\cdot\left[t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}\right]dt} \tag{8}$$

The equations provided above can easily be integrated within the limits from $V_N = 0$ to $V_N = V$, which gives the following equation for t values from t = 0 to t = t:

$$\Delta P\cdot t=t'\cdot\frac{\mu}{A_F}\cdot V+\frac{\mu}{b_O}\cdot\frac{\beta_N}{2\cdot A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}\cdot V^2 \tag{9}$$

or from $\Delta P = 0$ to $\Delta P = \Delta P$, which gives the following equation for t values from t = 0 to t = t:

$$\ln(t)=\frac{\Delta P}{a'_t\cdot\left[t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}\right]} \tag{10}$$

THEORETICAL ANALYSIS OF THE FILTRATION PROCESS WITH COMPRESSIBLE SEDIMENTATION

Theoretical analysis of the filtration process with compressible sedimentation accumulating on a filter mesh ($s_O \neq 0$) under fluctuating pressure ($\Delta p_O \neq 0$) has been the subject of a few considerations, e.g. by Palica and Kocurek [10, 23] where they established the correlation between porosity and pressure (introducing the notion of the so-called contact pressure) and divided the sediments accumulating on a filter mesh into layers.

However, for the purpose of their consideration, the authors of the present work decided to use the filtration equation directly according to its form presented in (4). Therefore, through the introduction of constant values into the general mathematical formula, that is:

$$A=t'\cdot\frac{\mu}{A_F} \tag{11}$$

and:

$$C=\frac{\mu\cdot V_N\cdot\beta_N}{b_O\cdot A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S} \tag{12}$$

and:

$$B=s \text{ (established value of sediment compressibility)} \tag{13}$$

the mathematical formula of the filtration equation where the integration variable of dx will be pressure (x = ΔP) in the x^B notation is obtained:

$$I=F(x)=\int \frac{dx}{A+Cx^B} \tag{14}$$

Version 1

If the sediment compressibility is theoretically maximal, that is s = 1, and B = 1, the integrated mathematical expression, takes the form:

$$I=F(x)=\int \frac{dx}{A+Cx}=\frac{1}{C}\int \frac{(A+Cx)'}{A+Cx}dx=\frac{1}{C}\ln|A+Cx|+D \tag{15}$$

where D – constant of integration, where it is possible to point out constant D from initial conditions.

Thus, the physical formulation of the equation where the sediment compressibility is full (s = 1), takes the following form:

$$V\cdot\ln(t)=\frac{b_O\cdot A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}{\mu\cdot V_N\cdot\beta_N}\cdot\ln\left|t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}\cdot\Delta P\right|+D \tag{16}$$

Version 2

Assuming the sediment compressibility to be s = 0.5 (B = 0.5), the general integral representing the filtration equation has the following form:

$$I=F(x)=\int \frac{dx}{A+C\cdot\sqrt{x}} \tag{17}$$

for $x^B = x^{0,5} = \sqrt{x}$.

Solving the integral (17) through substituting $\sqrt{x}$ = u, and thus x = u^2, that is dx = 2udu. Introducing an exchange integral into (17):

$$\int \frac{dx}{A+C\cdot\sqrt{x}}=2\cdot\int \frac{u\cdot du}{A+C\cdot u} \tag{18}$$

the integrand:

$$f(u)=\frac{u}{A+C\cdot u} \tag{19}$$

which is an improper rational function, can be presented as follows:

$$\frac{u}{A+C\cdot u}=\frac{\frac{1}{C}\cdot(A+C\cdot u)-\frac{A}{C}}{A+C\cdot u}=\frac{1}{C}-\frac{A}{C}\cdot\frac{1}{A+C\cdot u} \tag{20}$$

and thus:

$$\int\frac{u\cdot du}{A+C\cdot u}=\frac{1}{C}\cdot\int du-\frac{A}{C}\cdot\int\frac{du}{A+C\cdot u}=\frac{1}{C}\cdot u-\frac{A}{C^2}\cdot\ln|A+C\cdot u| \tag{21}$$

Finally, including (21) and (18), the following function formula is obtained:

$$I=F(x)=\int\frac{dx}{A+C\cdot\sqrt{x}}=\frac{2}{C}\cdot\sqrt{x}-\frac{2\cdot A}{C^2}\cdot\ln|A+C\cdot\sqrt{x}|+D \tag{22}$$

where D – constant of integration.

Thus, the physical formulation of the equation where the sediment compressibility equals s = 0.5 takes the following form:

$$\begin{gathered}V\cdot\ln(t)=\frac{2\cdot b_O\cdot A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}{\mu\cdot V_N\cdot\beta_N}\cdot\sqrt{\Delta P}-\\ -\frac{2\cdot t'\cdot A_F^3\cdot b_O^2\cdot\rho_S^2\cdot(1-\varepsilon_O)^2}{\mu\cdot V_N^2\cdot\beta_N^2}\cdot\ln\left|t'\cdot\frac{\mu}{A_F}+\frac{\mu}{b_O}\cdot\frac{V_N\cdot\beta_N}{A_F^2\cdot(1-\varepsilon_O)\cdot\rho_S}\cdot\sqrt{\Delta P}\right|+D\end{gathered} \tag{23}$$

Version 3

Assuming that the sediment compressibility is s = 2/3 (B = 2/3, and thus $x^B=\sqrt[3]{x^2}$), the general integral representing the filtration equation has the following form:

$$I=F(x)=\int\frac{dx}{A+C\cdot\sqrt[3]{x^2}}=3\cdot\int\frac{u^2\cdot du}{A+C\cdot u^2} \tag{24}$$

through the substitution $\sqrt[3]{x}=u$, and thus $x=u^3$, $dx=3u^2du$, takes the form of a rational function integral (24). The integrand:

$$f(u)=\frac{u^2}{A+C\cdot u^2} \tag{25}$$

which is an improper rational function, can be presented as follows:

$$\frac{u^2}{A+C\cdot u^2}=\frac{\frac{1}{C}\cdot(A+C\cdot u^2)-\frac{A}{C}}{A+C\cdot u^2}=\frac{1}{C}-\frac{A}{C}\cdot\frac{1}{A+C\cdot u^2} \tag{26}$$

and on the basis of the formula:

$$\int \frac{du}{k^2+u^2}=\frac{1}{k}\cdot \text{arctg}\left(\frac{u}{k}\right) \tag{27}$$

for k>0 the following equation is obtained:

$$\int \frac{u^2\cdot du}{A+C\cdot u^2}=\frac{1}{C}\cdot \int du-\frac{A}{C}\cdot \int \frac{du}{A+C\cdot u^2}=\frac{1}{C}\cdot \int du-\frac{A}{C^2}\cdot \int \frac{du}{\left(\sqrt{\frac{A}{C}}\right)^2+u^2}=$$
$$=\frac{1}{C}\cdot u-\frac{\sqrt{A}}{C\cdot\sqrt{C}}\cdot \text{arctg}\left(\sqrt{\frac{C}{A}}\cdot u\right)+D \tag{28}$$

where D – constant of integration, and the values of the A and C constants are positive.

Finally, including (28) and (24), the following function formula is obtained:

$$I=F(x)=\int \frac{dx}{A+C\cdot\sqrt[3]{x^2}}=\frac{3}{C}\cdot\sqrt[3]{x}-\frac{3\cdot\sqrt{A}}{C\cdot\sqrt{C}}\cdot \text{arctg}\left(\sqrt{\frac{C}{A}}\cdot\sqrt[3]{x}\right)+D \tag{29}$$

where D – constant of integration.

Thus, the physical formulation of the equation where the sediment compressibility equals s = 2/3, takes the following form:

$$V\cdot \ln(t)=\frac{3\cdot b_O\cdot A_F^2\cdot \rho_S\cdot(1-\varepsilon_O)}{\mu\cdot V_N\cdot \beta_N}\cdot\sqrt[3]{\Delta P}-\frac{3\cdot\sqrt{t'\cdot\mu\cdot A_F^{-1}}}{\sqrt{\left(\frac{\mu\cdot V_N\cdot \beta_N}{b_O\cdot A_F^2\cdot \rho_S\cdot(1-\varepsilon_O)}\right)^3}}\cdot$$
$$\cdot \text{arctg}\left(\sqrt{\frac{V_N\cdot \beta_N}{b_O\cdot A_F\cdot \rho_S\cdot t'\cdot(1-\varepsilon_O)}}\cdot\sqrt[3]{\Delta P}\right)+D \tag{30}$$

Version 4

Assuming that the sediment compressibility is s = 1/3 (B = 1/3, and thus $x^B=\sqrt[3]{x}$) and through the substitution $\sqrt[3]{x}=u$, and thus $x = u^3$ and $dx = 3u^2du$, the general integral representing the filtration equation has the following form:

$$I=F(x)=\int \frac{dx}{A+C\cdot\sqrt[3]{x}}=3\cdot\int \frac{u^2\cdot du}{A+C\cdot u} \tag{31}$$

The integrand is an improper rational function so when the numerator is divided by the denominator, the function can take the following form:

$$\frac{u^2}{A+C\cdot u}=\frac{1}{C}\cdot u-\frac{A}{C^2}+\frac{A^2}{C^2}\cdot\frac{1}{A+C\cdot u} \tag{32}$$

and thus:

$$\int \frac{u^2\cdot du}{A+C\cdot u}=\int\left(\frac{1}{C}\cdot u-\frac{A}{C^2}+\frac{A^2}{C^2}\cdot\frac{1}{A+C\cdot u}\right)du=\frac{1}{2\cdot C}\cdot u^2-\frac{A}{C^2}\cdot u+\frac{A^2}{C^3}\cdot\ln|A+C\cdot u|+D \tag{33}$$

where D – constant of integration.

Including the result of (33) in (31) and returning to the substitution $u=\sqrt[3]{x}$, the general integral form of the filtration equation for the sediment compressibility value of s = 1/3 is obtained:

$$I=F(x)=\frac{3}{2\cdot C}\cdot\sqrt[3]{x^2}-\frac{3\cdot A}{C^2}\cdot\sqrt[3]{x}+\frac{3\cdot A^2}{C^3}\cdot\ln\left|A+C\cdot\sqrt[3]{x}\right|+D \tag{34}$$

where D – constant of integration.

Thus, the physical formulation of the equation where the sediment compressibility equals s = 1/3 takes the form:

$$\begin{aligned} V\cdot\ln(t)= &\frac{3\cdot b_O\cdot A_F^2\cdot\rho_S\cdot(1-\varepsilon_O)}{2\cdot\mu\cdot V_N\cdot\beta_N}\cdot\sqrt[3]{\Delta P^2}-\frac{3\cdot t'\cdot b_O^2\cdot A_F^3\cdot\rho_S^2\cdot(1-\varepsilon_O)^2}{\mu\cdot V_N^2\cdot\beta_N^2}\cdot\sqrt[3]{\Delta P}+ \\ &+\frac{3\cdot t'^2\cdot b_O^3\cdot A_F^4\cdot\rho_S^3\cdot(1-\varepsilon_O)^3}{\mu\cdot V_N^3\cdot\beta_N^3}\cdot\ln\left|t'\cdot\frac{\mu}{A_F}+\frac{\mu\cdot V_N\cdot\beta_N}{b_O\cdot A_F^2\cdot\rho_S\cdot(1-\varepsilon_O)}\cdot\sqrt[3]{\Delta P}\right|+D \end{aligned} \tag{35}$$

SUMMARY – CONCLUSIONS

Taking all of the above into consideration, if the sediment compressibility denoted as “s” in the physical formulation and as “B” constant in the present mathematical consideration can take different values within the following range 0 < B < 1, that is B = m/n, where m and n are the numbers that constitute the fraction between the extreme values that represent no compressibility (zero) and full compressibility (one) which means that if

$m < n$ and $x^B = x^{m/n} = \sqrt[n]{x^m}$, the mathematical formulation of the general integral of the filtration equation (here with fluctuating filtration pressure and constant flow) takes the form:

$$I = \int \frac{dx}{A + C \cdot \sqrt[n]{x^m}} \tag{36}$$

and through the substitution $\sqrt[n]{x} = u$, thus $x = u^n$, $dx = n \cdot u^{n-1} du$. The integral (36) takes the following form of a rational function integral:

$$\int \frac{dx}{A + C \cdot \sqrt[n]{x^m}} = n \cdot \int \frac{u^{n-1}}{A + C \cdot u^m} du \tag{37}$$

The integrand takes the form:

$$f(u) = \frac{u^{n-1}}{A + C \cdot u^m} \tag{38}$$

and it is an improper rational function because $n-1 \geq m$. The result of this is that when the numerator is divided by the denominator, the function can be presented in the form of a polynomial and of a proper rational function. Such a function can be integrated in accordance with the common rules pertaining to the integration of rational functions (that is – partial fractions of degree 1 or partial fractions of degree 2). In such a situation, the right side of the mathematical formulation of the filtration equation (37) is always a function that is a polynomial sum, a natural logarithm, or an arcustangens function.

The following general conclusions can be drawn from the conducted analysis:

1. Establishing one general filtration equation with the exponent "s" (which here is a power exponent) is not possible.
2. The filtration equation for compressible sediment must be separately calculated every time by means of integrating the general filtration equation in its rudimentary form where a given value of the sediment compressibility factor "s" (within the range from 0 to 1) is assumed (substituted).
3. The difficulty with solving the complex general integral of the mathematical formulation of the filtration equation is the reason why, in many pieces of work, the presentation of the problem is limited with the assumption that the sediment is not compressible, which in turn leads to the removal of the Δp^0 formulation because it equals one.

Symbols

R	total resistance of the porous barrier	$[N \cdot s/m^5]$
ΔP	pressure drop	$[N/m^2]$
t'	average resistance of the filter mesh	$[m^{-1}]$
μ	dynamic viscosity factor for the emulsion	$[N \cdot s/m^2]$

A_F	layer area	$[m^2]$
b_O	sediment constant	[N]
s_O	sediment compressibility factor	[-]
V_N	feed volume	$[m^3]$
β_N	solid-phase condensation in the inflow	$[kg/m^3]$
ε_O	sediment porosity	[-]
ρ_S	density of the screened solid phase	$[kg/m^3]$
α	specific resistance of compressible sediment layer	$[N\cdot s/m^4]$
k	permeability coefficient	$[m^2]$
L	height of sediment layer	[m]

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RÓWNANIE FILTRACJI Z UTWORZENIEM OSADU ŚCIŚLIWEGO NA SIATCE FILTRACYJNEJ

Filtracja należy do podstawowych i głównych procesów w układach technologicznych oczyszczania ścieków miejskich, gminnych i przemysłowych. Proces filtracji jest przedmiotem ogromnej ilości publikowanych badań oraz rozważań teoretycznych. Niniejsza publikacja dotyczy analizy teoretycznej o charakterze podstawowym, stanowi uzupełnienie i rozwinięcie klasycznej teorii filtracji. Oryginalność tych rozważań teoretycznych polega na tym, że Autorzy przyjmują, iż współczynnik przepuszczalności „k" opisany równaniem (2) jest funkcją nie ciśnienia zgniotu jak m.in. (4) lecz ogólnego ciśnienia filtracji m.in. [26÷31]. Takie założenie umożliwia rozwiązanie ogólnego równania filtracji (4) odniesionego do wariantu różniczkowego równania filtracji przy stałym ciśnieniu (7) poprzez całkowanie dla różnych wartości przyjętych współczynników ściśliwości osadu. Efektem tych obliczeń są końcowe równania (16), (23), (30) oraz (35).