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Arch. Min. Sci., Vol. 58 (2013), No 3, p. 691–704

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2013-0048

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NUMERICAL STUDY OF THE HYDRODYNAMIC EFFICIENCY OF THE MULTI-STAGE FILTER SETTING TECHNOLOGY

STUDIUM NUMERYCZNE EFEKTYWNOŚCI HYDRODYNAMICZNEJ OTWORÓW EKSPLOATACYJNYCH Z ZASTOSOWANIEM WIELOSTOPNIOWYCH SELEKTYWNYCH FILTRÓW

In this work the numerical study of the hydrodynamic efficiency of the multistage filters setting technology is carried out on the basis of mathematical simulation. Obtained results of a flow of solution in porous media near a wellbore qualitatively conform to the experimental data.

In calculations the well is considered as the high-permeability channel with the fictitious permeability coefficient depending on a filter construction (porosity, form of perforations). The results of calculation show that the fictitious permeability coefficient has deep influence on the fluid influx to the well and the distribution of flow rate on well height is not uniform. The developed model is used for the axisymmetric case.

Calculations were carried out for a single well; however it can be easily applied to solve the 3D problem with various sets of wells.

Keywords: well, wellbore, porosity, permeability, Darcy law, Dupuit law, multi-stage filter setting

W artykule omówiono efektywność hydrodynamicznych filtrów na podstawie modelowania matematycznego. Decyzję doboru stref filtracji w sposób jakościowy są potwierdzone badaniami eksperymentalnymi. W obliczeniach numerycznych założono istnienie wysoko-przepuszczalnych kanałów z założoną fikcyjną przepuszczalnością zależną od konstrukcji (powierzchni otwarcia). Wyniki obliczeń pokazują, że współczynnik przepuszczalności ma duże znaczenie dla dopływu płynu do otworu i niejednorodnego profilu pionowego natężenia przepływu. Model numeryczny dotyczy symetrii osiowej.
Obliczenie wykonano dla pojedynczego otworu. Rozwiązanie może być zastosowane do modelowania trójwymiarowego z uwzględnieniem otworów.

Słowa kluczowe: otwór, porowatość, przepuszczalność, prawo Darcy'ego, prawo Dupuita, posadowienie otworów filtrów selektywnych

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1. Introduction

During the consideration the problem in the bottomhole formation zone wellbore is considered as a surface with a constant reduced pressure. In work (Brovin et al., 1997) it's shown that such assumption does not show a qualitative picture of the fluid flow in the bottomhole zone.

To construct an accurate mathematical model it's necessary to use Navier-Stokes equation for the interior of a vertical wellbore, and the Darcy's law for modeling the flow in porous media in the reservoir. Strictly speaking, it would have had to sew two laws on the contact surface of a rock and filter. Such approach requires enormous computing, as computational grid must be sufficiently refined to cover the interior of the wellbore (Tolpayev & Zaharov, 2003).

Therefore, the fluid flow in the wellbore is approximately considered as a flow in a fictitious porous medium with an apparent permeability k_2, which allows using Darcy's law at the bottomhole zone instead of the Navier-Stokes equations. In practice, the value of k_2 is determined by many factors (type of construction of the filter, its porosity, the shape of the perforations, etc., Fig. 1.).

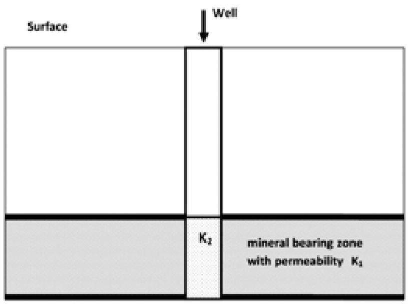

Fig. 1. Bottomhole zone with permeability of k_1. Producing well with permeability of k_2 noted by red color

2. Mathematical formulation of the problem

The law of fluid motion in the wellbore is defined by following relation

$$\frac{dP}{dz} = -f(u) \tag{1}$$

where $f(u)$ – given function satisfying the condition $f(0) = 0$. For the function $f(u)$ it's possible to use linear, binomial or power-law motion, depending on the intensity of well work (Tolpayev & Zaharov, 2003).

Let's consider the fluid flow to the well with multistage scheme of filters setting (Fig. 2). The law of motion in the wellbore will be taken by the law (1), and it is considered that the movement in the wellbore obeys a linear law and

$$f(u) = \frac{\mu u}{k_2} \tag{2}$$

where

μ — liquid viscosity,

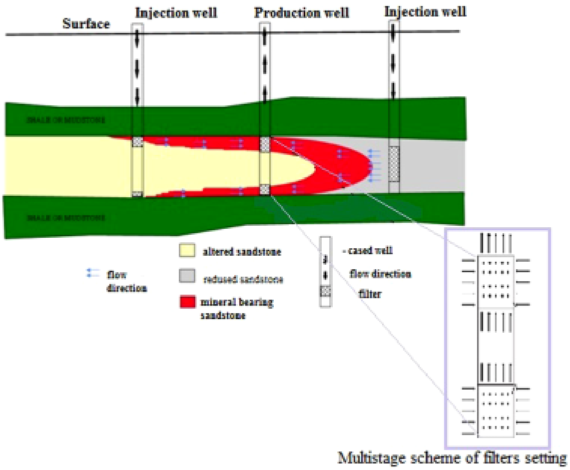

Fig. 2. Multistage scheme of filters setting

k_2 — fictitious permeability in the wellbore,
u — is vertical speed in the wellbore.

Radial inflow of the solution to the well filters is defined by Dupuit equation.

$$Q = \frac{2\pi k_1 h}{\mu}\left(\frac{P_0 - P_w}{\ln\left(\frac{R}{r_c}\right)}\right) \tag{3}$$

where
k_1 — rock permeability,
P_0 — pressure on external boundary (reservoir pressure),
P_w — pressure on well,
R — radius of external boundary,
r_c — well radius,
h — seam thickness.

The flow in the bore between the planes z and $z + dz$ is considering. From (3) we obtain the fluid flow to the side of the well (Fig. 3).

$$dq(z) = \frac{2\pi k_1 dz}{\mu}\left(\frac{P_0 - P(z)}{\ln\left(\frac{R}{r_c}\right)}\right) \tag{4}$$

Here $P(z)$ – reduced pressure on well height. From the mass conservation law it's clear that the amount of flow passing through the lateral surface of the volume and through a section of z should be equal to the flow through the cross section $z + dz$. In case of the well sites, where no filters we will not consider the radial flow to the well, but only flow within the channel is considered, i.e. set $dq = 0$ (Fig. 4).

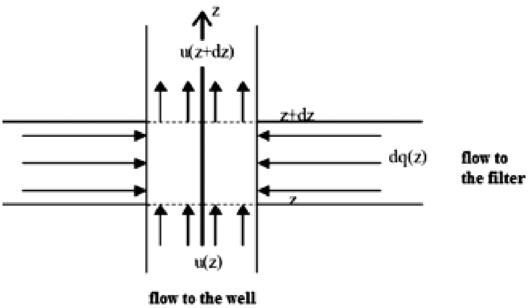

Fig. 3. Scheme of fluid flow inside the wellbore in a filter zone

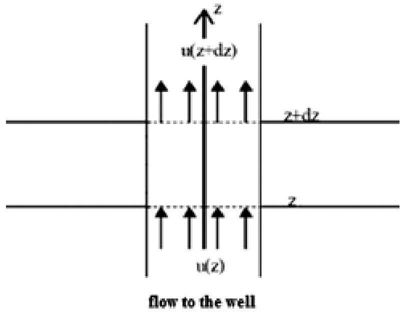

Fig. 4. Scheme of fluid flow inside the wellbore in a zone without filter

From the mass conservation law we obtain

$$\begin{cases} \pi r_c^2 u(z+dz) = dq(z) + \pi r_c^2 u(z), & \text{for a zone with filter} \\ \pi r_c^2 u(z+dz) = \pi r_c^2 u(z), & \text{for a zone without filter} \end{cases} \tag{5}$$

or

$$\begin{cases} u(z+dz) - u(z) = \dfrac{dq(z)}{\pi r_c^2}, & \text{for a zone with filter} \\ u(z+dz) - u(z) = 0, & \text{for a zone without filter} \end{cases} \tag{6}$$

Since we defined the law of motion in the wellbore through the relation (1)

$$\frac{dP(z+dz)}{dz} = -f(u(z+dz)) \tag{7}$$

Subtracting from (7) the relevant parts of (1) we obtain

$$\frac{dP(z+dz)}{dz} - \frac{dP(z)}{dz} = -\left[f(u(z+dz)) - f(u(z))\right] \tag{8}$$

We write the equation (8) in the form

$$\frac{dP(z+dz)}{dz} - \frac{dP(z)}{dz} = -\left[u(z+dz) - u(z)\right]\frac{\left[f(u(z+dz)) - f(u(z))\right]}{u(z+dz) - u(z)} \tag{9}$$

applying Lagrange's theorem, and $dz \to 0$ then we get

$$\begin{cases} \dfrac{d^2P(z)}{dz^2} = -\left[\dfrac{dq(z)}{\pi r_c^2}\right] f'(u), & \text{for a zone with filter} \\ \dfrac{d^2P(z)}{dz^2} = 0, & \text{for a zone without filter} \end{cases} \tag{10}$$

Here, the derivatives on the right side of the equation are taken with respect to u. Equation (10) describes the motion of the fluid along the bore, with the assumption that the radial flow inside the bore can be neglected. If we define the function $f(u)$ in the form (2), then from (10) and (4) we'll obtain

$$\begin{cases} \dfrac{d^2P(z)}{dz^2} = -\dfrac{2k_1}{r_c^2 k_2}\left(\dfrac{P_0 - P(z)}{\ln\left(\dfrac{R}{r_c}\right)}\right), & \text{for a zone with filter,} \\ \dfrac{d^2P(z)}{dz^2} = 0, & \text{for a zone without filter} \end{cases} \tag{11}$$

The system equation (11) takes into account the motion of the fluid within the wellbore the radial flow to the well, the pressure distribution within the well on height and heterogeneity on setting the filters. In addition, the solution of this problem can be applied as a boundary condition for the 3D case as far as the inflow to the well taken into account by the Dupuit equation. The k_2 – can be varied in section of the well, where the filters is located. In this case we take k_2 = const over the entire height of the well.

The upper and bottom boundary conditions for the equations (11) are as following

$$P\big|_{z=b} = P_c \quad \text{and} \quad \left.\frac{dP}{dz}\right|_{z=0} = 0 \tag{12}$$

The solution of (11), (12) can be obtained using software for the solution of systems of ordinary differential equations or directly numerically solving.

The flow to the well with multi-stage planted filter in the reservoir with a seam thickness b = 28 m is considered. Filters are defined on the heights $z \in$ [5 m; 11 m] and $z \in$ [17 m; 21 m] and the coordinate z directed vertically upwards. In this case the system (11) is as following

$$\begin{cases} \dfrac{d^2P(z)}{dz^2} = -\dfrac{2k_1 dz}{r_c^2 k_2}\left(\dfrac{P_0 - P(z)}{\ln\left(\dfrac{R}{r_c}\right)}\right), & \text{at } z \in [5m; 11m] \text{ or } z \in [17m; 21m] \\ \dfrac{d^2P(z)}{dz^2} = 0, & \text{at } z \notin [5m; 11m] \text{ or } z \notin [17m; 21m] \end{cases} \tag{13}$$

with boundary conditions $P\big|_{z=b} = P_c$ and $\left.\dfrac{dP}{dz}\right|_{z=0} = 0$.

After solving the problem (13) the pressure distribution on well height is obtained then using (4) flow rate can be finding at each level of z.

Results of the solution of (13) is presented at different ratios of the coefficients k_1/k_2 and pressures P_0/P_c.

The graphics are shown for the flow rate $q(z)$ for the case when filters are set through the production zone and for the multilevel setting at various ratios of filtration coefficients (Fig. 5-8).

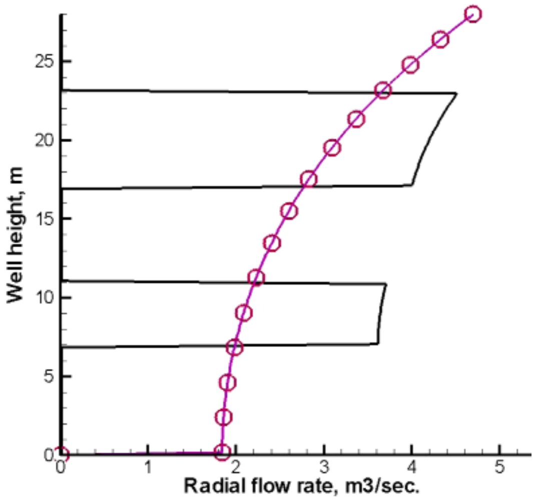

Fig. 5. Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter through height of z $\frac{k_1}{k_2} = 0.0001$, $\frac{P_0}{P_c} = 1.3$

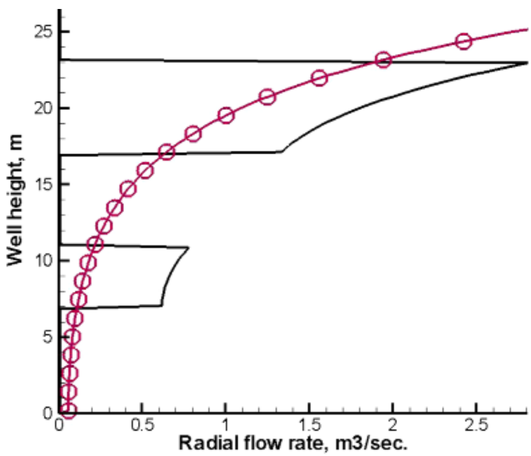

Fig. 6. Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter through height of z $\frac{k_1}{k_2} = 0.001$, $\frac{P_0}{P_c} = 1.3$

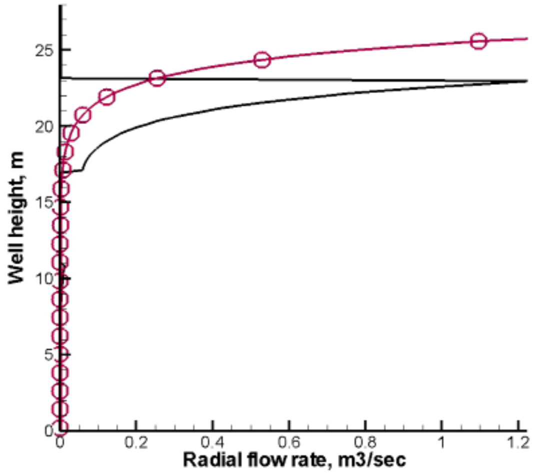

Fig. 7. Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter through height of z $\frac{k_1}{k_2} = 0.01$, $\frac{P_0}{P_c} = 1.3$

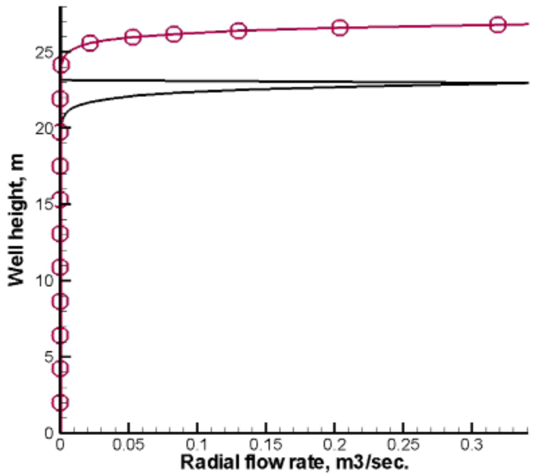

Fig. 8. Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter through height of z $\frac{k_1}{k_2} = 0.1$, $\frac{P_0}{P_c} = 1.3$

Fig. 9 shows a qualitative comparison of the calculation results with the data from the industrial experiment. The solid curve shows the flow rate $q(z)$ at multi-stage filter set, circled curve for fully filter through the height of z.

Problem has been treated in a cylindrical coordinate system. Like the previous problem well is modeled as a medium with an apparent permeability depending on the porosity of the filter.

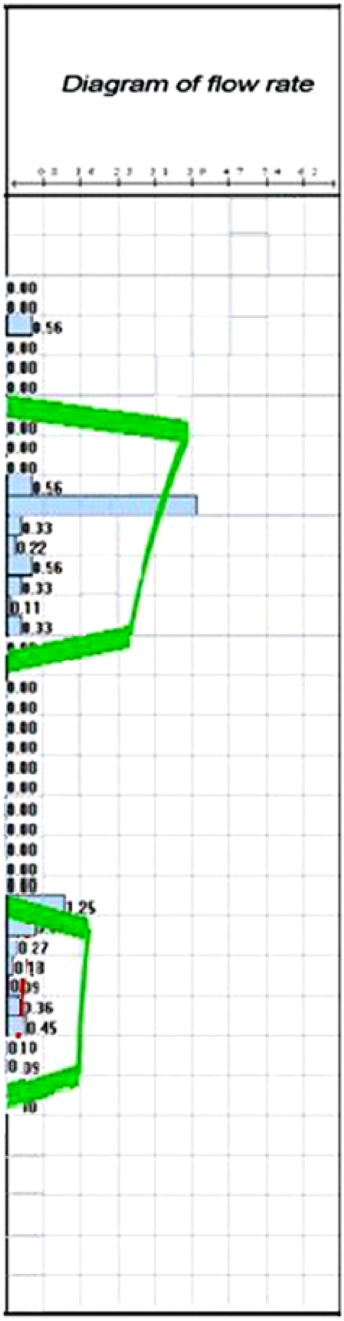

Fig. 9. Qualitative comparison of the calculation results with the data from the industrial experiment. The green curve shows the flow rate $q(z)$ at multi-stage setting filters for the problem (13)

3. 3D model of the process

Fluid filtration in porous media with certain permeability is considering. It is assumed that the medium is homogeneous and isotropic, the densities of the liquid and the reservoir are constant, and solution cross-flow doesn't exist on the upper and lower boundaries of the reservoir. Then using Darcy law and conservation law these processes are described by the following equations

$$div\left(\frac{K}{\mu}grad\ p\right)=0 \tag{14}$$

$$\vec{V}=-\frac{K}{\mu}grad\ p \tag{15}$$

with boundary conditions

$$\left.\frac{\partial p}{\partial n}\right|_S=0\ ,\ \ p|_S=p_0+\rho gz \tag{16}$$

where

p — hydraulic head in reservoir;
$\vec{V}$ — filtration rate;
K — permeability coefficient;
μ — fluid viscosity.

The problems (1), (2) and (3) are considered in the cylindrical formulation due to the isotropy of characteristics (Fig. 1). It is reduced to the solution of the Laplace equation with variable permeability – k. Within the wellbore some fictitious permeability coefficient is taken, and on the walls of the well chosen a coefficient depending on filter porosity. On this basis for accounting of fluid movement within the wellbore computational grid is constructed as non-uniform (Fig. 10). The results of calculation for the pressure and velocity fields are shown in Fig. 11-19.

As the pressure and velocity potential differ only by a constant ($\phi=-\frac{K}{\mu}p$), the flow at the junction of perforated and non-perforated zone of well corresponds to the inviscid flow around the edge of the wafer. This phenomenon corresponds to the model considered in present work.

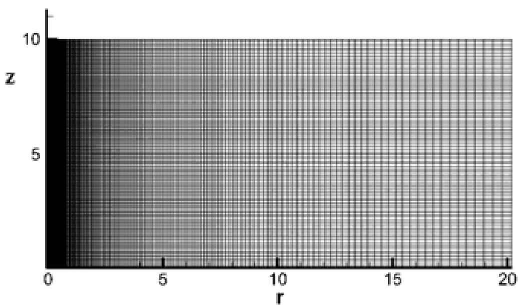

Fig. 10. Computational grid for 2D calculations

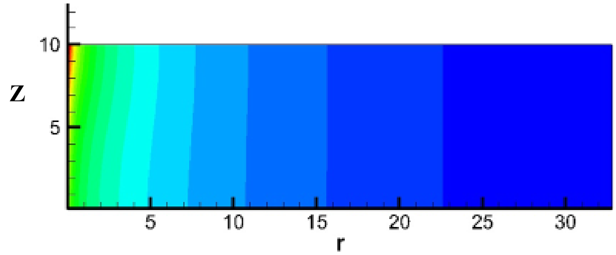

Fig. 11. Pressure distribution in reservoir for the case of fully setting filter

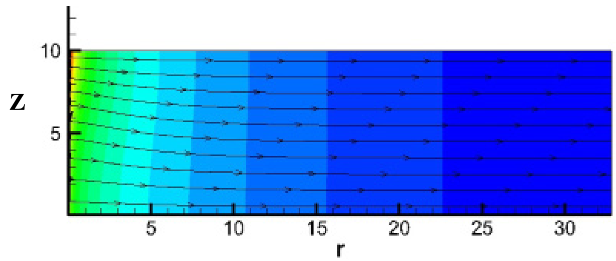

Fig. 12. Stream lines from injecting well with fully setting filter

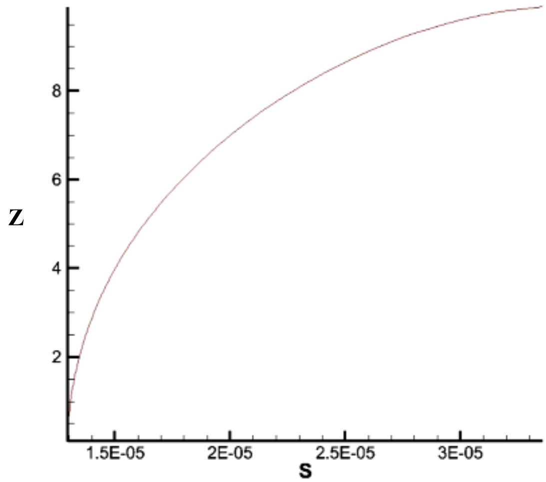

Fig. 13. Rate on height on the well wall for the case of fully setting filter

Because of the dimensionality of the problem a non-uniform grid is used (Fig. 10). On this grid the pressure in the reservoir is calculated (Fig. 11) as well as the streamlines (Fig. 12) for a fully setting filter. The flow rate at well wall on depth for the case of full setting filter is shown in Fig. 13.

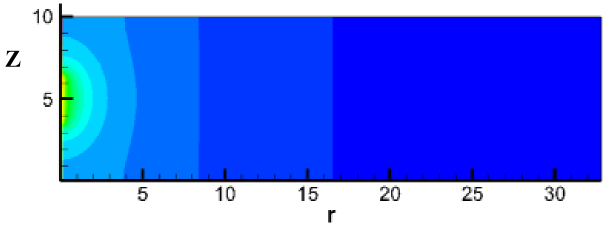

Fig. 14. Pressure distribution in the reservoir for the case of imperfect well

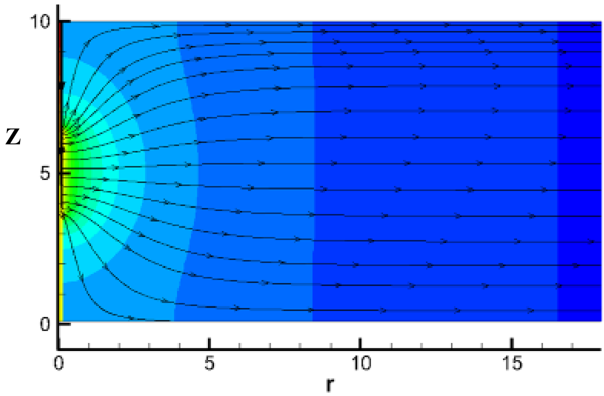

Fig. 15. Streamlines from injecting well for the case of imperfect well

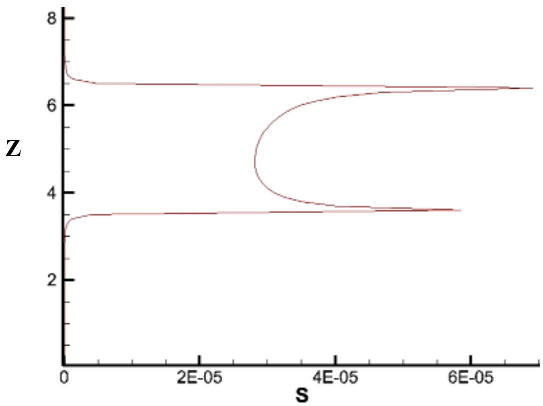

Fig. 16. Flow rate on height on the well wall for the case of imperfect well

A pressure in the reservoir is obtained (Fig. 14) as well as the streamlines (Fig. 15) for the imperfect well. The flow rate at well wall on depth for the case of imperfect well is shown in Fig. 16.

A pressure in the reservoir is obtained (Fig. 17) as well as the streamlines (Fig. 18) for the multi-stage setting filter. The flow rate at well wall on depth for the case of multi-stage setting filter is shown in Fig. 19.

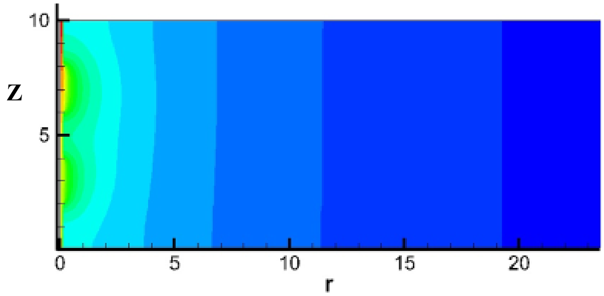

Fig. 17. Pressure distribution in the reservoir for the case of multi-stage setting filter

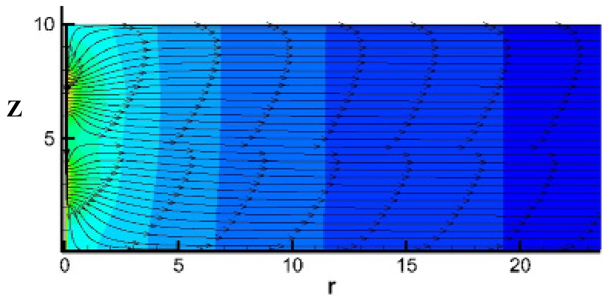

Fig. 18. Streamlines from injecting well with a multi-stage setting filter

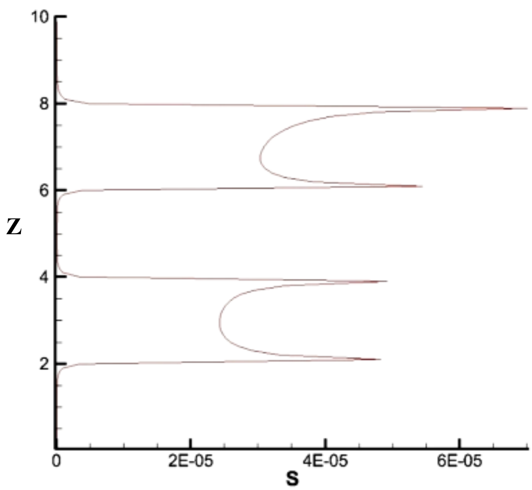

Fig. 19. Flow rate on depth on the well wall for the case of multi-stage setting filter

4. Conclusion

Due to the value increase of non-renewable mineral resources and the rapid growth of cost of wells repair in around the world pay special attention to the correct initial well completion. Maximum reliability and productivity have an especially importance for wells located in hard-to-reach places (sand, sand dunes). To achieve reliably and well productivity is particularly difficult where the reservoir sands are not cemented or otherwise, tend to destruction. Mechanism of carrying out of the sand is unusually complex, and it turns out influence each well completion operation (from primary drilling of layer to develop the wells for sampling or injection).

The results of calculations show that the distribution of flow (inflow) on well height is not uniform. In the calculations the well considered as high-permeability channel, depending on the construction of the filter (porosity and shape of the perforations). Coefficient of fictious permeability has a strong influence on the flow of liquid to the well (see Fig. 5-8). Calculations were carried out for a single well, but can easily be applied to solve three-dimensional problem with sets of wells.

Based on the results of the solution of this problem, we can conclude that in case of stagnation of the lower zone of the well it is appropriate to apply the multi-stage setting of filters, as the using of such filters in stagnation zone appears non-zero radial flow.

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Received: 08 January 2013