www.czasopisma.pan.pl PAN www.journals.pan.pl

ARCHIVES of FOUNDRY ENGINEERING

DOI: 10.1515/afe-2015-0058

Published quarterly as the organ of the Foundry Commission of the Polish Academy of Sciences

ISSN (2299-2944)
Volume 15
Issue 3/2015

47 – 50

Data Simulation with the Use of Weibull Analysis for Evaluation of Reliability of Chosen Assortment

J. Piątkowski [a], *, A. Mesjasz [b]

[a] Silesian University of Technology, Faculty of Materials Science, Krasińskiego 8, 40-019 Katowice, Poland

[b] Fortum Power and Heat Poland Company – Maintenance Department, Walońska 3-5, 50-413 Wroclaw, Poland

*Corresponding author. E-mail address: jaroslaw.piatkowski@polsl.pl

Received 19.03.2015; accepted in revised form 29.05.2015

Abstract

In the dissertation the data modeling has been shown for the data that regards the damages, which value is above zero. With the use of Weibull distribution, with prior regression and correlation analysis chosen parameters that defines the life time and failure level of two populations of AlSi17Cu5 were defined. The calculation sheet of reliability allows to create so called survival diagram, and on the basis of durability data the average warrantee can be determined, on the pre-exploitation period.

Keywords: Reliability, Weibull Analysis, Hypereutectic Al-Si cast alloys, Regression and correlation analysis

1. Introduction

In the works about reliability there can be, in general, distinguished two categories: of quality and of quantity. In the quantity analysis estimates the measure of reliability, the probability indicators of failure and models the properties. Using the mathematical functions the efficiency structure is being built in order to numeric description and representation of the occurring phenomenon. The quality category on the other hand consists certain methods of evaluation, formalized procedures of recognizing the damages influence on stability and acting standards of the analysis of damages consequences. The result is the expert evaluation and guidelines for improvement of reliability, defined on the basis of appropriate standards [1, 2]. With the reliability, the concept of material stability or construction endurance are connected, those are the features, which decides about failure-free operational use time of the system [3, 4]. However conducting the reliability testing on the actual objects is very costly, therefore the alternative in this case is statistical analysis, with the use of the part of calculation sheet, as for example MS Excell [5÷7] and process "tools", that define statistic distribution of technological stability (Weibull analysis) based on the strength properties of the material [8÷10].

2. Scope and purpose of research

The aim of the study is to demonstrate the modeling of the data that defines material reliability on the basis of initial testing of the samples for R_m. For the research two populations (A and B) of AlSi17Cu5 were chosen, of different technological history of charge materials. In order to achieve the assumed goal, the scope of this study consist, in between of the other, of the following:

- construction of the data base, and Excell calculation sheet,
- conducting variance, correlation and regression analysis,
- estimation of the two parameter Weibull distribution,

- development of the calculation sheet for reliability,
- graphic development of the „survival” function diagram.

3. Research method and materials

The AlSi17Cu5 alloy, chosen for the research, was casted to steel mould, where 3 samples were created at the same time. Samples that were subjected to mechanical machining, to reach the dimensions as per the standard PN-EN 10002-1. The inside of the casting mould, with the dimensions of the static tensile test is shown in Figure1.

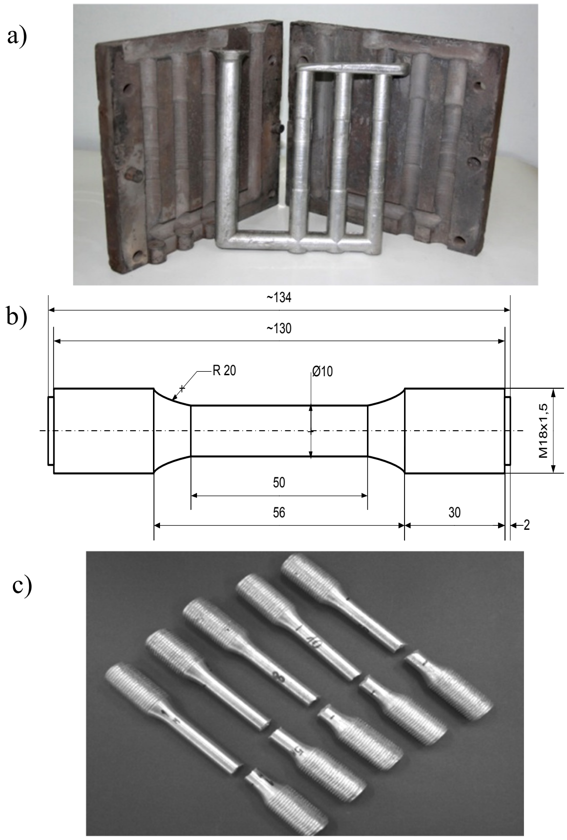

Fig. 1. Metal mould along with the casts (a), die diagram of static tensile test (b) and samples after rupture (c)

The static tensile test in ambient temperature (20°C) was conducted as per the standard PN-EN ISO 6892-1 with the use of the Instron 3382 machine, using the gear ratio 20:1 and constant tensile speed of 5 mm/min. The tensile strength R_m of the alloy was defined. Twelve durability tests was conducted (for each population), discarding two extreme, and the balance 10 were used as input data for further statistical tests.

In order to unify and to conduct comparative analysis, the level of significance was assumed as $p(\alpha) \leq 0{,}05$ being the result reliability of statistical calculation. The Neyman trust range was estimated as $1\text{-}\alpha=0.95$, so the maximal risk of possible acceptable error is 5%, and verified results are significant on the level of 95%. The data for statistical research were prepared as per the standards [11÷13], with prior verification of the hypothesis, that dependable variable (R_m) is compliant with normal distribution.

4. The results of investigations

The chemical content of the AlSi17Cu5 alloy is shown in Table 1.

Table 1.
Chemical content of the AlSi17Cu5 cast alloy (wt%)

Alloy	Chemical content of the AlSi17Cu5 alloy						
	Si	Cu	Fe	Mn	Mg	Ni	Al
AlSi17Cu5	16.81	4.71	0.54	0.03	0.04	0.13	rest

The results of tensile strength test for two populations of the AlSi17Cu5 alloy are shown in Table 2.

Table 2.
The entry data for statistical analysis

No sample	Results R_m of AlSi17Cu5 alloy[MPa]	
	Population A	Population B
1	219	222
2	222	220
3	218	218
4	223	233
5	220	225
6	223	219
7	218	229
8	226	236
9	228	222
10	219	240

On „first sight” it is visible, that both populations are close one to another; the results of R_m of population A are within the range between 218 and 228, and of population B between 218 and 240 MPa.

On the basis of the results that has been shown, it is difficult to define, which population has higher reliability.

In order to conduct the data modeling with Weibull analysis, the results has to be sorter in ascending order, assigning them the ranks from 1 to 10. Then the estimated value of proportional population part, that will get damaged with assigned value of tensile strength R_m should be calculated. It can be done with “median rate” method, as per the below dependence:

$$m = \frac{n_i - 0{,}3}{n + 0{,}4} \tag{1}$$

for i ∈ from 1 to 10, n – number of observations.

Then the standardizing of the dependence should be done (1) as per the following rule:

$$s = \frac{1}{1 - m} \tag{2}$$

And apply double logarithm as per the rule:

$$\ln(\ln(\frac{1}{1-m})) \tag{3}$$

In the final phase the R_m values should be subjected to logarithm (as variable „x" – the axis of abscissae), and the dependence (3) as variable „y" – the axis of ordinates and draw the correlation diagram (Figure 2).

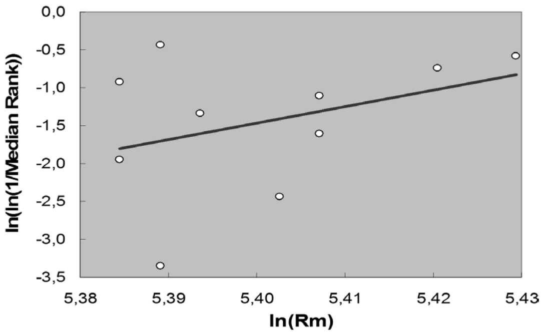

Fig. 2. The correlation diagram of population A of the AlSi alloy

The function of cumulative Weibull distribution is being defined by the formula [10]:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \tag{4}$$

where:
x – material feature, e.g. R_m,
α – characteristic life-time of the material,
β – shape parameter (Weibull modulus).

Applying the logarithm on both sides of the formula (4) and substituting for:

$$\left.\begin{array}{l} \ln\left[\ln\left(\frac{1}{1-F(x)}\right)\right] = y \\ \beta = m\,;\ \beta \cdot \ln x = x\,;\ \beta \cdot \ln \alpha = b \end{array}\right\} \tag{5}$$

We may receive the formula of a straight line as below:

$$y = m \cdot x + b \tag{6}$$

So, if the linear regression has been done, the estimation of the Weibull modulus ($m=\beta$) comes directly from the inclination of the regression line (correlation diagram), and the characteristic life-time (α) is being calculated from the following formula [11]:

$$\alpha = \exp\left[\left(-\frac{b}{\beta}\right)\right] \tag{7}$$

The Weibull analysis is best started with conducting linear regression in order to allow the inference of reliability. The dialog window of the base statistics of regression and variance analysis for population A has been shown in Figure 3.

On the basis of regression and variance analysis (Fig. 3) and on dependence (7), the α and β parameters of the AlSi17Cu5 alloy for populations A and B were calculated. The result of the calculation of characteristic life-time (α) and shape parameter (β) for both populations are shown in Table 3.

Regression Statistic						
Multiple R	0,3690701					
R square	0,1362127					
Adiusted R square	0,0282393					
Standard Error	0,9032552					
Observations	10					
Anova						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	1	1,029251985	1,029252	1,2615394	0,293933	
Residual	8	6,526959122	0,81587			
Total	9	7,556211107				
	Coefficiencs	*Standard Error*	*t Stat*	*p - value*	*Lover 95%*	*Upper 95%*
Intercept	-119,6924555	105,2743317	-1,13696	0,28846238	-362,4555	123,0706
ln(Rm)	21,89353632	19,49240885	1,123183	0,2939332	-23,05604	66,84311

Fig. 3. Results of Linear Regression for population A

Table 3.
Characteristic life-time (α) and shape parameter (β) of AlSi17Cu5 alloy for A and B populations

Parameter	Population A	Population B
α	236.7542	250.5684
β	21.8935	14.2292

At this stage of calculations, still is not certain, which population of results gas higher reliability. In order to achieve it, the Reliability formula $R_{(Rm)}$ needs to be estimated with assumption of Weibull distribution as per the following formula [11]:

$$R_{(Rm)} = \exp\left[-\left(\frac{x}{\alpha}\right)\right]^{\beta} \tag{8}$$

where:
R_m – represents the durability (or time) until break down.

For the needs of the task, it was assumed, that required reliability of the castings (on the basis of minimal tensile strength of 220 MPa) should be 0.90. It means, that 90% of the castings should „survive" the tensile strength of at least 220 MPa.

The results of the calculations for chosen levels of probability has been shown in Table 4.

Table 4.
Reliability (R) and values of R_m for different levels of probability of the AlSi17Cu5 alloy for populations A and B

	Parameter	Population A	Population B
	$R_{(220\ MPa)}$	0.8183	0.8547
		R_m [MPa]	
Probability level	1%	253.8	278.9
	10%	245.9	265.7
	50%	232.8	244.2
	90%	213.6	214.0
	99%	191.8	181.3

Out of the data shown in Table 4 comes, that population B has slightly higher reliability, but still it is not certain which, out of two populations, has higher reliability for full range of data. In order to ascertain it, the calculation sheet has been prepared for reliability, using the function =WEIBULL() for the range of tensile strength given in the Table 2.

For both population the life-time diagram has been drawn, with the use on axis of abscissae the tensile strength, and on axis of ordinates the probability of survival in the full range of R_m. This diagram, with assumed tensile strength value (220 MPa) is shown in Figure 4.

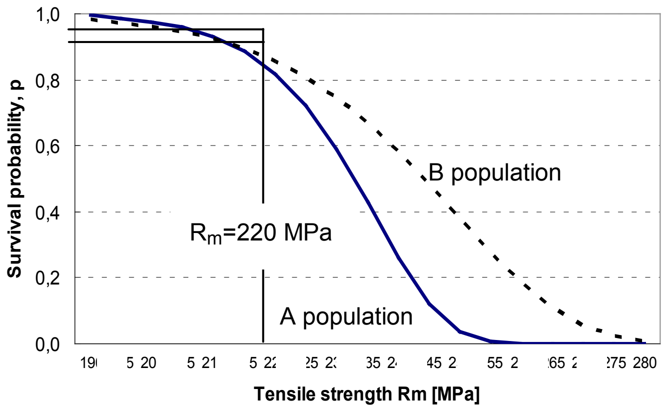

Fig. 4. Survival diagram for A and B population of AlSi alloy

5. Summary

In the study, the estimation of bi-parameter Weibull distribution for population (A and B), that differs in technological history of obtaining the AlSi17Cu5 alloy, has been shown. The study had an objective of showing the methodology of estimation of material reliability, that is determine with survival probability (p) on the basis of static tensile test in the temperature of 20°C. The aim was to indicate, which population of results has higher castings reliability, on the basis of tensile strength R_m. For ten results of tensile strength with the median rank method and using the relations (2) to (5), the linear regression was applied, to define the shape parameter (β), and with the use of the formula (7) the characteristic life-time of the material (α) was calculated. Using the Weibull distribution (8), the reliability determined with survival probability was defined for assumed durability of 220 MPa, out of which comes, that population B has higher reliability.

Using calculation sheet of reliability and function =WEIBULL() the diagram was created that shows the survival probability as a function of tensile strength in the whole data range (Table 2). From Figure 4 it comes, that up to the tensile strength R_m of around 210 MPa, the reliability of casts from AlSi17Cu5 alloy of both populations A and B is very similar. For the R_m=220 MPa (assumed durability level), the reliability of population B is slightly higher: $R_{(220\ MPa)}$=0,8547 than population A: $R_{(220\ MPa)}$=0,8183. However it deserves the attention, how the „survival" line is positioned (Fig. 4). As it can be seen, for population A it is more "vertical", which means, that in the range of R_m from 220 to 250 MPa the result distribution is close, so the "survival" probability in this range is higher. Population B has, as a matter of fact, higher tensile strength, but due to more stretched "survival" line, the survival probability is slightly lower.

One more thing is worth attention, if the parameter (β<1), then the level of mortality of the product gets lower. It is typical for products, which got damaged at the pre-exploitation stage (burn-in-period). If the indicator (β=1), then the level of mortality is on constant level. It applies to the products, that „survived" the initial testing period, and during the normal exploitation show constant level of mortality. The casts, for which (β>1) are characterized by increasing mortality, and that is typical for the products that are subjected to wear-off. From the works [11, 12] it comes, that if (β>6,0), then for proper data modeling the third parameter is required, and function gets the name of tri-parameter Weibull distribution. This parameter (γ) makes the distribution shift to the right and it can be interpreted as the earliest, possible break-down occurrence time [5], but then, the Weibull analysis, as it's interpretation becomes more complicated [13÷15].

References

[1] PN-EN 62309:2005 Reliability of products containing parts used repeatedly. Functionality and research. (in Polish).

[2] PN-EN 61703:2005 Ready mathematical expressions of reliability and provide a means of handling. (in Polish).

[3] Piątkowski, J. (2011). Estimation of Materials Reliability on Al-Si with Cu, Ni and Mg Alloy Used in the Automotive Industry. *Research and Teaching Technologies.* 3, 7-16.

[4] Bucior, J. (2004). *Basic theory and reliability engineering.* Rzeszów: Publ. University of Technology (in Polish).

[5] www.qualitydigest.com/html/weibull.html W.W. Dorner, Using Excel for Weibull analysis. (2010), 1-9.

[6] Greber, T. (2000). *Statistical process control-improving the quality of Statistic*. Cracow: StatSoft (in Polish).

[7] Gierek, A., Piątkowski, J. & Szymszal, J. (2001). Influence of technical process reliability of AlSi alloy as a structural material. *Archives of Foundry.* 1(2/2), 109-116.

[8] Hallinan Jr., A. (1993). Review of the Weibull Distribution. *Journal of Quality Technology.* 25, 85-93.

[9] Weibull, W. (1939). A statistical theory of the strength of materials. Generalstabens Litografiska Anstalts Förlag, 45-59.

[10] Weibull, W. (1951). A statistical distribution function of wide applicability. *J. of Applied Mechanics. 18,* 293-298.

[11] McCool, J. (1998). Inference on the Weibull Location Parameter. *Journal of Quality Technology.* 30, 119-126.

[12] Piątkowski, J. (2012). Influence of overheating degree on material reliability of A390.0 alloy. *Sol. St. Phen.* 191, 23-28.

[13] Mackisack, M. & Stillman, R. (1996). A Cautionary Tale About Weibull Analysis. *IEEE Reliab. Trans.* 45, 244-248.

[14] Nelson, W. (1982). *Applied Life Data Analysis*. John Wiley & Sons

[15] Meeker W. & Escobar L. (1998). *Statistical Methods for Reliability Data*. John Wiley & Sons.