# MEASUREMENT OF SMALL VALUES OF HYDROSTATIC PRESSURE DIFFERENCE 

## POMIAR MAŁYCH WARTOŚCI RÓŻNICY CIŚNIEŃ HYDROSTATYCZNYCH

In order to describe the fluid flow through the porous centre, made of identical spheres, it is necessary to know the pressure, but in fact - the pressure distribution. For the flows in the range that was traditionally called laminar flow (i. e. for Reynolds numbers (Bear, 1988; Duckworth, 1983; Troskolański, 1957) from the range 0,01 to 3 ) it is virtually impossible with the use of the tools directly available on the market. Therefore, many scientists who explore this problem have concentrated only on the research of the velocity distribution of the medium that penetrates the intended centre (Bear, 1988) or pressure distribution at high hydraulic gradients (Trzaska \& Broda, 1991, 2000; Trzaska et al., 2005). It may result from the inaccessibility to the measurement methods that provide measurement of very low hydrostatic pressures, such as pressure resulting from the weight of liquid located in the gravitational field (Duckworth, 1983; Troskolański, 1957). The pressure value c. 10 Pa (Troskolański, 1957) can be generated even by 1 mm height difference between the two levels of the free water surface, which in fact constitutes the definition of gauging tools of today measuring the level of the hydrostatic pressure.

Authors proposed a method of hydrostatic pressure measurement and devised a gauging tool. Then a series of tests was conducted aiming at establishing what is the influence of various factors, such as temperature, atmospheric pressure, velocity of measurement completion, etc. on the accuracy and method of measurements. A method for considerable reduction of hysteresis that occurs during measurement was also devised.

The method of measurement of small hydrostatic difference measurements allows for the accuracy of measurement of up to 0.5 Pa . Measurement results can be improved successfully by one order of magnitude, which for sure would entail necessary temperature stabilization of the tool. It will be more difficult though to compensate the influence of atmospheric pressure on the measurement process.

Keywords: small values of pressure difference, measurements, filtration, manometers

Aby opisać przepływ płynu przez ośrodek porowaty konieczna jest znajomość ciśnienia, a tak naprawdę jego rozkładu. Dla przepływów w zakresie umownie nazwanym laminarnym (tj. dla liczb Re (Bear, 1988; Duckworth, 1983; Troskolański, 1957) z zakresu 0.01 do 3) jest to praktycznie niemożliwe posługując się bezpośrednio dostępnymi na rynku przyrządami. Dlatego wielu naukowców zajmujących się tym

[^0]zagadnieniem skoncentrowało się tylko na badaniach rozkładu prędkości medium przenikającego przez zadany ośrodek (Bear, 1988) lub rozkładu ciśnienia przy wysokich spadkach hydraulicznych (Trzaska i Broda, 1991, 2000; Trzaska i in., 2005). Jako przyczynę można wskazać niedostępność metod pomiarowych umożliwiających pomiar bardzo niskich ciśnień hydrostatycznych, czyli ciśnienia wynikającego z ciężaru cieczy znajdującego się w polu grawitacyjnym (Duckworth, 1983; Troskolański, 1957). Wartość ciśnienia około 10 Pa (Troskolański, 1957) może już wygenerować różnica 1 mm wysokości między dwoma poziomami swobodnego zwierciadła wody, co stanowi tak naprawdę rozdzielczość dzisiejszych przyrządów pomiarowych mierzaccych wielkość ciśnienia hydrostatycznego.

Problem mierzenia tak niskich ciśnień dla cieczy nie tkwi w tym, że technicznie nie jest możliwe zbudowanie takiego czujnika, ale wynika z faktu, że możemy mierzyć ciśnienie wewnątrz ograniczonego obszaru z rozdzielczością np. 0.1 Pa tylko wtedy, gdy zmiana ciśnienia związana z położeniem lub przemieszczeniem przyrządu pomiarowego nie wpływa na dokładność. Dlatego czujniki o takim zakresie pomiarowym, budowane są tylko dla mediów gazowych.

Przystępując do projektowania i konstruowania stanowiska pomiarowego założono, że pomiar ciśnienia w zakresie od 0 do 25 Pa ma być wykonany z dokładnością do $2 \%$, czyli z rozdzielczością wynoszącą $0,5 \mathrm{~Pa}$. Konstrukcję oparto o czujnik (różnicowy) model MS-321-LCD firmy Magnesense przeznaczony do mierzenia mediów gazowych. Zasadę pomiaru niskich ciśnień hydrostatycznych przedstawiono na Rys. 2.

Na rys. 4 pokazano zdjęcie fragmentu autorskiej aparatury do pomiaru bardzo niskich wartości różnicy ciśnień hydrostatycznych. W artykule opisano tylko najważniejsze zagadnienia będące podstawą umożliwiającą skonstruowanie takiego urządzenia. Przystępując do pomiarów nie znano, w jaki sposób będzie zachowywać się aparatura i jaka metodyka pomiarów będzie najwłaściwsza i pozwalająca na osiągnięcie zakładanej dokładności.

Zaprezentowana nowatorska metoda pomiaru małych różnic ciśnień hydrostatycznych pozwala mierzyć różnicę ciśnień hydrostatycznych z dokładnością do 0.5 Pa , co przedstawiono w artykule. Należy tu zwrócić uwage, że przedstawiona metodologia pomiarów przy zastosowaniu doskonalszego czujnika pozwoli w przyszłości na zwiększenie dokładności pomiaru co najmniej o jeden rząd. Z pewnością wiązałoby się to $z$ koniecznością zastosowania stabilizacji temperatury urządzenia. W trakcie pomiarów niekorzystnym zjawiskiem okazało się pojawienie histerezy, co pociągnęło za sobą konieczność opracowania metodologii wydatnego jej zmniejszenia poniżej klasy zastosowanego czujnika, co udało się osiagnąć a rezultaty przedstawiono na rys. $5,6,8,9$. Trudniej będzie natomiast skompensować wpływ ciśnienia atmosferycznego na proces pomiarowy.

Słowa kluczowe: małe wartości różnicy ciśnień, pomiary, filtracja, manometry

In order to describe the fluid flow through the porous centre, made of identical spheres, it is necessary to know the pressure, but in fact - the pressure distribution. For the flows in the range that was traditionally called laminar flows (i. e. for Reynolds numbers ${ }^{1}$ (Bear, 1988; Duckworth, 1983; Troskolański, 1957) from the range 0,01 to 3 ) it is virtually impossible with the use of the tools directly available on the market. Therefore, many scientists who explore this problem have concentrated only on the research of the velocity distribution of the medium that penetrates the intended centre (Bear, 1988) or pressure distribution at high hydraulic gradients (Trzaska \& Broda, 1991, 2000; Trzaska et al., 2005). It may result from the inaccessibility to the measurement methods that provide measurement of very low hydrostatic pressures, such as pressure resulting from the weight of liquid located in the gravitational field (Duckworth, 1983; Troskolański, 1957). The pressure value c. 10 Pa (Troskolański, 1957) can be generated even by 1 mm height difference between the two levels of the free water surface, which in fact constitutes the definition of gauging tools of today measuring the level of the hydrostatic pressure, e.g. the

[^1]so called piezometers ${ }^{2}$. Admittedly, it is possible to slightly improve the range of the values being measured by setting the piezometer pipe at a specific angle from the vertical (Duckworth, 1983; Troskolański, 1957), it will not improve considerably the methodology of measurement though because the accuracy of piezometer indications is affected by such phenomena as surface tension or capillarity (Adamson, 1997). In the case of hydrostatic pressure increase the rise of water column is retarded in relation to the occurring changes. Surface tension and capillarity result in the situation where moistening of the surface is retarded against the pressure increase and thus the measured value is lowered. In the opposite situation the value can be overstated for free falls of pressure. The said phenomenon can be noticed easily if the capillary tube (glass tube), which was earlier carefully dried, is immersed in the vessel with water. The water column will be in this case significantly lower than it could be concluded from theoretical considerations. However, when we moisten the internal surface of the capillary pipe, the water column will reach the theoretical value. This phenomenon is termed as contact angle hysteresis. The value of the moistening angle of the fluid advancing along the surface exceeds the value of the angle of the fluid receding on this surface (Adamson, 1997).


Fig. 1. Diagram showing the range of measured values of pressure for various types of sensors (Raymond, 1997, 1998)

The problem of measuring such low pressures for fluid does not mean that it is technically impossible to build such a measurement sensor (Raymond, 1997, 1998). What is the purpose of designing a sensor measuring with 0.1 Pa density for fluid if we place it in the medium where the position change by 0.01 mm will equal the measurement accuracy? It means obviously that we can measure pressure inside the limited area with such accuracy only if the pressure change connected with the position or replacement of the gauging tool does not affect accuracy. Therefore, the sensors with such measurement range are constructed only for gas media.

Micromanometers are used for the measurement of small pressures. One of the most popular tool of the type is Askania micromanometer (Filek et al., 1990) for measurement of static pres-

[^2]sure difference. Depending on the model, the measured pressure of the gas medium in Askania micromanometer is balanced with hydrostatic pressure of the manometric water column (most often distilled water) with accuracy of measurement from $0,2 \mathrm{~Pa}$ to $0,5 \mathrm{~Pa}$.

Idea of measurement of low values of hydrostatic pressure difference presented in the paper is reversing the principle of Askania micromanometer operation described above. Hydrostatic fluid pressure is balanced with the measured gas medium pressure. Askania manometer measures only the difference of static pressures whereas it is possible to measure dynamic changes of the hydrostatic pressure in the proposed method. It depends only on the used sensor - pressure converter.

Since the outset of the designed construction of gauging unit it has been assumed that the measure of the pressure in the range of 0 do 25 Pa is to be made with accuracy of up to $2 \%$, so with the definition amounting to 0.5 Pa . It approximately corresponds to 0.05 mm difference of height between two levels of free flow of water. The construction was based on sensor (differential) MS-321-LCD model made by Magnesense, designed for gas medium measurement.

The principle of low hydrostatic pressures measurement was presented on the scheme below (Fig. 2).


Fig. 2. The scheme presenting the idea of measurement of small values of hydrostatic pressures

In the closed A vessel a sound of MS-321-LCD sensor was placed in such a way that it is above the free water surface. In turn, the second sound is placed outside the A vessel. Inside the A vessel there is $P_{x}$ pressure. A is connected with B vessel and B vessel is open. There is $P_{o}$ pressure over the free surface of water in B vessel and outside A vessel (Fig. 2).

$$
\begin{equation*}
\Delta P=P_{x}-P_{o} \tag{1}
\end{equation*}
$$

If the valve in A vessel is open, the pressure $P_{x}$ tends to reach the value of $P_{o}$ of the environment, i. e. atmospheric pressure. Free position of fluid surfaces in both the vessels is being balanced. After the stabilisation of the process, the valve in A vessel is closed. At this moment the measured value $\Delta P$ should theoretically amount to 0 and this value has been assumed for further analysis. If the height of the position of free water surface changes now in B vessel, by $H$ value, then the value of pressure $P_{x}$ must change. It will make the free surface of water move by $h$ in A vessel. On the basis of the Bernoulli theorem (Bear, 1988; Duckworth, 1983; Troskolański, 1957) we can provide the following relation (2).

$$
\begin{equation*}
P_{o}+\rho g H=P_{x}+\rho g h \tag{2}
\end{equation*}
$$

where:
$\rho$ - fluid density in A vessel,
$g$ - gravitational acceleration.
After transformation we receive the following equation (3).

$$
\begin{equation*}
\Delta P=P_{x}-P_{o}=\rho g(H-h) \tag{3}
\end{equation*}
$$

Additionally, it has been assumed (4) that the equation (3) can be presented as (5). In order to make the hydrostatic pressure measurement with the accuracy assumed in (4), suitable geometry of A vessel should be applied. It can be concluded that theoretically the pressure value depends on the class of the gauging tool.

$$
\begin{equation*}
\frac{h}{H} \leq 0.1 E \Rightarrow H-h \approx H \tag{4}
\end{equation*}
$$

It allows us to skip h parameter in determining hydrostatic pressure in B vessel since its influence will be lesser than the error of gauging tool. In consequence we can provide the following form for equation (3):

$$
\begin{equation*}
\Delta P=\rho g H \tag{5}
\end{equation*}
$$

It can be obviously concluded from the discussion presented above that it is the A vessel that is the most important element of the process in measuring very low hydrostatic pressure values. Gases, including air, are known to be compressible fluids, unlike liquids. By forcing liquid into a measuring vessel we diminish the volume of gas contained in the closed space over its free surface, which results in pressure change inside the gauging tool. The value of this change must be applied in such a way in relation to the geometry of the vessel that condition (4) is maintained. With such small change values of pressure amounting to a few pascals, we can assume that the gas inside the tool is almost ideal gas because in relation to the atmospheric pressure of $10^{5} \mathrm{~Pa}$ these changes constitute its thousands of parts (Górniak \& Szymczyk, 1999; Szargut, 1997). In connection with this, in accordance with the Boyle-Mariotte law (Górniak \& Szymczyk, 1999; Szargut, 1997), pressure change for the assumed values results in such a small change of volume that the influence of the change can be skipped while designing A vessel geometry.

Bellows pressure gauges are used in the measurement of such low pressures (Troskolański, 1957). The change of the bellows volume is here a determining element of the geometry of the whole tool. This volume was marked as $V_{m}$. Now it is possible to determine the value of S surface of A gauging vessel, in the function of maximum pressure measured $P[\mathrm{~Pa}]$. In accordance with relation (6), the greater S surface is, than smaller the range of the measured hydrostatic pressure.

$$
\left.\begin{array}{rl}
\frac{h}{H} \leq 0.1 E \Rightarrow \frac{h \rho g}{P} \leq 0.1 E  \tag{6}\\
V_{m} & =h S
\end{array}\right\} \Rightarrow S \geq \frac{10}{E} \rho g \frac{V_{m}}{P}
$$

Temperature changes - in accordance with the Gay - Lussac's law (Górniak \& Szymczyk, 1999; Szargut, 1997) - may exert considerable influence on the appropriate operation of the gauging tool. $V_{o}$ gas volume contained in a given area at the temperature of the measurement

## 162

onset point $t$ will change with the change of the temperature in accordance with relation (7). It will entail the position change of the free water surface and thus the value being measured. We can conclude from the discussion presented above that, during the measurement procedure, the temperature should remain constant and unchanged. Table 1 shows the maximum range of temperature changes $\Delta t$ during the hydrostatic pressure measurement, depending on the temperature at the onset measurement when condtion (4) is fulfilled. Obviously, the range of these changes should be smaller than that presented in the table. The influence of temperature on liquid expandability, however, can be skipped because it is by one order of magnitude smaller than that for gases (Górniak \& Szymczyk, 1999; Szargut, 1997).

$$
\begin{equation*}
\Delta V=V_{o} \frac{\Delta t}{273,15} \tag{7}
\end{equation*}
$$

TABLE 1
Maximum temperature change $\Delta t$ during hydrostatic pressure measurement in relation to onset measurement temperature $t$

| Onset measurement temperature | ${ }^{\circ} \mathrm{C}$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Maximum temperature change $\Delta t$ | ${ }^{\circ} \mathrm{C}$ | 2.83 | 2.85 | 2.87 | 2.89 | 2.91 | 2.93 | 2.95 | 2.97 | 2.99 | 3.01 |

In order to conduct accurate measurement of the hydrostatic pressure, pressure $P_{o}$ should be kept constant. However, because of the fact of the change of the environment pressure, colloquially referred to as atmospheric pressure in the function of time (which was presented as an example for the period from 13 till 14/4/2011 in Fig. 3), we cannot use it as a reference point for A and B vessels. From Fig. 3 we can conclude that environment pressure changes are by one order of magnitude greater than the value of pressures which we intend to measure. The earlier use of this term resulted from an easier description of the idea of measurement.


Fig. 3 Pressure reduced to sea level of the city of Cracow. http://new.meteo.pl/

Figure 4 presents picture of fragments of the author's gauging tool for measurement of very low differences of hydrostatic pressures. It has been decided that it is not vital to go into technology and automation matters allowing for accurate preparation, in accordance with the discussion presented above, of the tool and performance of measurement procedure. A detailed description of the said problems was included in the doctoral dissertation (Filipek, 2011). Thus, only the most important matters, constituting the basis for the construction of such a mechanism, were


Fig. 4. Gauging tool for small values of hydrostatic pressures
described herein. Among others, the explanation of how the relation pressure $P_{o}$ was stabilised during the time of measurement and the description of the module regulating the level of free surface of fluid in B vessel were omitted.

Test results of measurements were presented in figures 5-9. At the outset of measurement neither the future response of the gauging tool nor the future best and accurate methodology of measurement has been known.

In the first test measurements (Fig. 5 and 6) hysteresis was found, which resulted from the retardation of the water free surface in the gauging tool against the position change of the free surface in the test column.

It can be concluded from the diagrams presented above (Fig. 5 and 6) that the value of the change caused by hysteresis can be adjusted by changing the rapidity of change in the time of a given pressure). By changing the gradient we reduce hysteresis and obtain more accurate distribution of the measured pressure. In subsequent tests there was an attempt made to reduce the influence of hysteresis on measurement accuracy to the maximum (Fig. 7, 8, 9).

In all the presented diagrams illustrating completed measurements (Fig. 4-8) the phrase 'The pressure increases' means that the measurement was performed for the increasing hydrostatic pressure from 0 to 25 Pa approximately by 1 Pa . The situation is analogical for the phrase 'The pressure decreases'. In order to determine a given value 500 measurements were being registered in c. 10 minutes. The mean of obtained results was calculated and then marked on the graph. On the basis of two mean values of increasing and decreasing pressure additional mean was calculated which was also marked on the graph. The measurements were preformed in 5 series $\{(5,10,15,20$, $25 \mathrm{~Pa}), \ldots,(1,6,11,16,21 \mathrm{~Pa})\}$. Each series took c. 9 h. Although there was c. 36-hour interval
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Fig. 5. Hysteresis of measurement of the hydrostatic pressure value for rapid changes of measured value


Fig. 6. Hysteresis of measurement of hydrostatic pressure for small changes of measured value



Fig. 7 Influence of atmospheric pressure on measurement process at long times of measurement
between the measurement of values, e. g. 5 Pa and 6 Pa , it can be noticed that the measurement was performed correctly (Fig. 8). The visible sinusoid results from disproportional increase of pressure resulting in turn from inappropriate manufacture of power unit (c. 0.043 mm ), which could not be controlled and repaired (Filipek, 2011). However, this effect additionally supports the accuracy of measures and shows high sensitivity of the presented method.

The inventive method of measurement of small differences of hydrostatic pressures that has been presented allows for measurement of the difference of pressures with accuracy of up to 0.5 Pa , which was presented in the paper. It should be emphasised that the methodology of measurements with the use of more precise sensor will allow for an improvement of measurement accuracy in the future by at least one order of magnitude. Of course, it would require the use of temperature stabilisation for the tool. During measurement hysteresis turned out to be an adverse phenomenon, which entailed the need for creating an efficient methodology of hysteresis reduction below the class of the applied sensor, which was successfully managed and the results were presented in figures $5,6,8,9$. It will be more difficult to compensate the influence of atmospheric pressure on the measurement process.



Fig. 8 Influence of atmospheric pressure on measurement process at short times of measurement

The research on the gauging tool has been continued and the tool has been technically improved, which suggests appropriateness of the assumed guidelines for methodology and construction.

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Fig. 9 Graph presenting measurement of small values of hydrostatic pressures at optimal setting of the gauging

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[^1]:    ${ }^{1}$ Reynolds number is determined on the basis of filtration velocity and diameter of the spheres.

[^2]:    ${ }^{2}$ Piezometer - pipe, mainly made of glass, with a scale on the surface with definition most often of 1 mm designer for optical measurement of water column inside the pipe. With fluid density and gravitational acceleration given, we can define hydrostatic pressure (Duckworth, 1983; Troskolański, 1957).

