Mathematical modelling of heat transfer in liquid flat-plate solar collector tubes

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Abstract   The paper presents a one-dimensional mathematical model for simulating the transient processes which occur in the liquid flat-plate solar collector tubes. The proposed method considers the model of collector tube as one with distributed parameters. In the suggested method one tube of the collector is taken into consideration. In this model the boundary conditions can be time-dependent. The proposed model is based on solving the equation describing the energy conservation on the fluid side. The temperature of the collector tube wall is determined from the equation of transient heat conduction. The derived differential equations are solved using the implicit finite difference method of iterative character. All thermo-physical properties of the operating fluid and the material of the tube wall can be computed in real time. The time-spatial heat transfer coefficient at the working fluid side can be also computed on-line. The proposed model is suitable for collectors working in a parallel or serpentine tube arrangement. As an illustration of accuracy and effectiveness of the suggested method the computational verification was carried out. It consists in comparing the results found using the presented method with results of available analytic solutions for transient operating conditions. Two numerical analyses were performed: for the tube with temperature step function of the fluid at the inlet and for the tube with heat flux step function on the outer surface. In both cases the conformity of results was very good. It should be noted, that in real conditions such rapid changes of the fluid temperature and the heat flux of solar radiation, as it was assumed in the presented computational verification, do not occur. The paper presents the first part of the study, which aim is to develop a mathematical model for simulating the transient processes which occur in liquid flat-plate solar collectors. The experimental

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verification of the method is a second part of the study and is not presented in this paper. In order to perform this verification, the mathematical model would be completed with additional energy conservation equations. The experimental verification will be carry out in the close future.

Keywords: Solar collector; Mathematical model; Transient state; Exact solution

Nomenclature

A	–	area, flow area, m^2
c	–	specific heat, J(kgK)
d	–	diameter, m
g	–	thickness, m
h	–	specific enthalpy, J/kg
k	–	thermal conductivity, W/(mK)
L	–	length of the analysed collector tube, m
$\dot{m}$	–	mass flow, kg/s
M	–	number of cross-sections
p	–	tube pitch, m
r	–	radius, m
t	–	temperature, °C
T	–	temperature, K
w	–	flow velocity, m/s
z	–	spatial coordinate, m

Greek symbols

α	–	heat transfer coefficient, $\mathrm{W/(m^2K)}$
η	–	dimensionless time
θ	–	wall temperature, °C
μ	–	dynamic viscosity, kg/(sm)
ρ	–	density, $\mathrm{kg/m^3}$
τ	–	time, s
ζ	–	dimensionless coordinate
$\Delta\tau$	–	time step, s
Δz	–	spatial size of control volume, m

Subscripts

in	–	inner
j	–	subsequent control volume
l	–	loss
m	–	middle
o	–	outer
w	–	wall

1   Introduction

Due to environmental issues and limited fossil fuel resources, more and more attention is being given to renewable energy sources (RES) [1]. The energy of solar radiation is one form of RES. It can be widely used for example for water heating in hot water systems, swimming pools as well as a supporting energy sources for central heating installations. The energy of the solar radiation is in this case converted to heat with the use of solar panel. Currently the most often used solar collectors are liquid flat-plate (working in parallel or serpentine tube arrangement) and the vacuum based structures. The vacuum based collectors, despite their higher efficiency, are rarely used by the individual users (due to the high investment cost).

In order to obtain the appropriate heat power for the particular installation, the knowledge of the collector operation parameters is necessary. Those parameters, which include e.g. fluid temperature at the panel outlet, depend on the level of sun exposure, thus they are variable. The solar collector works in transient conditions. The variability of the collector working conditions depends also on the transient energy consumption by the users.

Existing attempts to model solar collectors are based on steady state conditions, much simplified models or models with lumped parameters, e.g. [2–4]. Moreover, the majority of models do not take into consideration the dependency of thermophysical properties of the fluid and the material, of which the tubes and the absorber are made, on the temperature. Fan *et al.* [2] presented a numerical and experimental investigation of the flow and temperature distribution in a solar collector panel. Numerically, the flow distribution through the tubes was investigated with CFD (a simplified model was built using the CFD code Fluent 6.1). They achieved high agreement between the calculation results and measurements for large mass flows. For smaller flows some inconsistencies occurred, resulting most likely from the oversimplification of the collector model. Zueva and Magiera [4] developed a mathematical model of heat exchange in the arrangement consisting of solar collector – heat exchanger. In this arrangement, both the collector and the exchanger are modelled as lumped parameters structures. The dependencies allowing to determine the temperature of the working fluid at the collector outlet and the heat flux transferred to the fluid were developed.

The published results describing the work of the solar collectors were obtained mainly from the experiments and measurements, e.g. [2,5,6]. Razavi *et al.* [5] analysed the heat fluxes in solar water heaters with poly-propylene

piping within the determined range of the Reynolds number. The obtained results were used to determine the Nusselt number and can be used to evaluate the heat fluxes transferred in poly-propylene collectors for similar working conditions. Morrison *et al.* [6] investigated the efficiency of the vacuum based collector and the mass flow distribution. They determined the influence of various factors on the collector operation. The numerical model of the vacuum based collector was proposed. On the basis of this model Morrison *et al.* determined the critical values of the Rayleigh number and the collector inclination angle for which the stagnation area occurred in the collector tube.

Augustus and Kumar [7] developed a mathematical model of the unglazed transpired collector (UTC), also known as perforated collectors – an innovation in the solar technology. On the basis of the proposed model the influence of the main parameters on the efficiency of the UTC collector can be determined. The analyses were performed for different values of porosity, air mass flows, solar radiation and emissivity. The obtained results allowed to develop nomograms, which can be a valuable tool for the design and optimisation of the UTC collectors.

The thermal performance of flat-plate collectors is strongly related to the flow distribution through the absorber tubes [8]. The more uniform the flow distribution, the higher the collector efficiency. However, uniform flow distributions are not always present in solar collectors [9,10]. Weitbrecht et al. [9] investigated the mass flow distribution of the "Z" configuration flat collector working in a laminar flow regime. They determined the flow distribution and the pressure losses in the collector. The proposed analytical solution of the flow through the collector, based on the values of the pressure losses coefficients, allows to determine the flow distribution for different cases and for variable boundary conditions.

In this paper a mathematical model of heat transfer in flat-plate solar collector tubes, being a model with distributed parameters, is proposed. It enables the on-line analysis of collector operation under transient boundary conditions. In order to correctly model the dynamics of the collector using the proposed method, the knowledge of the solar radiation intensity, the effective transmittance-absorption coefficient of the collector and the value of heat losses to the environment is necessary. Knowing from the measurement the value of the solar radiation, heat losses can be determined using the energy balance of the collector. The method presented in this paper allows to compute or to predict the transient fluid temperature. All the

thermophysical properties of the operating fluid and the material of the absorber can be computed in the on-line mode. The model can be also useful to compute monthly average hourly and daily utilizability [11].

2   Description of the proposed method

In this section the mathematical model for simulating the heat transfer in the liquid flat-plate solar collector tubes is presented. One tube of the collector is taken into consideration. In the proposed model, which is one with distributed parameters, the computations are carried out along the way of the flow of the operating fluid in one tube. That tube is equal in size to ones existing in the real object. The mass flow of fluid is also related to a single tube. The proposed method is based on the assumption that the operating fluid flows uniformly through all tubes of the collector working in a parallel channel arrangement. All the thermo-physical properties of the fluid and the material of the tube wall (absorber) can be computed in real time. The suggested one-dimensional model is proposed for modelling the dynamics of the liquid flat-plate solar collectors, considering time-dependent boundary conditions. The presented method of determining the time-spatial temperature distributions of the fluid and wall is based on the implicit finite difference scheme of iterative character.

The computational verification of the method is presented in Section 4. In order to carry out this verification, the energy conservation equations for the glass cover, insulation, and air gap between cover and absorber are omitted in this paper. One tube of the collector, working in a parallel channel arrangement, is taken into consideration (Fig. 1a).

The proposed model shows the same transient behaviour as the real solar collector tube if:

- the tube has the same inside and outside diameter, the same length and the same mass as the real one;

- mass flow of the operating fluid is given by:

$$\dot{m}_1 = \frac{\dot{m}_t}{n_{ct}} \,, \tag{1}$$

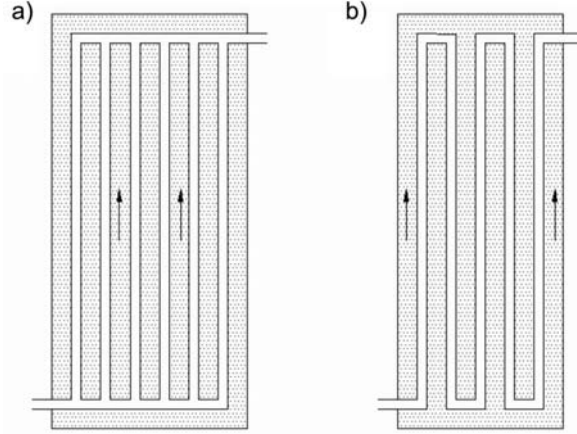 where: $\dot{m}_t$ – total mass flow of operating fluid, n_{ct} – number of solar collector tubes;

Figure 1. Liquid flat-plate solar collectors working in: a) parallel tube arrangement,
b) serpentine tube arrangement.

- the linear heat load of the collector tube equals:

$$\Delta q = \left(G_\beta \left(\tau a \right) - \frac{\dot{Q}_l}{A} \right) p \, , \qquad (2)$$

where: G_β – heat flux of solar radiation, $\mathrm{W/m^2}$, $\dot{Q}_l$ – collector heat
loss, W, τa – effective transmittance-absorption coefficient;

- all the thermophysical properties of the operating fluid and the tube
wall material are computed in real time;

- heat transfer coefficient at the fluid side is computed in on-line mode.

The temperature of the collector tube wall is determined from the equation of transient heat conduction:

$$c_w \left(\theta \right) \rho_w \left(\theta \right) \frac{\partial \theta}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left[r k_w \left(\theta \right) \frac{\partial \theta}{\partial r} \right] \, . \qquad (3)$$

After some transformation, Eq. (3) takes the following form:

$$c_w \left(\theta \right) \rho_w \left(\theta \right) \frac{\left(r_o^2 - r_{in}^2 \right)}{2} \frac{\partial \theta}{\partial \tau} = \left[r k_w \left(\theta \right) \frac{\partial \theta}{\partial r} \right] \bigg|_{r=r_o} - \left[r k_w \left(\theta \right) \frac{\partial \theta}{\partial r} \right] \bigg|_{r=r_{in}} \, . \quad (4)$$

Taking into consideration the boundary conditions:

$$k_w\left(\theta\right)\left.\frac{\partial \theta}{\partial r}\right|_{r=r_o} = G_\beta\left(\tau a\right)\ ,\tag{5}$$

$$k_w\left(\theta\right)\left.\frac{\partial \theta}{\partial r}\right|_{r=r_{in}} = \alpha\left(\left.\theta\right|_{r=r_{in}} - t\right) = \alpha\left(\theta - t\right)\ ,\tag{6}$$

the following ordinary differential equation was obtained:

$$D\frac{d\theta}{d\tau} = t - \theta + E\Delta q\ .\tag{7}$$

In the above formula:

$$D = \frac{c_w\left(\theta\right)\rho_w\left(\theta\right)\,d_m\,g_w}{\alpha d_{in}},\quad E = \frac{1}{\alpha\pi d_{in}}\ \text{and}\ d_m = \frac{d_o + d_{in}}{2}\ .$$

Coefficients D and E are derived from solving the heat transfer equation (3) for the specific boundary conditions (5) and (6). Coefficient D is the time constant characterising the thermal inertia of the tube.

On the side of the fluid energy balance equation, taking into consideration the change in time of the total energy in the control volume, the flux of energy entering and exiting from control volume, and the heat flux transferred to it through its surface, is used. The mass and momentum balance equations are omitted. Such a model results in fewer final equations and in their simpler form. Their solution is thereby reached faster. The omission of the mass and momentum balance equations does not generate errors in the computations and does not constitute the limitation of the proposed method.

The transient temperature of the operating fluid is evaluated iteratively, using relation derived from the equation of the energy balance (Fig. 2):

$$\Delta z A c\left(t\right)\rho\left(t\right)\frac{\Delta t}{\Delta \tau} = \dot{m}\left.h\right|_z - \dot{m}\left.h\right|_{z+\Delta z} + \alpha\pi d_{in}\Delta z\left(\theta - t\right)\ .\tag{8}$$

After rearranging and assuming that $\Delta \tau \rightarrow 0$ and $\Delta z \rightarrow 0$, Eq. (8) takes the following form:

$$B\frac{\partial t}{\partial \tau} = \theta - t - F\frac{\partial t}{\partial z}\ .\tag{9}$$

In the above equation:

$$B = \frac{Ac\left(t\right)\rho\left(t\right)}{\alpha\pi d_{in}},\quad F = \frac{\dot{m}c\left(t\right)}{\alpha\pi d_{in}}\ \text{and}\ A = \frac{\pi d_{in}^2}{4}\ .$$
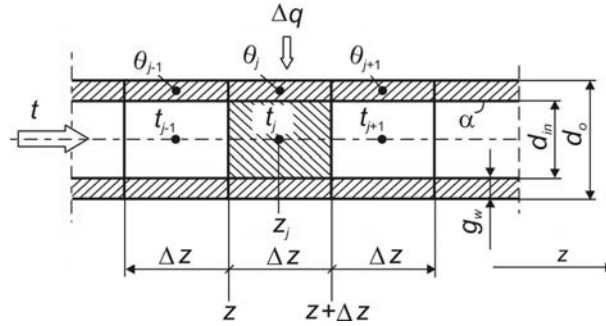
Figure 2. Analysed control volume of the solar collector tube.

Coefficient B is a time constant of the working fluid and coefficient F describes the relation between the fluid heat capacity and the thermal power of the surface of the 1m long channel.

The implicit finite differential method was used to solve the Eqs. (7) and (9). The time derivatives were replaced by the forward difference scheme, whereas the dimensional derivative in Eq. (9) was replaced by the backward difference scheme:

$$\frac{d\theta}{d\tau} = \frac{\theta_j^{\tau+\Delta\tau} - \theta_j^\tau}{\Delta\tau}, \quad \frac{\partial t}{\partial \tau} = \frac{t_j^{\tau+\Delta\tau} - t_j^\tau}{\Delta\tau}, \quad \frac{\partial t}{\partial z} = \frac{t_j^{\tau+\Delta\tau} - t_{j-1}^{\tau+\Delta\tau}}{\Delta z} . \quad (10)$$

After some transformations the following formulas were obtained:

$$\theta_j^{\tau+\Delta\tau} = \left(\frac{D_j^\tau}{D_j^\tau + \Delta\tau}\right)\theta_j^\tau + \left(\frac{\Delta\tau}{\Delta\tau + D_j^\tau}\right)\left(t_j^{\tau+\Delta\tau} + E_j^\tau \Delta q_j^{\tau+\Delta\tau}\right), j = 1, ..., M; \quad (11)$$

$$t_j^{\tau+\Delta\tau} = \frac{\theta_j^{\tau+\Delta\tau} + \frac{B_j^\tau}{\Delta\tau}t_j^\tau + \frac{F_j^\tau}{\Delta z}t_{j-1}^{\tau+\Delta\tau}}{\frac{B_j^\tau}{\Delta\tau} + \frac{F_j^\tau}{\Delta z} + 1} , \quad j = 2, ..., M . \quad (12)$$

In view of the iterative character of the suggested method, the computations should obey the following expression:

$$\frac{\left|Y_{j,(k+1)}^{\tau+\Delta\tau} - Y_{j,(k)}^{\tau+\Delta\tau}\right|}{Y_{j,(k+1)}^{\tau+\Delta\tau}} \leq \delta , \quad (13)$$

where: Y – currently evaluated temperature in node j, δ – assumed iteration tolerance, $k = 1, 2, ...$ – subsequent iteration counter over the single time step.

The heat transfer coefficient is calculated from the relation:

$$\alpha = \frac{\text{Nu}k}{d_{in}} \ . \tag{14}$$

Velocity of the fluid flowing within the tubes of the solar collector is quite small – from a few mm/s up to just more than a dozen of cm/s. Considering the small diameters of the collector tubes, the achieved values of the Reynold's number – from a few to a few hundred – are way below the critical value $\text{Re}_{cr} = 2300$ [12]. Thus the flow is of a laminar character. To determine the Nusselt number for the laminar flow in short channels the empirical Heaton formula can be used [8]:

$$\text{Nu} = \text{Nu}_\infty + \frac{a \left(\text{RePr}\frac{d_{in}}{L}\right)^m}{1 + b \left(\text{RePr}\frac{d_{in}}{L}\right)^n} \ ; \quad 1 < \text{RePr}\frac{d_{in}}{L} \le 1000 \ , \tag{15}$$

where: Re and Pr – Reynolds and Prandtl numbers, respectively; a, b, m, n – coefficients.

In the proposed model the boundary conditions can be time-dependent:

$$\dot{m}\big|_{z=0} = \dot{m}(\tau), \quad t\big|_{z=0} = t(\tau) \text{ and } \Delta q = \Delta q(\tau) \ . \tag{16}$$

The method allows to compute the transient temperature distribution for any selected cross section beginning from time $\tau = 0$, that is from the start of the process. Moreover, the following condition should be obeyed – the Courant-Friedrichs-Lewy stability condition over the time step [13]:

$$|\beta| \le 1, \quad \Delta\tau \le \frac{\Delta z}{w} \ , \tag{17}$$

where $\beta = \frac{w\Delta\tau}{\Delta z}$ is the Courant number. When satisfying this condition, the numerical result is achieved with the speed $\Delta z/\Delta\tau$, greater than the physical speed w.

The efficiency of the proposed method is verified in this paper by the comparison of the results obtained using the suggested method and from the corresponding analytical solutions.

3 The exact solutions for transient states

Available in literature exact solutions for transient heat transfer are developed only for the simplest cases, such as non-heated tube for step function

change of the fluid temperature at the inlet or for a step function heating on the outer surface of the tube [14]. The proposed numerical method allows to solve the problems of transient flows both for those selected configurations for which the analytical solutions exist, as well as for the more complex cases.

The available analytical dependencies allow to determine:

- time-spatial temperature distribution of the tube wall, insulated on the outer surface, as the tubes response to the temperature step function of the fluid at the tube inlet;

- time-spatial temperature distribution of the fluid in case of a heat flux step function on the outer surface of the tube (the case closest to the real conditions in which solar collectors work).

3.1   Temperature step function of the fluid at the tube inlet

The analysed step function assumed the following form (Fig. 3):

$$\Delta T\left(\tau\right) = \begin{cases} 0 & \text{for} \quad \tau < 0\,, \\ 1 & \text{for} \quad \tau \geq 0\,. \end{cases} \tag{18}$$
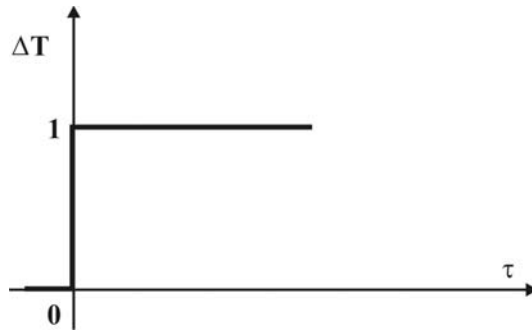

Figure 3. Temperature step function of the fluid at the collector tube inlet.

For this step function, the dimensionless dependency determining the increase of the tube wall temperature takes the following form [14]:

$$\frac{\Delta\Theta}{\Delta T} = V_1 - V_0\,, \tag{19}$$

where

$$V_1 = e^{-(\zeta+\eta)} U(\zeta, \eta) , \tag{20}$$

$$V_0 = e^{-(\zeta+\eta)} I_0 \left(2\sqrt{\zeta\eta}\right) . \tag{21}$$

The $U(\zeta, \eta)$ function is described by the following relation:

$$U(\zeta, \eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{\eta^{n} \zeta^{k}}{n! k!} , \tag{22}$$

and the Bessel function:

$$I_0 \left(2\sqrt{\zeta\eta}\right) = \sum_{k=0}^{\infty} \frac{(\zeta\eta)^k}{(k!)^2} . \tag{23}$$

Values ζ and η present in formulas (20)–(23) are the dimensionless variables of length and time respectively expressed by the following dependencies:

$$\zeta = \frac{z}{F}, \qquad \eta = \frac{\tau - \tau_{TP}(z)}{D} , \tag{24}$$

where

$$\tau_{TP}(z) = B\zeta = \frac{z}{w} . \tag{25}$$

Coefficients B, D and F were described in Section 2 of this paper.

3.2   Heat flux step function on the outer surface of the tube

A dimensionless time-spatial function describing the increase of the fluid temperature ΔT, caused by the heat flux step function Δq on the outer surface of the tube, is expressed by the following formula [14]:

$$\varphi_1 = \frac{\Delta T}{-\frac{c}{1-c} E \Delta q} = \frac{\tau}{D} - \frac{1}{1-c} \varphi_0 - V_2 , \tag{26}$$

where $c = -D/B$ and E are the coefficients described in Section 2.

Functions φ_0 and V_2 are described by the following relations:

$$\varphi_0 = 1 - e^{-(1-c)\frac{\tau}{D}} - V_1 + V_{00} , \tag{27}$$

$$V_2 = e^{-(\zeta+\eta)} \left[(\eta - \zeta) U(\zeta, \eta) + \zeta\, I_0 \left(2\sqrt{\zeta\eta}\right) + \sqrt{\zeta\eta} I_1 \left(2\sqrt{\zeta\eta}\right) \right] , \tag{28}$$

where

$$I_1\left(2\sqrt{\zeta\eta}\right) = \sum_{k=0}^{\infty} \frac{(\zeta\eta)^{\frac{2k+1}{2}}}{(k!)\ (k+1)\ !}\ . \tag{29}$$

Function V_{00} present in formula (27) takes the following form:

$$V_{00} = e^{-(\zeta+\eta)}U\left(\frac{\zeta}{c},\ c\eta\right). \tag{30}$$

The presented above analytical dependencies (19) and (26) allow to determine the time-spatial temperature increases, respectively $\Delta\Theta$ for the tube wall and ΔT for the fluid, for any selected cross-section. The results are obtained beginning from time $\tau_{TP}(z) = z/w$, that is from the moment this cross-section is reached by the fluid flowing with velocity w. For example, if the flow velocity equals 1m/s, than the analytical solutions allow to determine the temperature changes for the cross-section located 10 m away from the inlet of the tube only after 10 s.

4   Computational verification

As an illustration of accuracy and effectiveness of the suggested method the following numerical analyses were performed:

- for the tube with temperature step function of the fluid at the tube inlet,

- for the tube with heat flux step function on the outer surface.

In both cases the operating fluid was assumed to be a weighed 40% water propylene glycol solution. Because the exact solutions do not allow to consider the temperature dependent thermophysical properties, the following values of the water glycol solution were assumed for the computations (for temperature 40 °C): $\rho = 1020$ kg/m^3, $c = 3750$ J/(kgK), $k = 0.447$ W/(mK) and $\mu = 0.0013$ kg/(sm). For both cases it was also assumed, that the collector tube is 1.9 m long, its external diameter equals 0.01 m, the wall thickness is 0.0005 m, and that the tube is made of copper of the following properties: $\rho = 8960$ kg/m^3 and $c = 390$ J/(kgK).

Considering the before mentioned properties the following values of coefficients described in Section 2 were obtained: $D = 9.97$ s, $E = 0.191$ (Km)/W, $B = 46.52$ s, $F = 0.4653$ m and the heat transfer coefficient $\alpha = 185$ W/(m^{2}K).

Satisfying the Courant condition (17) for the hyperbolic equations, the following was assumed for the numeric calculations: $\Delta z = 0.005$ m, $\Delta \tau = 0.1$ s and $w = 0.01$ m/s. The collector tube of the 1.9 m length was divided into 381 cross-sections. Such spatial division allowed to avoid the effects of dissipation and dispersion causing the errors in the computations.

In the first numerical analysis it was assumed that the water glycol solution of the initial temperature $t = 10\,^{\circ}\mathrm{C}$ flows through the collector tube. Also, the tube wall for the initial time $\tau = 0$ has the same initial temperature. Beginning from the next time step, the fluid with temperature $t = 80\,^{\circ}\mathrm{C}$ appears at the inlet. The temperature step function is thus $\Delta T = 70$ K. This is a challenging test for the proposed method. Such large temperature step functions do not occur in solar collectors in the real conditions.

The results of the computations are presented in Fig. 4. The presented dimensionless coordinates $\zeta = 0, 1.289, 2.579$ and 4.083 correspond with the dimensional coordinates $z = 0, 0.6$ m, 1.2 m and 1.9 m respectively. Analysis of the comparison shows satisfactory convergence of the exact solution results and the results obtained using the presented method. Additionally temperature histories of the tube wall and the fluid at the analysed cross-sections are presented in Fig. 5. These histories were obtained using the proposed model.

In the second case it was assumed that the working fluid and the tube at time $\tau = 0$ take the initial temperature $t = \theta = 10\,^{\circ}\mathrm{C}$. Starting from the next time step, the heat flux step function $(\Delta q = G_\beta \cdot p)$ on the outer surface of the tube was assumed. The assumed heat load is the heat flux equals $G_\beta = 500$ W/m^2. The tube pitch $p = 0.12$ m.

Omitting heat losses included in formula (2) and assuming that $(\tau a) = 1$ following was obtained:

$$\Delta q = 500 \cdot 0.12 = 60\ \mathrm{W/m} \ . \tag{31}$$

The selected results of the numerical calculations for the same cross-sections as in the first case are shown in Figs. 6–8. Figure 6 shows the comparison of the dimensionless histories of the fluid temperature increase at the analysed cross-sections. These histories begin from the time instant, respectively: $\eta = 6.018$ ($\tau = 60$ s), $\eta = 12.036$ ($\tau = 120$ s) and $\eta = 19.057$ ($\tau = 190$ s), that is from the moment the analysed cross-sections were reached by the fluid flowing with the velocity $w = 0.01$ m/s. A satisfactory convergence of the results of the analytical calculations with the results obtained using the method suggested in this paper was achieved.
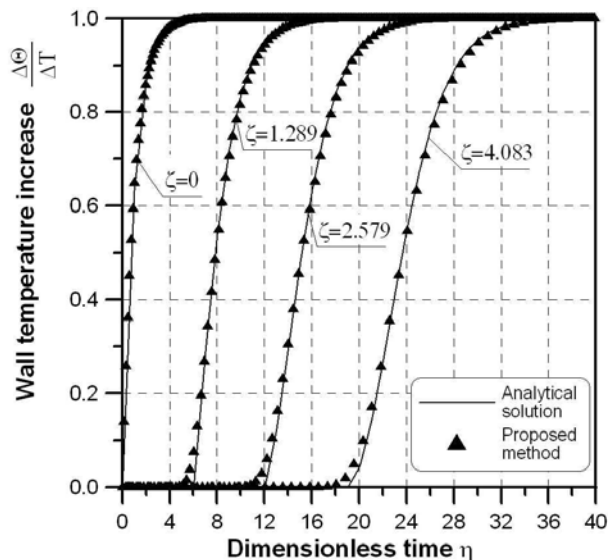
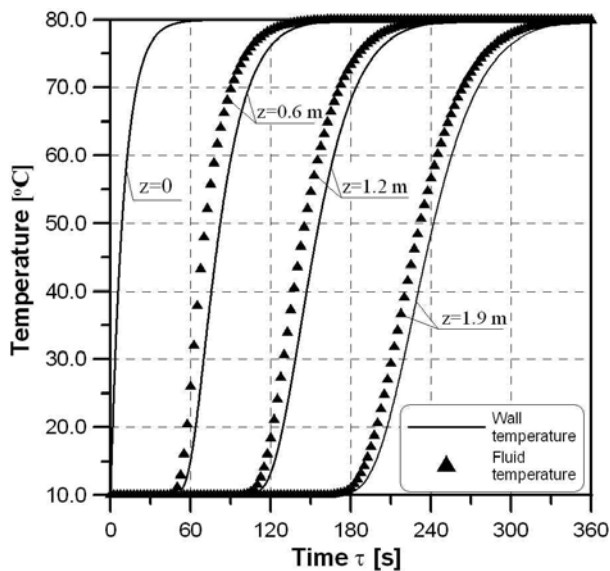Figure 4. Dimensionless histories of the tube wall temperature increase.

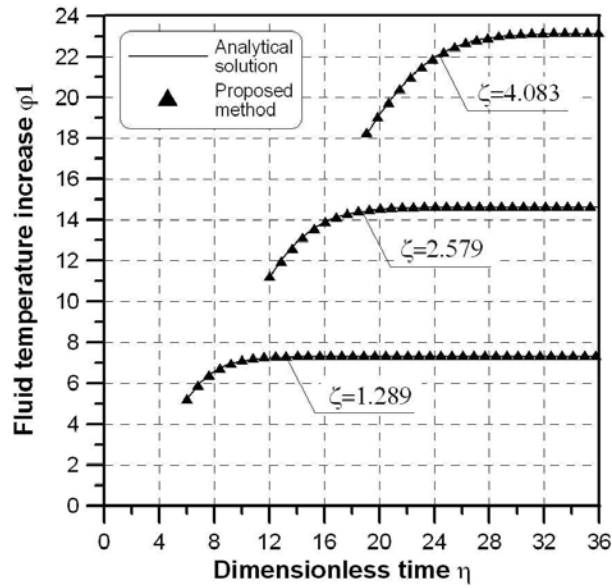Figure 5. Tube wall and fluid temperature histories for the analysed cross-sections.

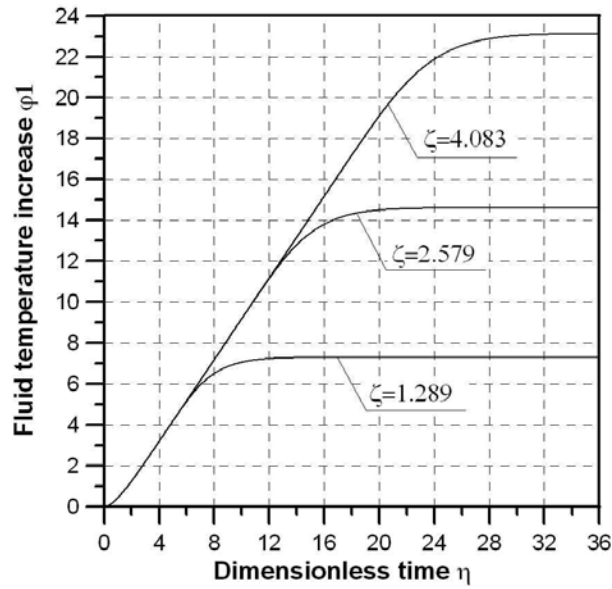Figure 6. Histories of the dimensionless fluid temperature increase.

Figure 7. Dimensionless histories of the fluid temperature increase for the analysed cross-
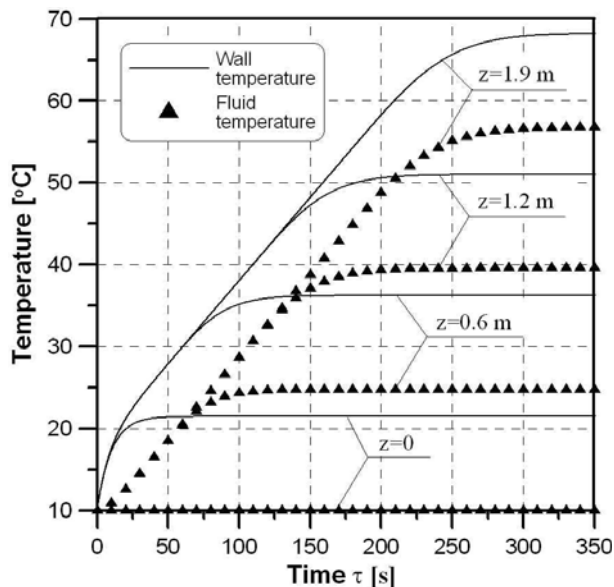sections (results obtained using the proposed method).

Figure 8. Fluid and tube wall temperature histories for the analysed cross-sections (results obtained using the proposed method).
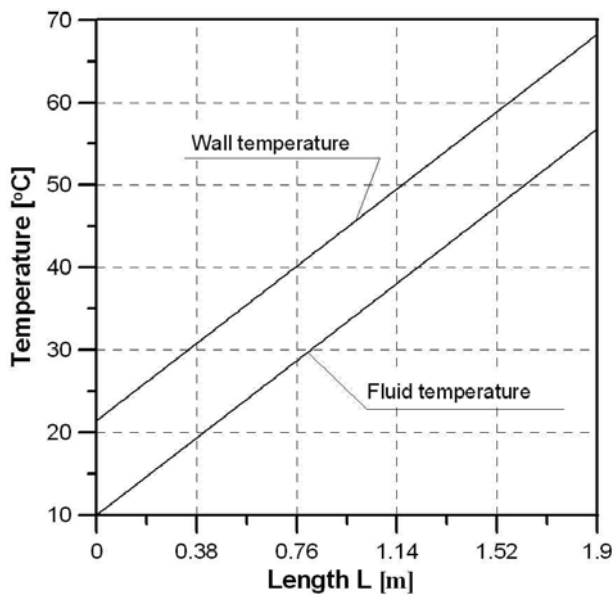

Figure 9. Distributions of the fluid and tube wall temperature after stabilization of the heat transfer conditions.

As distinct from the analytical method, the proposed model allows to determine temperature histories from the beginning of the process (from time $\tau = 0$) and the temperature increase can be dimensionless (Fig. 7) or dimensional (Fig. 8). Figure 7 shows the dimensionless fluid temperature increase for analysed cross-sections from the beginning of the process. These histories were obtained using the proposed method. The dimensionless temperature increase $\varphi 1 = 0$ corresponds to temperature $t = 10\,^{\circ}\mathrm{C}$. Figure 9 shows the fluid and the tube wall temperature distributions at the length of the tube. These distributions were achieved after stabilization of the heat transfer conditions, which occurred after ~ 350 s.

5   Summary

The proposed in this paper mathematical model of heat transfer in the liquid flat-plate solar collector tubes can be characterised by large precession and efficiency. This was proven by the comparison of the results obtained using this model with the results of the exact solutions for the transient states. Two numerical verifications were carried out. The inlet fluid temperature and the heat flux on the outer tube surface were set as a step function. In the real conditions the fluid temperature changes and the changes of the solar radiation heat flux do not occur that rapidly. To reach a stable solution to the difference equations, optimal time-spatial steps were determined ($\Delta\tau = 0.1$ s and $\Delta z = 0.005$ m). The obtained time-spatial division allows dissipation and dispersion at the grid to be avoided and at the same time allows the Courant condition to be satisfied.

The performed comparisons relate to the collector working in a parallel channel arrangement, however this method can also be used for solar collectors working in serpentine tube arrangement.

From the computational verification results, obtained by presented simplified model, follow that the fluid temperature distribution is correct. That fact allows to create complex mathematical model, for simulating the dynamics of liquid flat-plate solar collectors, equipped with additional energy conservation equations. These equations will be derived for glass cover, insulation, and for air gap. The complex model will be verified experimentally in the close future.

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